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Introduction to the Large-Scale AMRO Global Macro-Financial (DSGE) Model

Alex Liyang Tang

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Introduction to the Large-Scale AMRO Global Macro-Financial (DSGE) Model

Prepared by Alex Liyang Tang\textsuperscript{1,2}

Authorized by Hoe Ee Khor (Chief Economist)

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Abstract

This paper provides a comprehensive and systematic introduction to the Large-Scale AMRO Global Macro-Financial DSGE Model (AGMFM). It supports impulse response analysis, shock decomposition analysis, forecasting, optimal policy solving, sensitivity and identification analysis, and other in-depth, DSGE analyses for 48 economies, including the ASEAN+3. This paper describes the theoretical structure underlying the formulation of the various components of the AGMFM and the calibration or estimation of all parameters required by the model. In general, this is a large-scale DSGE model with 48 economies and 45 industries, incorporating rich microfoundations of the optimizing behavior of homogenous and heterogenous economic agents, which are subject to various imperfections and frictions. The model can be used for conducting macroeconomic, financial and policy scenarios at the global, national, and even industry levels.

JEL classification: C67, E60, F21, F37, G28

Keywords: AMRO Global Macro-Financial (DSGE) Model, ASEAN+3, Dynamic Stochastic General Equilibrium model, Inter-Country Input-Output table, industry

\textsuperscript{1} Author’s e-mail: Tang.Liyang@amro-asia.org

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### Abbreviations

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<tr>
<td>AGMFM</td>
<td>AMRO Global Macro-Financial (DSGE) Model</td>
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<td>DSGE</td>
<td>Dynamic Stochastic General Equilibrium Model</td>
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<tr>
<td>FDI</td>
<td>Foreign Direct Investment</td>
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<td>FTA</td>
<td>Free Trade Agreements</td>
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<tr>
<td>GFM</td>
<td>Global Macrofinancial (DSGE) Model</td>
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<tr>
<td>GVA</td>
<td>Gross Value Added</td>
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<td>ICIO</td>
<td>Inter-Country Input-Output Table</td>
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<td>IMF</td>
<td>International Monetary Fund</td>
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<td>ODI</td>
<td>Outbound Direct Investment</td>
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<tr>
<td>OECD</td>
<td>Organization for Economic Co-operation and Development</td>
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<td>RCEP</td>
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I. Introduction

The large-scale AMRO Global Macro-Financial (DSGE) Model (hereafter “AGMFM”) is an innovative, multi-economy, multi-sector, multi-industry DSGE model. It is designed to support rich macroeconomic, financial and policy scenarios, at the global, national and national industry levels. The AGMFM extends the Global Macrofinancial (DSGE) Model (hereafter “GFM”) described in Vitek (2015, 2018). The GFM covers as many as 40 economies, up to 9 sectors for each economy, and characterizes both individual behaviors within sectors, and mechanisms or relations between sectors as richly as possible. Compared with the GFM, the AGMFM:

- contains all 14 ASEAN+3 economies plus another 34 economies, which have important trade and financial connections with these ASEAN+3 economies, with some also representing very large economies within the global trade and financial system (Appendix I);
- further incorporates the structure of import tariff and the structure of international direct investment into the model;
- builds the intra-industry and inter-industry structure, with innovations including a detailed industry-level global supply chain structure.

The AGMFM has the additional capability to support various types of analyses. They include impulse response analysis, shock decomposition analysis, forecasting, optimal policy solving, sensitivity and identification analysis, and other in-depth, DSGE analyses into the 48 economies and 45 industries within each economy. For each of the 48 economies (Appendix I), which constitute more than 90 percent of total global GDP:

- Each economy contains a household sector, labor supply sector, construction sector, production sector, banking sector, foreign exchange sector, export sector, import sector, balance of payments sector, virtual “absorption sector” (an added sector to form a closed loop of national accounting, such as GDP accounting by expenditure), government sector that comprises monetary, fiscal, and macroprudential authorities, and domestic interbank, money, bond and equity markets. In addition, the model contains the international commodity market and the international financial market.
- All these sectors or markets within any specific economy optimize intertemporally, not only interacting with each other within the same economy, but also interacting with sectors or markets within other economies in an uncertain macroeconomic and financial environment, in order to determine equilibrium prices and quantities under rational expectations in the globally integrated commodity and financial markets.

Among the various options on model type, model hierarchy and sector combination within the model, there are three key considerations behind our modelling.

**First**, we choose to develop a DSGE model among various types of macroeconomic models, because it is theoretically more solid and can provide users with more in-depth policy insights. The advantages of a DSGE model include:
• the behaviors of homogeneous or heterogeneous representative individuals in all sectors, such as consumers, firms and financial intermediaries, are formally derived from as rich, explicit and solid microfoundations as possible;

• the underlying economic environment of the model is that of a competitive economy, but with a number of real world distortions added, from nominal rigidities to monopoly power to information problems (Blanchard 2018);

• the model is estimated as a system, rather than equation by equation as in the previous generations of macroeconomic models (Blanchard 2018).

While adaptive expectations in most other types of macroeconomic models are usually subject to the Lucas critique, and simulations based on them are considered flawed by a mistreatment of the formation of expectations, the DSGE model is less prone to the Lucas critique, since its treatment of expectations is typically forward-looking instead of adaptive (Lucas 1976; Vetlov and others 2010).

Second, we choose to develop a large-scale model with hierarchy and structure down to the national industry level, because it enables the model to conduct simulation at the more granular industry level. The characterization of industry details within each economy and the inter-country cross-industry input-output relations do not exist in almost all DSGE models developed by central banks and international organizations. However:

• These meticulous characterizations greatly enable our model to provide users with relatively rare industry-level granular simulation results, which usually make it easier to explain the dynamic performance of whole-economy-level macroeconomic variables, based on their decomposition at the industry level.

• Additionally, the intra-industry and inter-industry structure allows this model to innovatively incorporate the upstream-downstream transmission mechanism in the global supply chain, which makes our model more suitable for conducting useful simulation studies on global trade and finance related research topics.

• Moreover, the large enough scale of the model helps to accommodate and support the characterization of the significant heterogeneity between countries in reality as much as possible.

Third, we choose to develop a sector- and market-rich model, because the model would allow for the design of a broad range of macroeconomic, financial, and policy scenarios. We incorporate 11 sectors for each economy, and also their corresponding domestic and international commodity and financial markets into this model, which enables this model to tackle a broad range of macroeconomic, financial and policy issues. There are numerous parts of the model where whole-economy and industry-level shocks can be introduced. Besides, the richness of sectors and mechanisms also makes our model more capable of offering an appealing structural interpretation of the simulation results, compared with other models that include less abundant sectors and markets (Vetlov and others 2010).

This paper is organized as follows:

• Section II provides a simplified framework of the AGMFM, which is more friendly to readers who don’t want to get bogged down in too many technical details.
Specifically, this section describes the theoretical structure of not only all the above-
mentioned sectors or markets, but also various interactions between sectors or
markets, based on the characterization of the significant heterogeneity between
countries.

- Section III introduces the mathematical model as the realization of the above
theoretical framework, including the optimization problems and corresponding
constraints for all sectors of the economies, all settings for economy-specific markets
and international markets, and all market clearing conditions for economy-specific
markets and international markets in the large-scale AGMFM.

- Section IV, together with Appendix II, introduce the final computable large-scale
AGMFM, which could be considered as the approximate representation of the above
mathematical model. Compared with Appendix II, this section mainly focuses on all
industry-level equations of the AGMFM, where our innovative work is concentrated.

- Appendix I lists the industries and economies covered by the AGMFM.

- Appendix II lists all whole economy or international market equations of the AGMFM,
as a supplement to Section IV.

- Appendix III describes in detail the calibration or estimation methodology of all
parameters required by the AGMFM, such as which whole-economy-level or
industry-level parameters need to be calibrated or estimated, and what data sources
are used.

- Appendix IV lists all the whole-economy-level variables, industry-level variables and
international-market-level variables of the AGMFM, as a convenient and easy
reference for users, when they need to add various model codes to do various types
of DSGE analyses (Dynare Team 2022).

The complexity and large scale of the AGMFM also make it necessary to follow a multi-step
process to apply it properly to specific research problems. Hence, the whole process of
applying the AGMFM to a specific research topic should be divided into six steps, including
scenario setting, model equation modification, exogenous shock definition, recalibration or
reestimation of model parameters, simulation of the model, and analysis based on simulation
results.
Figure 1. Detailed Structure of the Original GFM

Sources: Vitek (2015) and Vitek (2018); and author’s visualization.
Figure 2. Detailed Structure of the AGMFM

Sources: Author’s visualization.
II. Theoretical Framework

This section provides a general impression of theoretical framework, through the purely literal and overall introduction of each sector or market of the model. As a comparison, we correspondingly provide the brief framework of the original GFM in Figure 1, and the brief framework of the new AGMFM in Figure 2, both of which take a representative economy and all its macroeconomic and financial relationship with other economies as an example:

- the first framework is for a representative economy without the detailed structure of 45 industries, as in the original GFM introduced in Vitek (2015, 2018);
- the second framework is for a representative economy—which characterizes the detailed structure of 45 industries within an economy plus the inter-country cross-industry input-output relations with other economies—in the new AGMFM, which is developed based on the GFM.

Comparison of these two frameworks is mainly used to help readers know about our innovative work around this large-scale AGMFM, in fact all those sectors marked with yellow background in Figure 2 are the sectors where our innovation is concentrated. Besides, the author also expects that these frameworks could assist readers in forming an overall picture about the application potential of the AGMFM. If readers need more detailed and diversified model comparison perspectives, it is strongly recommended to refer to Ljungqvist and Sargent (2004) and Stachurski and Sargent (2020). Besides the brief framework of the AGMFM in Figure 2, the theoretical framework for each sector or market and various interactions between sectors or markets, which are the characterization of the significant heterogeneity between countries, are explained in the following subsections.

A. Household Sector

The structurally more complex household sector consists of three types of continuums of households. All types of households purchase industry specific final consumption goods or services from the additional virtual “absorption sector” (which forms a closed loop of national accounting, such as GDP accounting by expenditure), purchase identical final housing services from the construction sector, sell indivisible differentiated intermediate labor services to the labor supply sector in order to get labor income, and pay labor income tax to the government sector. Each household is originally endowed with one share of each domestic real estate intermediate developer, domestic industry specific intermediate output good firm, domestic intermediate bank, domestic industry specific intermediate export good firm, and domestic economy specific and industry specific intermediate import good firm. However, there are also the following differences between different types of households:

- The bank intermediated households additionally accumulate household wealth in the form of bank deposit holdings, and real estate holdings (here household wealth balances in the form of real estate holdings are allocated across the values of the domestically traded real estate portfolio, and the value of this real estate portfolio is further allocated across the values of specific shares of domestic real estate intermediate developers), in order to get interest return, profit income or dividend payments.

- The capital market intermediated households additionally accumulate household wealth in the form of short term bond holdings (allocated across the domestic
currency denominated values of both domestic and foreign economy specific short term bonds), long term bond holdings (allocated across the domestic currency denominated values of both domestic and foreign economy specific and vintage specific long term bonds), and stock holdings (allocated across the domestic currency denominated values of shares of both domestic and foreign economy specific and industry specific intermediate output good firms), in order to get interest return, profit income or dividend payments. In addition, they need to bear the nondiscretionary lump sum transfer payments for the relatively poor credit constrained households implemented by the government sector, since they are relatively rich.

- **The credit constrained households** do not accumulate household wealth and only get dividend payments from holding originally endowed shares of domestic real estate intermediate developers, domestic industry specific intermediate output good firms, domestic intermediate banks, domestic industry specific intermediate export good firms, and domestic economy specific and industry specific intermediate import good firms. In addition, they receive both nondiscretionary and discretionary lump sum transfer payments implemented by the government sector, since they are relatively poor.

### B. Labor Supply Sector

The labor supply sector consists of two layers of agents. Here, the structure of multi-layer agents is a widely-accepted technical setting, rather than a setting to correspond one-to-one with reality, in order to take both the characterization of the optimization problem of representative agents and the characterization of the heterogeneity of agents into account.

- In the lower layer, continuums of **monopolistically competitive labor unions of workers** combine indivisible differentiated intermediate labor services from all types of households to supply differentiated intermediate labor services to final labor service firms.

- In the upper layer, a large number of **perfectly competitive final labor service firms** combine differentiated intermediate labor services supplied by labor unions of workers to produce final labor services needed by the production sector.

In addition, the settings of the labor supply sector incorporate the **employment and unemployment mechanism** and consider the **nominal wage rigidity** in the labor market.

### C. Construction Sector

The construction sector consists of two layers of agents based on considerations similar to those of the labor supply sector.

- In the lower layer, continuums of **monopolistically competitive real estate intermediate developers** produce and sell differentiated intermediate housing services to domestic real estate final developers, obtain identical mortgage loans from domestic final banks, accumulate the housing stock through purchasing identical final residential investment goods or services from the additionally added virtual “absorption sector”, and sell their specific shares to domestic bank intermediated households and pay dividends.
• In the upper layer, a large number of **perfectly competitive real estate final developers** combine differentiated intermediate housing services supplied by domestic real estate intermediate developers to produce final housing services needed by all types of domestic households.

### D. Production Sector

The **structurally more complex production sector**, which has the detailed structure of 45 industries and their inter-country cross-industry input-output relations characterized, is the most complex sector based on the innovative modelling work.

• First of all, the classification of 45 industries is based on the classification standard of ICIO tables released by OECD (Appendix I). Among all 45 industries, some are mainly considered as the industries that produce internationally homogeneous energy or nonenergy commodities, while others are mainly considered as the industries that produce internationally heterogeneous goods or services.

• Second, in order to take both the characterization of the optimization problem of representative agents and the characterization of the heterogeneity of agents into consideration, the structurally more complex production sector is composed of two layers of agents.

  o In the lower layer, continuuums of **monopolistically competitive industry specific intermediate output good firms** produce and sell industry specific differentiated intermediate output goods or services to domestic industry specific final output good firms, obtain domestic currency denominated corporate loans from domestic global final banks, accumulate the private physical capital stock through purchasing identical final business investment goods or services from the additionally added virtual "absorption sector", purchase final labor services from final labor service firms in the domestic labor supply sector, purchase industry specific final output goods or services not only from industry specific final output good firms in various industries of the domestic production sector, but also from industry specific final output good firms in various industries of the foreign production sector with the help of the domestic import sector, to serve as their intermediate inputs for production, and sell their specific shares to domestic and foreign capital market intermediated households and pay dividends.

  o In the upper layer, a large number of **perfectly competitive industry specific final output good firms** combine industry specific differentiated intermediate output goods or services supplied by industry specific intermediate output good firms to produce industry specific final output goods or services, and they’re either needed by industry specific intermediate output good firms in various domestic and foreign industries to serve as the intermediate inputs for production, or they directly become industry specific final output goods or services, which are needed by domestic and foreign industry specific final absorption good firms in the additionally added virtual “absorption sector” to produce industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services, while becoming intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector requires the help of the domestic export sector. Here
the proportions of various uses of the above industry specific final output goods or services are determined by the corresponding row in the OECD ICIO table.

• Third, the production function of industry specific intermediate output good firms:
  o utilizes final labor services with the economy specific and industry specific labor productivity that not only depends on the economy specific and industry specific per capita public physical capital stock to reflect the government's differentiated support for sunrise and sunset industries in different countries, but also relies on the labor productivity shocks of all domestic and foreign industries that have input-output relation with this domestic industry to reflect technology diffusion;
  o additionally utilizes a part of industry specific final output goods or services from all domestic and foreign industries that have input-output relation with this domestic industry, to serve as intermediate inputs, here the proportions of these uses of domestically produced industry specific final output goods or services and foreign produced industry specific final import goods or services are determined by the corresponding column in the OECD ICIO table.

• Fourth, the settings of the structurally more complex production sector not only consider the industry specific nominal output price rigidity in the industry specific output market, but also innovatively incorporate the transmission mechanism between upstream and downstream quantities and prices in the global supply chain.

E. Banking Sector

The banking sector has different layer structures for creating mortgage loans that are issued to domestic real estate intermediate developers and creating corporate loans that are issued to both domestic and foreign industry specific intermediate output good firms, due to their different domestic or international attributes.

• For risky mortgage loans with infrequently adjusted predetermined mortgage loan rates, the banking sector contains two layers of agents.
  o In the lower layer, continuums of monopolistically competitive domestic intermediate banks create and sell differentiated intermediate mortgage loans to domestic final banks, obtain funding from domestic bank intermediated households in the household sector via deposits and from the domestic interbank market via interbank loans, accumulate bank capital out of retained earnings given credit losses to satisfy a regulatory bank capital ratio requirement set by the government sector (the cost of satisfying the regulatory bank capital ratio requirement decreases with the increase of the ratio of bank capital to assets), and pay dividends to domestic households for holding their specific shares.
  o In the upper layer, a large number of perfectly competitive domestic final banks combine differentiated intermediate mortgage loans supplied by domestic intermediate banks to produce final mortgage loans needed by domestic real estate intermediate developers. In addition, the regulatory mortgage loan to value ratio limit applicable to borrowing by domestic real estate intermediate developers from domestic final banks must satisfy the mortgage loan to value limit rule set by
the government sector, and the mortgage loan default rate satisfies a default rate relationship that is related to both the contemporaneous domestic output gap and the intertemporal change in the price of domestic housing.

- For risky corporate loans with infrequently adjusted predetermined corporate loan rates, the banking sector contains three layers of agents.
  - In the lower layer, continuums of monopolistically competitive domestic intermediate banks also create and sell differentiated intermediate corporate loans to domestic final banks, obtain funding from domestic bank intermediated households in the household sector via deposits and from the domestic interbank market via interbank loans, accumulate bank capital out of retained earnings given credit losses to satisfy a regulatory bank capital ratio requirement set by the government sector (the cost of satisfying the regulatory bank capital ratio requirement decreases with the increase of the ratio of bank capital to assets), and pay dividends to domestic households for holding their specific shares.
  - In the middle layer, a large number of perfectly competitive domestic final banks combine differentiated intermediate corporate loans supplied by domestic intermediate banks to produce final corporate loans for both domestic and foreign global final banks.
  - In the upper layer, a large number of perfectly competitive global final banks combine economy specific local currency denominated final corporate loans from the banking sectors of all economies to produce domestic currency denominated final corporate loans needed by domestic industry specific intermediate output good firms. In addition, the regulatory corporate loan to value ratio limit applicable to borrowing by domestic industry specific intermediate output good firms from domestic global final banks must satisfy the corporate loan to value limit rule set by the government sector, and the corporate loan default rate satisfies a default rate relationship that is related to both the contemporaneous domestic output gap and the intertemporal change in the price of domestic corporate equity.

In addition, the settings of the banking sector consider both the nominal mortgage and corporate loan rate rigidity in the lending market.

**F. Foreign Exchange Sector**

The foreign exchange sector can help form the nominal bilateral exchange rate, the real bilateral exchange rate that is defined as the relative price of foreign output of the other economy in terms of domestic output, the nominal effective exchange rate that is defined as the trade weighted average price of foreign currency in terms of domestic currency, and the real effective exchange rate that is defined as the trade weighted average relative price of foreign output in terms of domestic output. These exchange rates are necessary for any domestic sector to conduct a transaction in either international commodity markets or international financial markets.

**G. Export Sector**

The structurally more complex export sector consists of two layers of agents based on considerations similar to those of the structurally more complex production sector.
In the lower layer, continuums of monopolistically competitive industry specific intermediate export good firms produce and sell industry specific differentiated intermediate export goods or services to domestic industry specific final export good firms, purchase industry specific final output goods or services from domestic industry specific final output good firms and utilize them for production (if they produce industry specific intermediate export goods or services that are internationally homogeneous energy or nonenergy commodities, they can purchase the industry specific final output goods or services from either domestic industry specific final output good firms or the international commodity markets, due to the law of one price for internationally homogeneous energy or nonenergy commodities), and pay dividends to domestic households for holding their specific shares.

In the upper layer, a large number of perfectly competitive industry specific final export good firms combine industry specific differentiated intermediate export goods or services supplied by domestic industry specific intermediate export good firms to produce industry specific final export goods or services, and they're eventually sent to foreign production sector (to be taken as intermediate inputs) or foreign absorption sector (to be taken as final output goods or services), with the help of the import sector of that foreign economy. Here the proportions of various import demand of other economies for the above industry specific final export goods or services are determined by the corresponding row in the OECD ICIO table.

In addition, the settings of the structurally more complex export sector not only consider the industry specific nominal export price rigidity in the industry specific export market, but also innovatively incorporate the transmission mechanism between upstream and downstream quantities and prices in the global supply chain.

H. Import Sector

The structurally more complex import sector consists of three layers of agents based on considerations similar to those of the structurally more complex production sector.

In the lower layer, continuums of monopolistically competitive economy specific and industry specific intermediate import good firms produce and sell economy specific and industry specific differentiated intermediate import goods or services to domestic economy specific and industry specific final import good firms, purchase economy specific and industry specific final export goods or services from industry specific final export good firms of the corresponding economy, and pay dividends to domestic households for holding their specific shares.

In the middle layer, a large number of perfectly competitive economy specific and industry specific final import good firms combine economy specific and industry specific differentiated intermediate import goods or services supplied by domestic economy specific and industry specific intermediate import good firms to produce economy specific and industry specific final import goods or services that are needed by domestic industry specific final import good firms.

In the upper layer, a large number of perfectly competitive industry specific final import good firms combine economy specific and industry specific final import goods or services supplied by domestic economy specific and industry specific final import good firms to produce industry specific final import goods or services, they're
either needed by domestic industry specific intermediate output good firms in various domestic industries to serve as the intermediate inputs for production, or needed by domestic industry specific final absorption good firms in the additionally added virtual “absorption sector”, in order to produce industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services. Here the proportions of various uses of the above industry specific final import goods or services are determined by the corresponding columns in the OECD ICIO table.

In addition, the settings of the structurally more complex import sector not only consider the economy specific and industry specific nominal import price rigidity in the economy specific and industry specific import market, but also innovatively incorporate the transmission mechanism between upstream and downstream quantities and prices in the global supply chain. Since we further incorporate the structure of import tariff into the import sector relative to the GFM introduced in Vitek (2015, 2018), the import price should distinguish between the pre-tariff import price and the post-tariff import price, which is very helpful for the analysis of the effect of tariff agreements or trade agreements.

I. Balance of Payments Sector

The balance of payments sector not only defines the US dollar denominated net foreign asset position, the US dollar denominated current account balance and the US dollar denominated trade balance, but also defines the relationship between them and the constraints they should meet (such as the clearing of the international commodity markets or multilateral consistency in nominal trade flows). In addition, we further incorporate the structure of international direct investment, including the structure of foreign direct investment (FDI) and outbound direct investment (ODI) into the balance of payments sector, compared with the original GFM introduced in Vitek (2015, 2018).

J. Virtual “Absorption Sector”

In order to form a closed loop of national accounting, two layers of virtual final absorption good firms are additionally added to each economy as a separate sector.

- In the lower layer, a large number of perfectly competitive industry specific final absorption good firms combine the industry specific final output goods or services (including domestically produced industry specific private consumption, residential investment, business investment, public consumption and public investment goods or services) from domestic industry specific final output good firms, with the industry specific final import goods or services from domestic industry specific final import good firms, in order to produce industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services.

- In the upper layer, a large number of perfectly competitive final absorption good firms combine the industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services from domestic industry specific final absorption good firms, in order to produce final private consumption, residential investment, business investment, public consumption and public investment goods or services, which are needed
correspondingly by the domestic household sector, the domestic construction sector, the domestic production sector, and the domestic government sector.

**K. Government Sector**

The government sector is composed of the monetary authority, the fiscal authority, and the macroprudential authority.

- **The monetary authority** implements monetary policies through control of the nominal policy interest rate, and the specific adjustment formula of nominal policy interest rate depends on the specific exchange rate and inflation targeting arrangement of a specific economy. Briefly speaking:

  o Under a free floating exchange rate and flexible inflation targeting arrangement, the nominal policy interest rate depends on a weighted average of the past nominal policy interest rate and the desired nominal policy interest rate, while the desired nominal policy interest rate responds to the expected future domestic consumption price inflation and the contemporaneous domestic output gap.

  o Under a managed exchange rate arrangement, the nominal policy interest rate depends on a weighted average of the past nominal policy interest rate and the desired nominal policy interest rate, while the desired nominal policy interest rate responds to the expected future domestic consumption price inflation, the contemporaneous domestic output gap, and the intertemporal change in the nominal effective exchange rate of the domestic currency.

  o Under a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union), the nominal policy interest rate tracks the contemporaneous nominal policy interest rate of the economy that issues the anchor currency for the domestic currency, and responds to the intertemporal change in the corresponding nominal bilateral exchange rate between currencies issued by these two economies.

- **The fiscal authority** implements fiscal policies through control of public consumption and industry specific public investment, adjustments of tax rates applicable to corporate earnings, household labor income and imports, the operation of a budget neutral nondiscretionary lump sum transfer program that redistributes national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households, the operation of a discretionary lump sum transfer program that provides income support only to credit constrained households, and the accumulation of industry differentiated public investment. Here the budgetary resources of the fiscal authority can be transferred intertemporally through transactions in the domestic money and bond markets.

  o The control of public consumption and industry specific public investment (final public consumption goods or services and industry specific final public investment goods or services come from the additionally added virtual “absorption sector”) satisfy countercyclical fiscal expenditure rules, in order to achieve public wealth stabilization objectives.
The adjustments of tax rates applicable to corporate earnings, household labor income and imports satisfy acyclical fiscal revenue rules. Since the structure of import tariff is further incorporated into this model relative to the GFM introduced in Vitek (2015, 2018), tariff revenue is also added to the tax revenue of the government sector accordingly.

The ratio of nondiscretionary lump sum transfer payments to nominal output satisfies a nondiscretionary transfer payment rule that stabilizes national financial wealth, in order to achieve national financial wealth stabilization objectives.

The ratio of discretionary lump sum transfer payments to nominal output satisfies a discretionary transfer payment rule that stabilizes public financial wealth, in order to achieve public financial wealth stabilization objectives.

The industry differentiated public investment is accumulated to form industry differentiated public physical capital stock, which further influences industry differentiated labor productivity to reflect the government's differentiated industry policies or differentiated support for sunrise and sunset industries.

The fiscal authority accumulates public financial wealth subject to the dynamic budget constraint function of the government.

The macroprudential authority implements macroprudential policies through control of a regulatory bank capital ratio requirement, and control of loan to value ratio limits.

The regulatory bank capital ratio requirement applicable to bank capital accumulation of domestic intermediate banks satisfies a countercyclical capital buffer rule.

The regulatory mortgage loan to value ratio limit applicable to borrowing by domestic real estate intermediate developers from domestic final banks satisfies the mortgage loan to value limit rule.

The regulatory corporate loan to value ratio limit applicable to borrowing by domestic intermediate output good firms from domestic global final banks satisfies the corporate loan to value limit rule.

L. Domestic Interbank, Money, Bond and Equity Markets

The domestic interbank, money, bond and equity markets are used to trade domestic currency denominated interbank loans, domestic currency denominated both domestic and foreign economy specific short term bonds, domestic currency denominated both domestic and foreign economy specific and vintage specific long term bonds, domestic currency denominated specific shares of domestic real estate developers, domestic currency denominated specific shares of both domestic and foreign economy specific and industry specific output good firms.

Domestically traded and domestic currency denominated interbank loans are traded between domestic intermediate banks.
Domestically traded and domestic currency denominated shares of domestic real estate intermediate developers are sold to domestic bank intermediated households.

Internationally traded and local currency denominated both domestic and foreign economy specific short term bonds, internationally traded and local currency denominated both domestic and foreign economy specific and vintage specific long term bonds, internationally traded and local currency denominated shares of both domestic and foreign economy specific and industry specific intermediate output good firms are sold to domestic capital market intermediated households.

Domestic currency denominated domestic short term bonds and domestic currency denominated domestic vintage specific long term bonds issued by the domestic government are also sold to the domestic fiscal authority to transfer its budgetary resources intertemporally.

M. International Commodity and Financial Markets

In addition to all economies, there exist the international commodity markets and the international financial markets in the world economic and financial system.

In the international commodity markets, industry specific final export good firms of one economy can sell economy specific and industry specific final export goods or services denominated in this economy’s currency to economy specific and industry specific intermediate import good firms of another economy. After this international transaction is completed, these internationally homogeneous or heterogeneous goods or services will change from the industry specific final export goods or services of the export sector of the original economy to the economy specific and industry specific differentiated intermediate import goods or services of the import sector of the new economy.

In the international financial markets:

- The capital market intermediated households of one economy can buy internationally traded and local currency denominated economy specific short term bonds, internationally traded and local currency denominated economy specific and vintage specific long term bonds, internationally traded and local currency denominated specific shares of economy specific and industry specific output good firms that come from other economies.

- Industry specific intermediate output good firms of one economy can sell their specific shares to capital market intermediated households of other economies.

- Global final banks of one economy can buy economy specific local currency denominated final corporate loans from the banking sectors of other economies, in order to combine them to produce domestic currency denominated final corporate loans that can be issued to domestic industry specific intermediate output good firms.

The international financial markets need to cooperate with the domestic interbank, money, bond and equity markets of any specific economy to help those internationally traded and local currency denominated economy specific short term bonds, internationally traded and
local currency denominated economy specific and vintage specific long term bonds, internationally traded and local currency denominated specific shares of economy specific and industry specific output good firms flow into various sectors of this economy.

III. Mathematical Model

This section introduces the mathematical model, which could be considered as the realization of the theoretical framework introduced in Section II. Specifically, the following subsections include the introduction of optimization problems and corresponding constraints for each sector of a representative economy, the settings for various domestic and international markets, and the settings for all market clearing conditions in the large-scale AGMFM. It is worth pointing out that the innovative work around the AGMFM has been inspired by a variety of modelling ideas and flexible modelling techniques, most of which come from Ljungqvist and Sargent (2004) and Stachurski and Sargent (2020).

First of all, it needs to be pointed out uniformly that variable subscript “i” or “j” or “i*” represents the i-th or j-th or i*-th economy, \( i \) or \( j \) \( \in \{1,2, \ldots, N\} \), \( N \) is the total number of economies contained in the large-scale AGMFM, and for most cases \( i^* \) is adopted to represent that the \( i^*-th \) economy issues the anchor currency for the currency of the i-th economy. Variable subscript “k” or “m” represents the k-th or m-th industry, and \( k \) or \( m \) \( \in \{1,2, \ldots, 45\} \).

In addition, all the parameters not specifically described below are either for specific setting in modelling or for specific adjustment in model derivation. Readers are also suggested to refer to Vitek (2015, 2018) for relevant details in the original mathematical model, based on which the AGMFM is further developed.

A. Household Sector

A continuum of domestic households indexed by \( h \in [0,1] \) is divided into the following three types: the bank intermediated households, the capital market intermediated households, and the credit constrained households. A household of any type consists of a continuum of members \( N_{h,fi} \), and there is full risk sharing among members within this household, who supply indivisible differentiated intermediate labor services \( L_{h,fi} \) indexed by \( f \in [0,1] \) to the labor supply sector, also incurring disutility from work indexed by \( g \in [0,1] \) if they are employed and zero otherwise. Each household is originally endowed with one share of each domestic real estate intermediate developer, domestic industry specific intermediate output good firm, and domestic economy specific intermediate import good firm.

Type I. The bank intermediated household chooses state contingent sequences in the current period \( s = t \) and all future periods \( s = t+1, t+2, \ldots \) for consumption \( C_{h,i}(s) \), housing \( H_{h,i}(s) \), labor force participation \( \{N_{h,fi}(s)\}_{f=0}^{1} \) or labor supply \( \{L_{h,fi}(s)\}_{f=0}^{1} \), and nominal property balances \( A_{h,i}^{B,H}(s+1) \) that consist of both bank deposit holdings \( B_{h,i}^{D,H}(s+1) \) and real estate holdings \( S_{h,i}^{H,H}(s+1) \), in order to maximize its following intertemporal utility function:
\[
U_{h,i}(t) = E_t \sum_{s=t}^{\infty} \beta^{s-t} u \left( C_{h,i}(s), H_{h,i}(s), \{ L_{h,f,i}(s) \} \right)\frac{A_{h,i}^{B,H}(s+1)}{P_t^i(s)} = \\
E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{1}{1-\sigma} \left( C_{h,i}(s) - \alpha C_{C_i}^{R(s-1)} \right)^{1-\frac{1}{\sigma}} + \frac{\nu_i^{H}(s)}{1-\xi} \left( H_{h,i}(s) \right)^{1-\frac{1}{\xi}} \right\}
\]

subject to the following dynamic budget constraint:

\[
A_{h,i}^{B,H}(s+1) = \left( 1 + i_{h,i}^{A,H} \right) A_{h,i}^{B,H}(s) + \Pi^{B}(s) + \left( 1 - \tau_{i}^{H}(s) \right) \int_{f=0}^{1} W_{f,i}(s)L_{h,f,i}(s) df - P_t^i(s)C_{h,i}(s) - \nu_i^{H}(s)H_{h,i}(s)
\]

and also subject to terminal nonnegativity constraints that \( A_{h,i}^{B,H}(s+1) \geq 0, \ B_{h,i}^{D,H}(s+1) \geq 0 \) and \( S_{h,i,e}^{H,H}(s+1) \geq 0 \) for \( s \to \infty \).

Here,

\( P_t^i(s) \) is the comprehensive consumption price level, which can be further expressed as the industry-level consumption weighted average of the consumption price level in the i-th economy for the final goods or services from the domestic and foreign k-th industry

\[
\sum_{k=1}^{45} \text{Contribution}_{C_k} P^{C}_{k}(s);
\]

\( v_t^{C}(s) \) is the consumption demand shock;

\( C_{i}^{B}(s) = \int_{h=0}^{\phi_i^{B}} C_{h,i}(s) dh \) is the aggregate consumption of all the bank intermediated households;

\( \phi_i^{B} \) is the percentage of bank intermediated households among all households;

\( L^{B}_{f,i}(s) = \int_{h=0}^{\phi_i^{B}} L_{h,f,i}(s) dh \) is the differentiated intermediate labor services supplied by all the bank intermediated households;

\( i_{h,i}^{A,H}(s) \) is the return rate of nominal property balances \( A_{h,i}^{B,H}(s) \);

\( \Pi^{B}(s) \) is the dividend payments for the bank intermediated household from holding various kinds of shares except shares of domestic real estate intermediate developers, since \( i_{h,i}^{A,H}(s) \) has already included the dividend payment per specific share of domestic real estate intermediate developer;

\( \tau_{i}^{H}(s) \) is the labor income tax rate;

\( W_{f,i}(s) \) is the nominal wage;

\( \nu_i^{H}(s) \) is the rental price of housing.

Endogenous preference shifter for housing \( \nu_i^{H,H}(s) \) depends on both the aggregate consumption of all the bank intermediated households \( C_{i}^{B}(s) \) and the aggregate consumption of all three types of households \( C_{i}(s) \), according to the following intratemporal subutility function:

\[
\nu_i^{H,H}(s) = \nu_i^{H,H} \left( \frac{C_{i}^{B}(s)}{\phi_i^{B}} - \alpha C_{C_i}^{R(s-1)} \right)^{1-\frac{1}{\sigma}} \left( C_{i}(s) \right)^{1-\frac{1}{\xi}}
\]
Endogenous preference shifter for labor supply $v_i^{LH}(s)$ depends on the trend labor productivity (which exhibits partial adjustment dynamics of labor productivity) $\tilde{A}_i(s)$, the aggregate consumption of all the bank intermediated households $C_i^B(s)$, the final labor services of all three types of households $L_i(s)$, and the labor supply shock $\nu_i^N(s)$, according to the following intratemporal subutility function:

$$v_i^{LH}(s) = \frac{1}{\eta} \left[ \left( \frac{C_i^B(s)}{\phi_B} - \alpha \frac{C_i^B(s-1)}{\phi_B} \right)^2 \right]^\frac{1}{2} \left[ \frac{L_i(s) - \alpha^4 L_i(s-1)}{\nu_i^N(s)} \right]^\frac{1}{\eta}$$

Endogenous preference shifter for nominal property balances $v_i^{B,H}(s)$ depends on both the aggregate consumption of all the bank intermediated households $C_i^B(s)$ and the aggregate consumption of all three types of households $C_i(s)$, according to the following intratemporal subutility function:

$$v_i^{B,H}(s) = \left( \frac{C_i^B(s)}{\phi_B} - \alpha \frac{C_i^B(s-1)}{\phi_B} \right)^\frac{1}{2} \left( C_i(s) \right)^\frac{1}{\eta}$$

Nominal property balances $\Delta_i^{B,H}(s+1)$ of the bank intermediated household are further distributed across both bank deposit holdings $B_{h,i}^{B,(s+1)}$ and real estate holdings $\{S_{h,i}^{H}(s+1)\}_{e=0}^1$. The bank intermediated household’s preferences over the real value of bank deposits $B_{h,i}^{B,(s+1)}$ and the real value of real estate portfolio $S_{h,i}^{H}(s+1)$ are represented by the following constant elasticity of substitution intratemporal subutility function:

$$\Delta_i^{B,H}(s+1) = \left( 1 - \phi^H \right)^\frac{1}{\psi^H} \left( \frac{B_{h,i}^{B,(s+1)}}{P_i^H(s)} \right)^\frac{1}{\psi} \left( \frac{1}{v_i^H(s)} \right)^\frac{1}{\psi} \left( \frac{S_{h,i}^{H}(s+1)}{P_i^H(s)} \right)^\frac{1}{\psi}$$

Here $v_i^H(s)$ is the housing risk premium shock.

Since the value of real estate portfolio is further allocated across the values of specific shares of domestic real estate intermediate developers, the bank intermediated household’s preferences over the real values of specific shares of domestic real estate intermediate developers $\left\{ \frac{V_i^H(s) S_{h,i}^{H}(s+1)}{P_i^H(s)} \right\}_{e=0}^1$ are represented by the following constant elasticity of substitution intratemporal subutility function:

$$\Delta_i^{H}(s+1) = \left( 1 - \phi^H \right)^\frac{1}{\psi^H} \left( \frac{V_i^H(s) S_{h,i}^{H}(s+1)}{P_i^H(s)} \right)^\frac{1}{\psi} \left( \frac{1}{V_i^H(s)} \right)^\frac{1}{\psi}$$

Here $V_i^H(s)$ is the price per specific share of domestic real estate intermediate developer.

The return rate of nominal property balances $i_{h,i}^{AB,H}(s)$ for the bank intermediated household can also be split accordingly as follows:
\[
\left(1 + i_{h,i}^{B,H}(s)\right)A_{h,i}^{B,H}(s) = \left(1 + i_{h,i}^P(s - 1)\right)B_{h,i}^{D,H}(s) + \left(1 + i_{h,i}^{S,H}(s)\right)S_{h,i}^{H,H}(s)
\]

Here,
\(i_{h,i}^P(s - 1)\) is the interest rate of bank deposits;
\(i_{h,i}^{S,H}(s)\) is the return rate of real estate portfolio.

The total return yielded by real estate portfolio for the bank intermediated household is split into returns yielded by various specific shares of domestic real estate intermediate developers:
\[
\left(1 + i_{h,i}^{S,H}(s)\right)S_{h,i}^{H,H}(s) = \int_{e=0}^{1}(\Pi_{k,e}^H(s) + \nu_{k,e}(s))S_{h,i,e}^{H,H}(s)de
\]

Here \(\Pi_{k,e}^H(s)\) is the dividend payment per specific share of domestic real estate intermediate developer.

Type II. The capital market intermediated household chooses state contingent sequences in the current period \(s = t\) and all future periods \(s = t + 1, t + 2, \ldots\) for consumption \(C_{h,i}(s)\), housing \(H_{h,i}(s)\), labor force participation \(\{N_{h,f,i}(s)\}_{s=0}^{s=s} - 1\) or labor supply \(\{L_{h,f,i}(s)\}_{s=0}^{s=s} - 1\), and nominal portfolio balances \(A_{h,i}^{A,H}(s + 1)\) that consist of domestic and foreign economy specific short term bond holdings \(\{B_{h,i}^{S,H}(s + 1)\}_{s=0}^{s=s} - 1\), domestic and foreign economy specific and vintage specific long term bond holdings \(\{B_{h,i}^{L,H}(s + 1)\}_{s=0}^{s=s} - 1\), and domestic and foreign economy specific and industry specific and firm specific stock holdings \(\{\{S_{h,i,kl}^{F,H}(s + 1)\}_{s=0}^{s=s} - 1\}\), in order to maximize its following intertemporal utility function:

\[
U_{h,i}(t) = E_t\sum_{s=t}^{\infty}\beta^{s-t}u\left(C_{h,i}(s), H_{h,i}(s), \{L_{h,f,i}(s)\}_{s=0}^{s=s} - 1\right)\frac{A_{h,i}^{A,H}(s + 1)}{P_{f}^i(s)} =
E_t\sum_{s=t}^{\infty}\beta^{s-t} \left\{v_{f}^{C}(s)\left[\frac{1}{\phi A} \left(C_{h,i}(s) - \alpha^{C} A_{h,i}^{A,H}(s + 1)\right)^{1 - \frac{1}{\phi A}} + \frac{\nu_{i}^{H}(s)}{1 - \frac{1}{\phi A}} H_{h,i}(s)^{1 - \frac{1}{\phi A}}\right]\right. -
\left. v_{f}^{L,H}(s)\int_{g=0}^{1} \frac{L_{h,f,i}(s)}{A_{h,i}^{A,H}(s + 1)} \frac{1}{\phi A} \left(g - \alpha^{L} A_{h,i}^{A,H}(s + 1)\right)^{\frac{1}{\phi A}} dg + \nu_{i}^{A,H}(s) \left(A_{h,i}^{A,H}(s + 1)\right)^{1 - \frac{1}{\phi A}}\right\}
\]

subject to the following dynamic budget constraint:

\[
A_{h,i}^{A,H}(s + 1) = \left(1 + i_{h,i}^{A,H}(s)\right)A_{h,i}^{A,H}(s) + \Pi_{i}^{A}(s) + \left(1 - \tau_{i}^{L}(s)\right)\int_{f=0}^{1} W_{f,i}(s)L_{f,i}(s) df +
T_{h,i}^{A,N}(s) - P_{f}^{C}(s)C_{h,i}(s) - i_{h,i}^{L,H}(s)H_{h,i}(s)
\]

and also subject to terminal nonnegativity constraints that \(A_{h,i}^{A,H}(s + 1) \geq 0\), \(B_{h,i}^{S,H}(s + 1) \geq 0\), \(B_{h,i}^{L,H}(s + 1) \geq 0\) and \(S_{h,i,kl}^{F,H}(s + 1) \geq 0\) for \(s \to \infty\).

Here,
\(P_{f}^{C}(s), \nu_{f}^{C}(s), \tau_{i}^{L}(s), W_{f,i}(s)\) and \(i_{h,i}^{L,H}(s)\) for the capital market intermediated household are the same as those for the bank intermediated household;
\(C_{f}^{A}(s) = \int_{f=0}^{1} \phi_{h}^{A} C_{h,i}(s) df\) is the aggregate consumption of all the capital market
intermediated households;
\( \phi_A \) is the percentage of capital market intermediated households among all households;
\( L_{h,i}^A(s) = \int_{h=\phi_h}^{\phi_A} L_{h,i}(s) dh \) is the differentiated intermediate labor services supplied by all the capital market intermediated households;
\( i_{h,i}^{A,H} (s) \) is the return rate of nominal portfolio balances \( A_{h,i}^{A,H} (s) \);
\( \Pi_i^A(s) \) is the dividend payments for the capital market intermediated household from holding various kinds of shares except shares of domestic industry specific intermediate output good firms, since \( i_{h,i}^{A,H} (s) \) has already included the dividend payment per specific share of domestic industry specific intermediate output good firm;
\( T_{h,i}^{A,N} (s) \) is the nondiscretionary lump sum transfer payments specifically for the capital market intermediated household, and the budget neutral nondiscretionary lump sum transfer program is operated by the fiscal authority in the government sector to redistribute national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households.

Endogenous preference shifter for housing \( u_i^{H,H} (s) \) depends on both the aggregate consumption of all the capital market intermediated households \( C_i^A(s) \) and the aggregate consumption of all three types of households \( C_i(s) \), according to the following intratemporal subutility function:

\[
u_i^{H,H} (s) = v_i^{H,H} \left( \frac{C_i^A(s)}{\phi_A} - \alpha C_i^{A(s-1)} \right)^{\frac{1}{\sigma}} \left( C_i(s) \right)^{\frac{1}{\eta}}
\]

Endogenous preference shifter for labor supply \( u_i^{L,H} (s) \) depends on the trend labor productivity (which exhibits partial adjustment dynamics of labor productivity) \( \bar{\lambda}_i (s) \), the aggregate consumption of all the capital market intermediated households \( C_i^A(s) \), the final labor services of all three types of households \( L_i(s) \), and the labor supply shock \( v_i^{N} (s) \), according to the following intratemporal subutility function:

\[
u_i^{L,H} (s) = \bar{\lambda}_i (s) \left( \frac{C_i^A(s)}{\phi_A} - \alpha C_i^{A(s-1)} \right)^{\frac{1}{\sigma}} \left[ \frac{L_i(s) - \lambda_i \lambda_i L_i(s-1)}{v_i^{N}(s)} \right]^{\frac{1}{\eta}}
\]

Endogenous preference shifter for nominal portfolio balances \( u_i^{A,H} (s) \) depends on both the aggregate consumption of all the capital market intermediated households \( C_i^A(s) \) and the aggregate consumption of all three types of households \( C_i(s) \), according to the following intratemporal subutility function:

\[
u_i^{A,H} (s) = v_i^{A,H} \left( \frac{C_i^A(s)}{\phi_A} - \alpha C_i^{A(s-1)} \right)^{\frac{1}{\sigma}} \left( C_i(s) \right)^{\frac{1}{\mu}}
\]

Nominal portfolio balances \( A_{h,i}^{A,H} (s+1) \) of the capital market intermediated household are further distributed across domestic and foreign economy specific short term bond holdings \( \{B_{h,i}^{S,H}(s+1)\}_{h=1}^{N} \), domestic and foreign economy specific and vintage specific long term bond holdings \( \{B_{h,i,j,v}^{L,H}(s+1)\}_{j=1}^{N} \), and domestic and foreign economy specific and industry
specific and firm specific stock holdings \( \{ \{ S^F_{h_i,j,k,l}(s+1) \}_{i=0}^{N} \}_{j=1}^{45} \). The capital market intermediated household’s preferences over the real value of internationally diversified short term bonds \( \frac{B_{i,j}^{H}(s+1)}{P^*_i(s)} \), the real value of internationally diversified and vintage diversified long term bonds \( \frac{B_{h_i,j}^{L}(s+1)}{P^*_i(s)} \), and the real value of internationally diversified and industry diversified and firm diversified stocks \( \frac{S^F_{h_i,j,k,l}(s+1)}{P^*_i(s)} \) are represented by the following constant elasticity of substitution intratemporal subutility function:

\[
A_{i,j}^{H}(s+1) = \left( 1 - \phi_{i,j}^{B} - \phi_{i,j}^{S} \right)^{1 - \psi_{A}} \left( \frac{B_{h_i,j}^{H}(s+1)}{P^*_i(s)} \right)^{\frac{\psi_{A}}{\psi_{A} - 1}} + \left( \phi_{i,j}^{B} \right)^{1 - \psi_{A}} \left( \frac{1}{\psi_{A} - 1} \frac{B_{h_i,j}^{H}(s+1)}{P^*_i(s)} \right)^{\frac{\psi_{A}}{\psi_{A} - 1}} + \\
\left( \phi_{i,j}^{S} \right)^{1 - \psi_{A}} \left( \frac{1}{\psi_{A} - 1} \frac{S^F_{h_i,j,k,l}(s+1)}{P^*_i(s)} \right)^{\frac{\psi_{A}}{\psi_{A} - 1}} \left( \frac{P^*_i(s)}{\psi_{A} - 1} \right)^{1 - \psi_{A}}
\]

Here, 
\( \psi_{i,j}^{B} \) is the weighted average of domestic and foreign duration risk premium for the i-th economy; 
\( \psi_{i,j}^{S} \) is the weighted average of domestic and foreign equity risk premium for the i-th economy.

The capital market intermediated household’s preferences over the domestic currency denominated real values of foreign economy specific short term bonds \( \left\{ E_{l,j}(s)B_{h_i,j}^{H}(s+1) \right\}_{j=1}^{N} \) are represented by the following constant elasticity of substitution intratemporal subutility function:

\[
B_{h_i,j}^{H}(s+1) = \left[ \sum_{i=1}^{N} \left( \phi_{i,j}^{B} \right)^{\frac{1}{\psi_{A}}} \left( \frac{E_{l,j}(s)B_{h_i,j}^{H}(s+1)}{P^*_i(s)} \right)^{\frac{\psi_{A}}{\psi_{A} - 1}} \right]^{\frac{1}{\psi_{A} - 1}}
\]

Here, 
\( \psi_{l,j}^{B} \) is the currency risk premium shock for the currency issued by the j-th economy; 
\( E_{l,j}(s) \) is the nominal bilateral exchange rate of the currency issued by the i-th economy to the currency issued by the j-th economy.

The capital market intermediated household’s preferences over the domestic currency denominated real values of foreign economy specific and vintage specific long term bonds \( \left\{ E_{l,j}(s)B_{h_i,j}^{L}(s+1) \right\}_{j=1}^{N} \) are represented by the following two constant elasticity of substitution intratemporal subutility functions:
The return rate of nominal portfolio balances intermediate output good firm. Here, term bonds; is the overall return rate of internationally diversified short term bonds; is the overall return rate of internationally diversified and vintage diversified long term bonds; is the overall return rate of internationally diversified and industry diversified and firm diversified stocks.

Since short term bonds are discount bonds, the total return yielded by internationally diversified short term bonds for the capital market intermediated household is split into
returns yielded by various domestic currency denominated values of domestic and foreign economy specific short term bonds \( \{E_{ij}(s)B_{h_{ij}}^{SH}(s)\}_{i=1}^{N} \):  
\[
\left(1 + i_{h_{ij}}^{SH}(s)\right)B_{h_{ij}}^{SH}(s) = \sum_{i=1}^{N} E_{ij}(s)(1 + i_{h_{ij}}^{SH}(s)-1)B_{h_{ij}}^{SH}(s)
\]

Here \( i_{h_{ij}}^{SH}(s) \) is the yield to maturity on economy specific short term bond.

Since long term bonds are perpetual bonds with coupon payments that decay exponentially at rate \( \omega^{B} \), the total return yielded by internationally diversified and vintage diversified long term bonds for the capital market intermediated household is split into returns yielded by various domestic currency denominated values of domestic and foreign economy specific and vintage specific long term bonds \( \{E_{ij}(s)V_{l,v}^{B}(s)B_{h_{ij,v}}^{LH}(s)\}_{v=1}^{N} \):  
\[
\left(1 + i_{h_{ij}}^{LH}(s)\right)B_{h_{ij}}^{LH}(s) = \sum_{i=1}^{N} E_{ij}(s)(1 + i_{h_{ij}}^{LH}(s))B_{h_{ij}}^{LH}(s)
\]
\[
\left(1 + i_{h_{ij}}^{LH}(s)\right)B_{h_{ij}}^{LH}(s) = \sum_{v=1}^{s-1}(\Pi_{j,v}^{B}(s) + V_{l,v}^{B}(s))B_{h_{ij,v}}^{LH}(s)
\]

Here \( \Pi_{j,v}^{B}(s) \) is the local currency denominated coupon payment per economy specific and vintage specific long term bond, which has the following expression:
\[
\Pi_{j,v}^{B}(s) = (1 + i_{j}^{L}(v) - \omega^{B})(\omega^{B})^{s-v}V_{l,v}^{B}(v)
\]

Here \( i_{j}^{L}(v) \) is the yield to maturity on economy specific and vintage specific long term bond at issuance, and \( V_{l,v}^{B}(v) = 1 \).

The total return yielded by internationally diversified and industry diversified and firm diversified stocks for the capital market intermediated household is split into returns yielded by various domestic currency denominated values of domestic and foreign economy specific and industry specific and firm specific stocks \( \{E_{ij}(s)V_{l,k,f}^{S}(s)S_{h_{ij,k,f}}^{FH}(s)\}_{k=1}^{N} \):  
\[
\left(1 + i_{h_{ij}}^{SF,H}(s)\right)S_{h_{ij}}^{FH}(s) = \sum_{i=1}^{N} E_{ij}(s)(1 + i_{h_{ij}}^{SF,H}(s))S_{h_{ij}}^{FH}(s)
\]
\[
\left(1 + i_{h_{ij}}^{SF,H}(s)\right)S_{h_{ij}}^{FH}(s) = \sum_{k=1}^{45} \sum_{l=0}^{1} (\Pi_{j,k,f}^{S}(s) + V_{l,k,f}^{S}(s))S_{h_{ij,k,f}}^{FH}(s)dl
\]

Here \( \Pi_{j,k,f}^{S}(s) \) is the dividend payment per specific share of economy specific and industry specific intermediate output good firm.

Type III. The credit constrained household chooses state contingent sequences in the current period \( s = t \) and all future periods \( s = t, t + 1, t + 2, \ldots \) for consumption \( C_{h_{i}}(s) \), housing \( H_{h_{i}}(s) \), and labor force participation \( \{N_{h,f_{i}}(s)\}_{f=0}^{1} \) or labor supply \( \{l_{h,f_{i}}(s)\}_{f=0}^{1} \), in order to maximize its following intertemporal utility function:
\begin{equation}
U_{h, i}(t) = E_t \sum_{s=t}^{\infty} \beta^{s-t} u \left( \left( C_{h, i}(s), H_{h, i}(s), \{L_{h, f, i}(s)\}_{f=0}^{1}\right) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ v_c(s) \left[ \frac{1}{1-\frac{1}{\sigma}} (C_{h, i}(s) - \right. \right.
\right.
\right.
\right.
\right.
\alpha C \frac{C^c(s-1)}{\phi^c} \right)^{1-\frac{1}{\gamma}} + \left. v_{H}^H(s) \left( H_{h, i}(s) \right)^{1-\frac{1}{\xi}} - u_{L}^L(s) \int_{f=0}^{1} \int_{g=0}^{1} \frac{L_{h, f, i}(s)}{\phi^c} \left( g - \alpha C \frac{L^c(s-1)}{\phi^c} \right)^{\frac{1}{\gamma}} dg df \right\}
\end{equation}

subject to the following dynamic budget constraint:

\begin{equation}
P^C(s)C_{h, i}(s) + \iota^H(s)H_{h, i}(s) = \Pi^C(s) + \left( 1 - \tau^L(s) \right) \int_{f=0}^{1} W_{h, f, i}(s) L_{h, f, i}(s) \, df + T_{h, i}^{CN}(s) + T_{h, i}^{CD}(s)
\end{equation}

Here,

\begin{align*}
P^C(s), v_c(s), \tau^L(s), W_{h, f, i}(s) \text{ and } v_{L}^L(s) \text{ for the credit constrained household are the same as those for the bank intermediated household;} \\
C^c(s) = \int_{h=\phi^B+\phi^A}^{\phi^B+\phi^A} C_{h, i}(s) \, dh \text{ is the aggregate consumption of all the credit constrained households;} \\
\phi^c \text{ is the percentage of credit constrained households among all households, and } \phi^B + \phi^A + \phi^c = 1; \\
L^L(s) = \int_{h=\phi^B+\phi^A}^{\phi^B+\phi^A} \int_{f=0}^{1} L_{h, f, i}(s) \, df \text{ is the differentiated intermediate labor services supplied by all the credit constrained households;} \\
\Pi^C(s) \text{ is the dividend payments for the credit constrained household from holding various kinds of originally endowed shares;}
\end{align*}

\begin{align*}
T_{h, i}^{CN}(s) \text{ is the nondiscretionary lump sum transfer payments specifically for the credit constrained household, and the budget neutral nondiscretionary lump sum transfer program is operated by the fiscal authority in the government sector to redistribute national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households;} \\
T_{h, i}^{CD}(s) \text{ is the discretionary lump sum transfer payments specifically for the credit constrained household, and the discretionary lump sum transfer program is operated by the fiscal authority in the government sector to provide income support only to credit constrained households.}
\end{align*}

Endogenous preference shifter for housing \( v_{L}^H(s) \) depends on both the aggregate consumption of all the credit constrained households \( C^c(s) \) and the aggregate consumption of all three types of households \( C_i(s) \), according to the following intratemporal subutility function:

\begin{equation}
v^H_{L}(s) = v^H_{L} \left( \frac{C_c(s)}{\phi^c} - \alpha C \frac{C^c(s-1)}{\phi^c} \right)^{\frac{1}{\gamma}} \left( C_i(s) \right)^{\frac{1}{\xi}}
\end{equation}

Endogenous preference shifter for labor supply \( v_{L}^L(s) \) depends on the trend labor productivity (which exhibits partial adjustment dynamics of labor productivity) \( \bar{A}_i(s) \), the aggregate consumption of all the credit constrained households \( C^c(s) \), the final labor services of all three types of households \( L_i(s) \), and the labor supply shock \( \iota^L(s) \), according to the following intratemporal subutility function:
The labor supply sector consists of two layers of agents. Variable subscript "f" is for labor unions of workers in the lower layer.

In the lower layer, continuums of monopolistically competitive labor unions of workers combine indivisible differentiated intermediate labor services from all types of households $L_{h,f,i}(s)$ to supply differentiated intermediate labor services $L_{f,i}(t)$ (unified symbol for $L^A_{f,i}(t)$, $L^B_{f,i}(t)$ or $L^C_{f,i}(t)$) mentioned in the above Subsection A) to final labor service firms as follows:

$$L_{f,i}(t) = \int_{s=0}^{1} L_{h,f,i}(s) \, dh$$

In the upper layer, a large number of perfectly competitive final labor service firms combine differentiated intermediate labor services supplied by labor unions of workers $L_{f,i}(t)$ to produce final labor services $L_i(t)$ needed by the production sector, according to the following constant elasticity of substitution production function:

$$L_i(t) = \left[ \int_{f=0}^{1} \left( \frac{L_{f,i}(t)}{\theta_{f}(t)} \right)^{1-\theta_{f}(t)} \right]^{\frac{\theta_{f}(t)}{\theta_{f}(t)-1}}$$

Here the exponent $\theta_{f}(t) = \frac{\theta_{f}(t)}{\theta_{f}(t)-1}$ has the meaning of wage markup shock $\theta_{f}(t) = \frac{W_{f,i}(t)}{W_{f,i}(t)}$.

"Perfectly competitive" means in equilibrium the final labor service firms maximize profits derived from production of the final labor services $L_i(t)$ with respect to inputs of intermediate labor services $L_{f,i}(t)$, implying the following labor service demand functions:

$$L_{f,i}(t) = \left( \frac{W_{f,i}(t)}{W_{f,i}(t)} \right)^{-\theta_{f}(t)} L_i(t)$$

Since in equilibrium the final labor service firms generate zero profits, then the aggregate wage index $W_i(t)$ can be expressed as the integral of the wage for intermediate labor services $W_{f,i}(t)$:

$$W_i(t) = \left[ \int_{f=0}^{1} \left( W_{f,i}(t) \right)^{1-\theta_{f}(t)} \right]^{\frac{1}{1-\theta_{f}(t)}}$$

In addition, each household consists of a continuum of members $N_{h,f,i}$ and there is full risk sharing among household members, they supply indivisible differentiated intermediate labor services $L_{h,f,i}$ to labor unions. The gap between the total labor force $N_i(t) = \int_{f=0}^{1} \int_{h=0}^{1} N_{h,f,i}(s) \, dhdf$ and the final labor services (also called the total employment) $L_i(t)$
The construction sector in the labor market.

In each period \( t \), a randomly selected fixed fraction \( \omega^L \) of labor unions adjust their wage \( W_{f_i}(t) \) to account for the past consumption price inflation \( \frac{P^c(t-1)}{P^c(t-2)} \) and the past trend labor productivity growth \( \frac{\tilde{A}(t-1)}{\tilde{A}(t-2)} \), according to the following partial indexation rule:

\[
W_{f_i}(t) = \left( \frac{P^c(t-1)}{P^c(t-2)} \right)^{1-\gamma^L} \left( \frac{P^c(t-1)\tilde{A}(t-1)}{P^c(t-2)\tilde{A}(t-2)} \right)^{\gamma^L} W_{f_i}(t-1)
\]

Here \( \frac{P^c(t-1)}{P^c(t-2)} \) and \( \frac{\tilde{A}(t-1)}{\tilde{A}(t-2)} \) are the steady state equilibrium values of \( \frac{P^c(t-1)}{P^c(t-2)} \) and \( \frac{\tilde{A}(t-1)}{\tilde{A}(t-2)} \).

The remaining fraction \( (1 - \omega^L) \) of labor unions adjust their wage optimally, in equilibrium they all choose a common wage \( W^*_i(t) \) given by the following necessary first order condition:

\[
\frac{W^*_i(t)}{W_i(t)} = \frac{E_t \sum_{s=t}^{\infty} (\omega^L)^s \frac{\partial u(s)}{\partial h^i_t(s)} \left( \frac{P^c(t-s)\tilde{A}(t-s)}{P^c(t-s-1)\tilde{A}(t-s-1)} \right)^{1-\gamma^L} \left( \frac{P^c(t-s)\tilde{A}(t-s)}{P^c(t-s-1)\tilde{A}(t-s-1)} \right)^{\gamma^L} W^*_i(t-s) \left( \frac{W^*_i(t-s)}{W_i(t-s)} \right) - \frac{\partial u(s)}{\partial h^i_t(s)} \right)}{E_t \sum_{s=t}^{\infty} (\omega^L)^s \frac{\partial u(s)}{\partial h^i_t(s)} \left( \frac{P^c(t-s)\tilde{A}(t-s)}{P^c(t-s-1)\tilde{A}(t-s-1)} \right)^{1-\gamma^L} \left( \frac{P^c(t-s)\tilde{A}(t-s)}{P^c(t-s-1)\tilde{A}(t-s-1)} \right)^{\gamma^L} W^*_i(t-s) \left( \frac{W^*_i(t-s)}{W_i(t-s)} \right) - \frac{\partial u(s)}{\partial h^i_t(s)}}
\]

Here \( u(s) \) is the brief symbol for the utility function of any type of household \( u \left( \left\{ \alpha_{h_i}^1(s), \alpha_{h_i}^2(s), \{ L_{h,f_i}(s) \} \right\} \right)^1_{s=0} \) or \( \frac{\alpha_{h_i}^1(s+1)}{P^c_i(s)} \), or \( \frac{\alpha_{h_i}^2(s+1)}{P^c_i(s)} \) or None.

Under this specification, although labor unions adjust their wage in every period, they infrequently do so optimally, and the time interval between optimal wage adjustments is a random variable. Then the aggregate wage index \( W_i(t) \) equals an average of the wage set by the randomly selected fixed fraction \( (1 - \omega^L) \) of labor unions that adjust their wage optimally, and the average of the wages set by the remaining fraction \( \omega^L \) of labor unions that adjust their wages according to the above partial indexation rule:

\[
W_i(t) = \left( 1 - \omega^L \right) \left( W^*_i(t) \right)^{1-\omega^L} + \omega^L \left( \frac{P^c(t-1)\tilde{A}(t-1)}{P^c(t-2)\tilde{A}(t-2)} \right)^{1-\gamma^L} \left( \frac{P^c(t-1)\tilde{A}(t-1)}{P^c(t-2)\tilde{A}(t-2)} \right)^{\gamma^L} W_i(t-1) \right)^{1-\omega^L} \frac{W^*_i(t)}{W_i(t)} \right] \}
\]

All the above intertemporal adjustment mechanism of wages cause nominal wage rigidity in the labor market.

### C. Construction Sector

The construction sector consists of two layers of agents. Variable subscript “e” is for real estate intermediate developers in the lower layer.

In the lower layer, continuums of monopolistically competitive real estate intermediate developers choose state contingent sequences in the current period \( s = t \) and all future
periods \( s = t + 1, t + 2, \ldots \) for residential investment \( I_{i,e}^H(s) \) and housing stock \( H_{i,e}(s + 1) \), in order to maximize the following pre-dividend stock market value:

\[
\Pi_{i,e}^H(t) + V_{i,e}^H(t) = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_i^H(s)}{\lambda_i^H(t)} \Pi_{i,e}^H(s)
\]

subject to the following housing accumulation function:

\[
H_{i,e}(s + 1) = (1 - \delta^H)H_{i,e}(s) + H^H(I_{i,e}^H(s), I_{i,e}^H(s - 1))
\]

and also subject to terminal nonnegativity constraint that \( H_{i,e}(s + 1) \geq 0 \) for \( s \to \infty \).

Here, \( \Pi_{i,e}^H(t) \) and \( V_{i,e}^H(t) \) are the dividend payment and price per specific share of domestic real estate intermediate developer; \( \lambda_i^H(s) \) is the Lagrange multiplier associated with the dynamic budget constraint for the bank intermediated household in period \( s \) in the above Subsection A.

\( H^H(I_{i,e}^H(s), I_{i,e}^H(s - 1)) \) is the effective residential investment function, which incorporates convex adjustment costs for the intertemporal change in residential investment as follows:

\[
H^H(I_{i,e}^H(s), I_{i,e}^H(s - 1)) = v^H_i(s) \left[ 1 - \frac{\delta^H}{2} \left( \frac{I_{i,e}^H(s)}{I_{i,e}^H(s - 1)} - 1 \right)^2 \right]
\]

Here \( v^H_i(s) \) is the residential investment demand shock.

The dividend payment per specific share of domestic real estate intermediate developer \( \Pi_{i,e}^H(s) \) equals to the total net profits of domestic real estate intermediate developer:

\[
\Pi_{i,e}^H(s) = \zeta_{i,e}^H(s)H_{i,e}(s) + \left[ B_{i,e}^{CD}(s + 1) - (1 - \delta^M(s))(1 + i^M(s - 1))B_{i,e}^{CD}(s) \right] - P_{i,e}^H(s)I_{i,e}^H(s)
\]

Here,

- \( \zeta_{i,e}^H(s)H_{i,e}(s) \) is the earning of real estate intermediate developer from selling differentiated intermediate housing services \( H_{i,e}(s) \) to real estate final developers at rental price \( \zeta_{i,e}^H(s) \);
- \( B_{i,e}^{CD}(s + 1) - (1 - \delta^M(s))(1 + i^M(s - 1))B_{i,e}^{CD}(s) \) is the net borrowing of real estate intermediate developer from the banking sector, which is defined as the increase in mortgage loans from domestic final banks \( B_{i,e}^{CD}(s + 1) \), net of writedowns at mortgage loan default rate \( \delta^M(s) \) and interest payments at mortgage loan rate \( i^M(s - 1) \);
- \( P_{i,e}^H(s)I_{i,e}^H(s) \) is the residential investment expenditure, identical final residential investment goods or services \( I_{i,e}^H(s) \) are purchased from the additionally added virtual “absorption sector”;
- \( P_{i,e}^H(s) \) is the price of both residential investment and housing stock.

Given regulatory mortgage loan to value ratio limit \( \phi^D_i(s) \) required by the macroprudential authority in the government sector, domestic real estate intermediate developer can only
maintain mortgage loans \( B_{i,e}^{CD}(s + 1) \) equal to a fraction of the value of its housing stock \( P_{i}^{H}(s + 1) \):

\[
\frac{B_{i,e}^{CD}(s + 1)}{P_{i}^{H}(s)H_{i,e}(s + 1)} = \phi_{i}^{D}(s)
\]

In addition, the Lagrange multiplier associated with the above-mentioned housing accumulation function \( Q_{i,e}^{H}(t) \) has the meaning of shadow price of housing:

\[
Q_{i,e}^{H}(t) = E_{t} \frac{\beta_{i}^{P}(t+1)}{\lambda_{i}^{H}(t)} \left( P_{i}^{H}(t + 1) \frac{\frac{i_{i,e}^{H}(t)}{p_{i}^{H}(t)}}{p_{i}^{H}(t+1)} - \phi_{i}^{D}(t) \frac{p_{i}^{H}(t)}{p_{i}^{H}(t+1)} \left( (1 - \delta_{i}^{M}(t + 1)) (1 + i_{i}^{M}(t)) - \lambda_{i}^{P}(t) \right) \right) + (1 - \delta_{i}^{H})Q_{i,e}^{H}(t + 1)
\]

In the upper layer, a large number of perfectly competitive real estate final developers combine differentiated intermediate housing services supplied by domestic real estate intermediate developers \( H_{i,e}(t) \) to produce final housing services \( H_{i}(t) \) that are needed by all types of domestic households, according to the following constant elasticity of substitution production function:

\[
H_{i}(t) = \left[ \int_{e=0}^{1} \left( H_{i,e}(t) \right)^{\frac{\theta_{i}^{H}(t)-1}{\theta_{i}^{H}(t)}} \text{de} \right]^{\theta_{i}^{H}(t) \over \theta_{i}^{H}(t)-1}
\]

Here the exponent \( \frac{\theta_{i}^{H}(t)}{\theta_{i}^{H}(t)-1} \) has the meaning of the endogenous rental price of housing markup shifter \( \theta_{i}^{H}(t) = \frac{\theta_{i}^{H}(t)}{\theta_{i}^{H}(t)-1} \).

"Perfectly competitive" means in equilibrium the real estate final developers maximize profits derived from production of the final housing services \( H_{i}(t) \) with respect to inputs of intermediate housing services \( H_{i,e}(t) \), implying the following housing service demand functions:

\[
H_{i,e}(t) = \left( \frac{i_{i,e}^{H}(t)}{p_{i}^{H}(t)} \right)^{-\theta_{i}^{H}(t)} H_{i}(t)
\]

Since in equilibrium the real estate final developers generate zero profits, then the aggregate rental price index of housing \( i_{i}^{H}(t) \) can be expressed as the integral of the rental price of intermediate housing services \( i_{i,e}^{H}(t) \):

\[
i_{i}^{H}(t) = \left[ \int_{e=0}^{1} \left( i_{i,e}^{H}(t) \right)^{1-\theta_{i}^{H}(t)} \text{de} \right]^{-1 \over 1-\theta_{i}^{H}(t)}
\]

In each period \( t \), all real estate intermediate developers adjust their rental price of housing optimally, in equilibrium they all choose a common rental price of housing \( i_{i}^{H,*}(t) \):
\[
\frac{i_{iH(t)}^{H^*}}{p_{i1H(t)}} = \frac{\phi_{i1}^D(t-1) \left( 1 - \delta_i^{M(t)}(t) \right) \left( 1 + i_i^{M(t-1)} \right) \left( p_{i1}^{H(t-1)} / p_{i1H(t)} \right) - (1 - \delta_i^H) \phi_i^D(t-1)}{\delta_i^{H(t-1)} - 1}
\]

Then the aggregate rental price index of housing \( i_{iH(t)} \) equals \( i_{iH^*}(t) \).

**D. Production Sector**

The *structurally more complex production sector with 45 industries* is innovatively constructed based on the structurally simple total production sector of the GFM introduced in Vitek (2015, 2018). Specifically, we not only build the intra-industry and inter-industry structure of this sector from nonexistence to existence, but also help this sector fully characterize the massive and accurate inter-country cross-industry input-output relations and utilize the rich industry-level information provided by the OECD ICIO tables as much as possible. Among all sectors it is the most complex sector that consists of 45 industries (Appendix I), while each industry further consists of two layers of agents. Variable subscript “I” is for industry specific intermediate output good firms in the lower layer of each industry. First of all, among 45 industries classified by the OECD ICIO tables:

- The 3rd industry-\{ Mining and quarrying, energy producing products \} and the 10th industry-\{ Coke and refined petroleum products \} are mainly considered as the industries that produce internationally homogeneous energy commodities, which would become intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector.

- The 1st industry-\{ Agriculture, hunting, forestry \}, the 2nd industry-\{ Fishing and aquaculture \}, the 4th industry-\{ Mining and quarrying, non-energy producing products \}, the 14th industry-\{ Other non-metallic mineral products \}, the 15th industry-\{ Basic metals \}, and the 16th industry-\{ Fabricated metal products \} are mainly considered as the industries that produce internationally homogeneous nonenergy commodities, which would be taken as intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector.

- All the remaining 37 industries are mainly considered as the industries that produce internationally heterogeneous goods or services, to be taken as intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector.

Since we further incorporate the structure of international direct investment into this AGMFM relative to the original GFM introduced in Vitek (2015, 2018), the domestic business investment for the production sector of the i-th economy \( I_{i,k1}(t) \) should always be replaced by the innovatively added domestic and foreign business investment inflow from all economies (including from the i-th economy itself) to the production sector of the i-th economy \( I_{i,k1}(t) \). Please refer to the following Subsection I for more details around the domestic and foreign business investment inflow.

In the lower layer of each industry, continuums of *monopolistically competitive industry specific intermediate output good firms* choose state contingent sequences in the current period \( s = t \) and all future periods \( s = t + 1, t + 2, \ldots \) for final labor services (also called employment) \( L_{i,k1}(s) \), the capital utilization rate \( u_{i,k1}(s) \), the domestic and foreign business
investment inflow $i_{k,l}^{K\text{in}}(s)$, and the private physical capital stock $K_{i,k l}(s + 1)$, in order to maximize the following pre-dividend stock market value:

$$\Pi_{k l}^S(t) + V_{k l}^S(t) = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda^A(t)}{\lambda^A(t)} \Pi_{i,k l}(s)$$

subject to the following production function:

$$Y_{i,k l}(s) = \left( u_{k l}^K(s) K_{i,k l}(s) \phi^K \right)^{\phi^K} \left( A_{i,k}(s) L_{i,k l}(s) \right)^{1-\phi^K} \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right) \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right)$$

subject to the following private physical capital accumulation function:

$$K_{i,k l}(s + 1) = (1 - \delta^K) K_{i,k l}(s) + H(i_{k l}^{K\text{in}}(s), i_{k l}^{K\text{in}}(s - 1))$$

and also subject to terminal nonnegativity constraint that $K_{i,k l}(s + 1) \geq 0$ for $s \to \infty$.

Here, $\Pi_{k l}^S(t)$ and $V_{k l}^S(t)$ are the dividend payment and price per specific share of domestic industry specific intermediate output good firm; $\lambda^A(t)$ is the Lagrange multiplier associated with the dynamic budget constraint for the capital market intermediated household in period $s$ in the above Subsection A; $Y_{i,k l}(s)$ is the industry specific differentiated intermediate output goods or services produced by industry specific intermediate output good firm.

Compared with the simple production function in the structurally simple total production sector of the original GFM introduced in Vitek (2015, 2018), here the innovatively improved production function:

- Utilizes final labor services $L_{i,k l}(s)$ with the economy specific and industry specific labor productivity $A_{i,k}(s)$ instead of the economy specific labor productivity $A_{i}(s)$, and $A_{i,k}(s)$ not only depends on the economy specific and industry specific per capita public physical capital stock $\frac{K_{i,k l}(s)}{N_{i,k l}(s)}$ to reflect the government's differentiated support for sunrise and sunset industries in different countries, but also relies on the labor productivity shifter for the k-th industry of the i-th economy $\nu_{i,k}(s)$ to reflect economy specific and industry specific technology diffusion among all domestic and foreign industries that have input-output relation with the k-th industry of the i-th economy, which is the weighted average of the labor productivity shock for any specific industry of any specific economy $\nu_{j,m}(s)$:

$$A_{i,k}(s) = \left( u_{i,k}(s) \phi^A \right)^{\phi^A} \left( \frac{K_{i,k l}(s)}{N_{i,k l}(s)} \right)^{1-\phi^A} \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right) \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right)$$

$$u_{i,k}(s) = \left[ \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \nu_{j,m}(s) \right) \phi^A \left( \frac{K_{i,k l}(s)}{N_{i,k l}(s)} \right)^{1-\phi^A} \right]^{\frac{\lambda^A}{\lambda^A}} \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right) \prod_{i=1}^{N} \prod_{m=1}^{45} \left( \frac{Y_{j,m-i,k l}(s)}{V_{l k}} \right)$$
Correspondingly, the economy specific and industry specific trend labor productivity \( \bar{\bar{\alpha}}_{i,k}(s) \) that exhibits partial adjustment dynamics is expressed as follows:

\[
\bar{\bar{\alpha}}_{i,k}(s) = (\bar{\bar{\alpha}}_{i,k}(s - 1))^{\rho_A} (\bar{\bar{\alpha}}_{i,k}(s))^{1 - \rho_A}
\]

- Additionally utilizes a part of industry specific final output goods or services from all domestic and foreign industries \( Y_{j,m\rightarrow i,k,l}(s) \), to serve as intermediate inputs, here the ratio of total output minus intermediate inputs of the k-th industry of the i-th economy to the total output of the k-th industry of the i-th economy

\[
\frac{Y_{i,k} - \sum_{m=1}^{N} Y_{j,m\rightarrow i,k}}{Y_{i,k}}
\]

and the ratio of industry specific final output goods or services from the m-th industry of the j-th economy that are adopted as intermediate inputs of the k-th industry of the i-th economy to the total output of the k-th industry of the i-th economy

\[
\frac{Y_{j,m\rightarrow i,k}}{Y_{i,k}}
\]

are all calculated based on the corresponding column in the OECD ICIO table.

- In order to reduce the large number of new intermediate variables \( \{Y_{j,m\rightarrow i,k,l}(s)\}_{m=1}^{N} \) that are introduced into the above expanded production function as various intermediate inputs, the input-output elasticity \( \gamma_{GVA\rightarrow i,k} \) specifically for the gross value added (GVA) created by capital and labor input

\[
\left( u_{i,k,l}(s)K_{i,k,l}(s) \right)^{\phi_i} \left( A_{i,k}(s)L_{i,k,l}(s) \right)^{1 - \phi_i}
\]

the input-output elasticity \( \gamma_{i,m\rightarrow i,k} \) specifically for the input from the m-th industry of the j-th economy to the k-th industry of the i-th economy \( Y_{j,m\rightarrow i,k}(s) \), and a fraction \( \omega_{j,m\rightarrow i,k}^{hf} \) of the total output of the m-th industry of the j-th economy \( Y_{j,m}(s) \) to serve as intermediate inputs, are additionally estimated based on the econometric estimation of time series that are composed of the corresponding column in the OECD ICIO tables, then

\[
(\omega_{j,m\rightarrow i,k}^{hf} Y_{j,m}(s))^{\gamma_{i,m\rightarrow i,k}}
\]

is adopted to replace \( Y_{j,m\rightarrow i,k,l}(s) \), ultimately the above-mentioned innovatively expanded production function is changed to the following form:

\[
Y_{i,k,l}(s) = \left[ \left( u_{i,k,l}(s)K_{i,k,l}(s) \right)^{\phi_i} \left( A_{i,k}(s)L_{i,k,l}(s) \right)^{1 - \phi_i} \frac{\gamma_{GVA\rightarrow i,k} Y_{i,k} - \sum_{m=1}^{N} Y_{j,m\rightarrow i,k}}{Y_{i,k}} \right]^{\gamma_{i,m\rightarrow i,k}} \prod_{j=1}^{N} \prod_{m=1}^{45} \left( \omega_{j,m\rightarrow i,k}^{hf} Y_{j,m}(s) \right)^{\gamma_{i,m\rightarrow i,k}} \frac{Y_{j,m\rightarrow i,k}}{Y_{i,k}}
\]

- Correspondingly, the potential output of the k-th industry of the i-th economy (the inferred industry-level total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \bar{\bar{Y}}_{i,k,l}(s) \) is calculated using private physical capital stock \( K_{i,k,l}(s) \) instead of utilized private physical capital stock \( u_{i,k,l}(s)K_{i,k,l}(s) \), effective labor force \( A_{i,k}(s)N_{i,k,l}(s) \) instead of effective employment \( A_{i,k}(s)L_{i,k,l}(s) \), and a fraction \( \omega_{j,m\rightarrow i,k}^{hf} \) of the potential output of the m-th industry of the j-th economy \( \bar{\bar{Y}}_{j,m}(s) \) to serve as intermediate inputs instead of a fraction \( \omega_{j,m\rightarrow i,k}^{hf} \) of the total output of the m-th industry of the j-th economy \( Y_{j,m}(s) \) to serve as intermediate inputs:
\[ \bar{V}_{i,k,l}(s) = \left[ (K_{i,k,l}(s))^\phi_i (A_{i,k}(s)N_{i,k,l}(s))^{1-\phi_i} \right]^{1-\phi_i} \]

\[ \prod_{i=1}^{N} \prod_{m=1}^{45} \left[ \left( \omega_{j,m,-i,k}^{h_f} \bar{V}_{j,m}(s) \right)^{\text{elasticity}_{j,m,-i,k}} \right]^{\nu_i(s) N_{i,k,l}(s)^{1-\phi_i}} \]

In the above private physical capital accumulation function, \( H(i_{i,k,l}(s), i_{k,l}(s - 1)) \) is the effective business investment function, which incorporates convex adjustment costs for the intertemporal change in business investment as follows:

\[ H(i_{i,k,l}(s), i_{k,l}(s - 1)) = v_i^K(s) \left[ 1 - \frac{\chi^K}{2} \left( \frac{i_{k,l}(s)}{i_{k,l}(s - 1)} - 1 \right) \right] i_{k,l}(s) \]

Here \( v_i^K(s) \) is the business investment demand shock.

The dividend payment per specific share of domestic industry specific intermediate output good firm \( \Pi_{i,k,l}(s) \) equals to the total net profits of domestic industry specific intermediate output good firm:

\[ \Pi_{i,k,l}(s) = \left( 1 - \tau^K(s) \right) \left( P_{i,k,l}(s) Y_{i,k,l}(s) - W_i(s) L_{i,k,l}(s) - \Phi_{i,k,l}(s) \right) + \left[ B_{i,k,l}(s + 1) - \left( 1 - \delta_C(s) \right) \left( 1 + i_{i,k,l}^{\text{CE}}(s) \right) B_{i,k,l}(s) \right] - P_i^K(s) \]

Here \( \tau^K(s) \) is the corporate income tax rate.

The pre-tax earnings of domestic industry specific intermediate output good firm \( \left( P_{i,k,l}(s) Y_{i,k,l}(s) - W_i(s) L_{i,k,l}(s) - \Phi_{i,k,l}(s) \right) \) equals to the revenue from selling industry specific differentiated intermediate output goods or services \( Y_{i,k,l}(s) \) to domestic industry specific final output good firms at output price \( P_{i,k,l}(s) \), less the expenditure on final labor services \( L_{i,k,l}(s) \) at wage price \( W_i(s) \), and less other variable cost \( \Phi_{i,k,l}(s) \) that increases as the capital utilization rate \( u_{i,k,l}(s) \) increases:

\[ \Phi_{i,k,l}(s) = P_i^K(s) \frac{\mu^K}{1-\tau_i} \left( e^{\eta^K(u_{i,k,l}(s) - 1)} - 1 \right) K_{i,k,l}(s) + P_i^K(s) \]

Here,

- \( P_i^K(s) \) is the price of both business investment and private physical capital stock;
- \( \tau_i \) is the i-th economy’s broad level of taxation (as a proportion of the i-th economy’s nominal GDP);
- \( B_{i,k,l}(s + 1) - \left( 1 - \delta_C(s) \right) \left( 1 + i_{i,k,l}^{\text{CE}}(s) \right) B_{i,k,l}(s) \) is the net borrowing of industry specific intermediate output good firm from the banking sector, which is defined as the increase in corporate loans from domestic global final banks \( B_{i,k,l}(s + 1) \), net of writedowns at corporate loan default rate \( \delta_C(s) \) and interest payments at effective corporate loan rate \( i_{i,k,l}^{\text{CE}}(s) \);
- \( P_i^K(s) u_{i,k,l}(s) \) is the business investment expenditure, identical final business investment goods or services \( u_{i,k,l}(s) \) are purchased from the additionally added virtual "absorption
Given regulatory corporate loan to value ratio limit $\phi_i^F(s)$ required by the macroprudential authority in the government sector, domestic industry specific intermediate output good firm can only maintain corporate loans $B_{ik}^{CF}(s + 1)$ equal to a fraction of the value of its private physical capital stock $P_{ik}(s)K_{ik}(s + 1)$:

$$\frac{B_{ik}^{CF}(s + 1)}{P_{ik}^K(s)K_{ik}(s + 1)} = \phi_i^F(s)$$

In addition, the Lagrange multiplier associated with the above-mentioned private physical capital accumulation function $Q_{ik}^K(t)$ has the meaning of shadow price of private physical capital:

$$Q_{ik}^K(t) = E_t^{\frac{\alpha_i^K(t+1)}{\lambda_i^K(t)}}\left\{P_{ik}^K(t+1)u_{ik}^K(t + 1)\right\}$$

In the upper layer of each industry, a large number of perfectly competitive industry specific final output good firms combine industry specific differentiated intermediate output goods or services supplied by industry specific intermediate output good firms $Y_{ik}(t)$ to produce industry specific final output goods or services $Y_{ik}(t)$, according to the following constant elasticity of substitution production function:

$$Y_{ik}(t) = \left[\int_{l=0}^{1} P_{ik}(t)Y_{ik}^{(t+1)} - 1 \right] \left(1 - \delta_i^{F}(t + 1)\right) \left(1 + i_i^{E}(t + 1) + \frac{i_A(t)}{\beta A_i(t + 1)}\right) + (1 - \delta K)Q_{ik}^K(t + 1)$$

Here the exponent $\frac{\theta_i^{Y}(t)}{\theta_i^{Y}(t)-1}$ has the meaning of the output price markup shock $\theta_i^{Y}(t) =$ $\frac{\theta_i^{Y}(t)}{\theta_i^{Y}(t)-1}$. "Perfectly competitive" means in equilibrium the industry specific final output good firms maximize profits derived from production of the industry specific final output goods or services $Y_{ik}(t)$ with respect to inputs of industry specific differentiated intermediate output goods or services $Y_{ik}(t)$, implying the following output goods or services demand functions:

$$Y_{ik}(t) = \left(p_{ik}(t)Y_{ik}^{(t+1)} - 1\right)\left(1 - \delta_i^{F}(t + 1)\right)\left(1 + i_i^{E}(t + 1) + \frac{i_A(t)}{\beta A_i(t + 1)}\right) + (1 - \delta K)Q_{ik}^K(t + 1)$$
Since in equilibrium the industry specific final output good firms generate zero profits, then the industry specific aggregate output price index $P_{i,k}^Y(t)$ can be expressed as the integral of the output price of industry specific intermediate output goods or services $P_{i,k,l}^Y(t)$:

$$P_{i,k}^Y(t) = \left[ \int_0^1 \left( P_{i,k,l}^Y(t) \right)^{1-\theta_i} \, dl \right]^{1-\theta_i}_{1-\theta_i}$$

In addition, a large number of perfectly competitive final output good firms that exist in the GFM introduced in Vitek (2015, 2018) are still retained in this new model, in order to facilitate the macroeconomic activities and maintain the macroeconomic relations between this production sector and other sectors or markets that do not distinguish between industries. However, compared with the settings of final output good firms in the GFM introduced in Vitek (2015, 2018), final output good firms in this structurally more complex production sector combine the industry specific final output goods or services supplied by domestic industry specific final output good firms $Y_{i,k}(t)$ to produce final output goods or services $Y_i(t)$ that do not distinguish between industries:

$$Y_i(t) = \prod_{k=1}^{45} \left( Y_{i,k}(t) \right)^{\text{Contribution}_{Y_{i,k}}}$$

Here $\text{Contribution}_{Y_{i,k}}$ represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy, they are calculated based on the corresponding column sum in the OECD ICIO table.

Correspondingly, the potential output of the i-th economy (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) $\bar{Y}_i(t)$ is calculated using the industry-level potential output of the i-th economy (the inferred industry-level total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) $\bar{Y}_{i,k}(t)$ instead of the industry-level total output of the i-th economy $Y_{i,k}(t)$:

$$\bar{Y}_i(t) = \prod_{k=1}^{45} \left( \bar{Y}_{i,k}(t) \right)^{\text{Contribution}_{Y_{i,k}}}$$

In each period $t$, a randomly selected fixed fraction $\omega_Y^Y$ of industry specific intermediate output good firms adjust their output price $P_{i,k,l}^Y(t)$ to account for the past industry specific output price inflation $\frac{P_{i,k,l}^Y(t-1)}{P_{i,k,l}^Y(t-2)}$, according to the following partial indexation rule:

$$P_{i,k,l}^Y(t) = \left( \frac{P_{i,k,l}^Y(t-1)}{P_{i,k,l}^Y(t-2)} \right)^{\gamma_k^Y} \left( \frac{P_{i,k,l}^Y(t-1)}{P_{i,k,l}^Y(t-2)} \right)^{1-\gamma_k^Y} P_{i,k,l}^Y(t-1)$$

Here $\frac{P_{i,k,l}^Y(t-1)}{P_{i,k,l}^Y(t-2)}$ is the steady state equilibrium value of $\frac{P_{i,k,l}^Y(t-1)}{P_{i,k,l}^Y(t-2)}$.

The remaining fraction $(1 - \omega_Y^Y)$ of industry specific intermediate output good firms adjust their output price optimally, in equilibrium they all choose a common industry specific output price $P_{i,k}^Y(t)$ given by the following necessary first order condition:
nominal output price rigidity

All the above intertemporal adjustment mechanism of output prices cause their output price in every period, optimal price adjustment opportunities arrive randomly, and the time interval between optimal price adjustments is a random variable. Then the aggregate industry specific output price index $P_i^Y(t)$ equals an average of the price set by the randomly selected fixed fraction $(1 - \omega_i^Y)$ of industry specific intermediate output good firms that adjust their output price optimally, and the average of the output prices set by the remaining fraction $\omega_i^Y$ of industry specific intermediate output good firms that adjust their output prices according to the above partial indexation rule:

$$P_i^Y(t) = \left\{ \left( 1 - \omega_i^Y \right) \left( P_{i,k}^Y(t) \right)^{1-\theta_i^Y} + \omega_i^Y \left\{ (P_{i,k}^Y(t-1))^{1-\gamma_i^Y} + \theta_i^Y \gamma_i^Y P_{i,k}^Y(t-1) \right\} \left( P_{i,k}^Y(t-2) \right)^{1-\gamma_i^Y} \right\}^{\frac{1}{1-\theta_i^Y}}$$

All the above intertemporal adjustment mechanism of output prices cause industry specific nominal output price rigidity in the industry specific output market.

Since we want to further include the transmission mechanism between upstream and downstream quantities and prices in the global supply chain in this model:

- We set the output quantity of the industry specific final output goods or services that would become intermediate inputs for domestic and foreign production sector, no matter whether they come from the domestic industry that produces internationally heterogeneous energy or nonenergy commodities or the domestic industry that produces internationally heterogeneous goods or services, equal to the distribution value weighted average of cross-border cross-industry downstream industry-level output quantity. While the output quantity of the industry specific final output goods or services that would become intermediate inputs for domestic and foreign production sector, no matter whether they come from the domestic industry that produces internationally heterogeneous energy or nonenergy commodities or the domestic industry that produces internationally heterogeneous goods or services, are not adjusted relative to the original GFM introduced in Vitek (2015, 2018).

- We set the output price of the industry specific final output goods or services that would become intermediate inputs for domestic and foreign production sector, no matter whether they come from the domestic industry that produces internationally heterogeneous energy or nonenergy commodities or the domestic industry that produces internationally heterogeneous goods or services, equal to the distribution value weighted average of exchange adjusted cross-border cross-industry downstream industry-level output price. While the output price of the industry specific final output goods or services that would become final output goods or services for domestic and foreign absorption sector, no matter whether they come from the...
domestic industry that produces internationally homogeneous energy or nonenergy commodities or the domestic industry that produces internationally heterogeneous goods or services, are not adjusted relative to the original GFM introduced in Vitek (2015, 2018).

As the above complex settings lead to the expression of industry-level real output and output price being too lengthy and complex, we will only present the derived linearized relationship equations in Section IV for further introduction.

E. Banking Sector

The banking sector has different layer structures between creating and issuing mortgage loans and creating and issuing corporate loans. The whole process of creating mortgage loans that are issued to domestic real estate intermediate developers is only related to the following lower layer and middle layer of the banking sector, while the whole process of creating corporate loans that are issued to both domestic and foreign industry specific intermediate output good firms is related to the following lower layer, middle layer and upper layer of the banking sector. Variable subscript “b” is for domestic intermediate banks in the lower layer.

In the lower layer, continuums of monopolistically competitive domestic intermediate banks choose state contingent sequences in the current period $s = t$ and all future periods $s = t + 1, t + 2, \ldots$ for deposit funding $B_{i,b}^{D}(s + 1)$, net interbank market funding $B_{i,b}^{B}(s + 1)$, retained earnings $I_{i,b}^{B}(s)$, and the bank capital stock $K_{i,b}^{B}(s + 1)$, in order to maximize the following pre-dividend stock market value:

$$\Pi_{i,b}^{C}(t) + V_{i,b}^{C}(t) = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_{i,b}^{B}(s)}{\lambda_{i,b}^{B}(t)} \Pi_{i,b}^{C}(s)$$

subject to the following balance sheet identity:

$$B_{i,b}^{D}(s + 1) + B_{i,b}^{B}(s + 1) = B_{i,b}^{D}(s) + B_{i,b}^{B}(s) + K_{i,b}^{B}(s) + K_{i,b}^{B}(s + 1)$$

subject to the following bank capital accumulation function:

$$K_{i,b}^{B}(s + 1) = (1 - \delta_{i,b}^{B}(s)) K_{i,b}^{B}(s) + B_{i,b}^{B}(s) + H^{B}(I_{i,b}^{B}(s), I_{i,b}^{B}(s - 1))$$

and also subject to terminal nonnegativity constraints that $B_{i,b}^{D,B}(s + 1) \geq 0$, $B_{i,b}^{R,B}(s + 1) \geq 0$ and $K_{i,b}^{R,B}(s + 1) \geq 0$ for $s \to \infty$.

Here, $\Pi_{i,b}^{C}(t)$ and $V_{i,b}^{C}(t)$ are the dividend payment and price per specific share of domestic intermediate bank; $\lambda_{i,b}^{B}(s)$ is the Lagrange multiplier associated with the dynamic budget constraint for the bank intermediated household in period $s$ in the above Subsection A.

The dividend payment per specific share of domestic intermediate bank $\Pi_{i,b}^{C}(s)$ equals to the total net profits of domestic intermediate bank:
\[ \Pi_{lb}^C(s) = \left[ B_{lb}^{DB}(s + 1) - \left( 1 + i^D_{1}(s - 1) \right) B_{lb}^{DB}(s) \right] + \left[ B_{lb}^{RB}(s + 1) - \left( 1 + i^B_{1}(s - 1) \right) B_{lb}^{RB}(s) \right] \]

Here the total net profits of domestic intermediate bank are defined as the sum of the increase in deposit funding from domestic bank intermediated households \( B_{lb}^{DB}(s + 1) \) net of interest payments at the deposit rate \( i^D_{1}(s - 1) \), and the increase in net interbank market funding from the domestic interbank market \( B_{lb}^{RB}(s + 1) \) net of interest payments at the interbank loans rate \( i^B_{1}(s - 1) \), less the increase in differentiated intermediate mortgage loans \( B_{lb}^{CD,B}(s + 1) \) net of writedowns at mortgage loan default rate (for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate intermediate developers) \( \delta_i^M(s) \) and interest receipts at mortgage loan rate \( i^M_{1,i}(s - 1) \), less the increase in differentiated intermediate corporate loans \( B_{lb}^{CF,B}(s + 1) \) net of writedowns at corporate loan default rate (for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks) \( \delta_i^{CF}(s) \) and interest receipts at corporate loan rate \( i^C_{1,i}(s - 1) \), less the cost of satisfying the regulatory bank capital ratio requirement \( \Phi_{lb}^B(s) \), and less the retained earnings \( I_{lb}^B(s) \).

The sum of differentiated intermediate mortgage loans \( B_{lb}^{CD,B}(s + 1) \) and differentiated intermediate corporate loans \( B_{lb}^{CF,B}(s + 1) \) of all domestic intermediate banks is defined as the aggregate bank assets (also called bank credit stock) of the banking sector of the i-th economy \( B_{i}^{CB}(s + 1) \). The sum of the bank capital stock \( K_{lb}^B(s + 1) \) of all domestic intermediate banks is defined as the aggregate bank capital of the banking sector of the i-th economy \( K_{i}^B(s + 1) \). The bank capital ratio \( \kappa_i(s + 1) \) is defined as the ratio of aggregate bank capital to aggregate bank assets:

\[ \kappa_i(s + 1) = \frac{K_{lb}^B(s+1)}{B_{i}^{CB}(s+1)} \]

The sum of deposit funding from domestic bank intermediated households \( B_{lb}^{DB}(s + 1) \) and net interbank market funding from the domestic interbank market \( B_{lb}^{RB}(s + 1) \) by all domestic intermediate banks is defined as the aggregate bank funding of the i-th economy \( M_{i}^{S}(s + 1) \). Since the sum of net interbank market funding from the domestic interbank market \( B_{lb}^{RB}(s + 1) \) by all domestic intermediate banks equals zero due to clearing of the domestic interbank market, the aggregate bank funding of the i-th economy \( M_{i}^{S}(s + 1) \) equals to the sum of deposit funding from domestic bank intermediated households \( B_{lb}^{DB}(s + 1) \) by all domestic intermediate banks.

The domestic intermediate bank’s cost of satisfying the regulatory bank capital ratio requirement \( \Phi_{lb}^B(s) \) decreases as the ratio of domestic intermediate bank’s capital to its assets \( \frac{K_{lb}^B(s)}{B_{lb}^{CD,B}(s)+B_{lb}^{CF,B}(s)} \) increases:
\[
\Phi_{ib}^B(s) = \mu C \left[ e^{\left(2+\eta C\right)} \left( 1 - \frac{1}{\eta C} \left( \kappa_{ib}^B(s) \left( \delta_{ib}^B(s) + \delta_{ib}^E(s) \right) \right) \right) - 1 \right] K_{ib}^B(s) + F_{ib}^B(s)
\]

Here, 
\( \kappa_{ib}^B(s) \) is the regulatory bank capital ratio requirement; 
\( F_{ib}^B(s) \) is the economy specific fixed cost to insure \( \Phi_{ib}^B(s) \) + \( I_{ib}^B(s) \). 

The overall bank capital destruction rate \( \delta_{ib}^B(s) \) is a weighted average of both the domestic mortgage loan default rate (for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate intermediate developers) \( \delta_{iM}^M(s) \) and the weighted average of domestic and foreign corporate loan default rate (for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks) \( \delta_{iC,E}^C(s) \):

\[
\delta_{ib}^B(s) = \chi C \left[ \omega C \delta_{iM}^M(s) + (1 - \omega C) \delta_{iC,E}^C(s) \right]
\]

\( H_{ib}^B(t) = \lambda_{ib}^B(t+1) \) is the effective retained earnings function, which incorporates convex adjustment costs for the intertemporal change in retained earnings as follows:

\[
H_{ib}^B(t) = \left[ 1 - \frac{\lambda_{ib}^B(t) - 1}{2} \right] I_{ib}^B(s)
\]

In addition, in equilibrium the deposit rate \( i_{ib}^D(t) \) equals to the interbank loans rate \( \delta_{ib}^B(t) \), and the Lagrange multiplier associated with the above-mentioned bank capital accumulation function \( Q_{ib}^B(t) \) has the meaning of shadow price of bank capital:

\[
Q_{ib}^B(t) = E_t \left( 1 + i_{ib}^D(t) \right) \left( 1 - \delta_{ib}^B(t+1) \right) Q_{ib}^B(t+1) - \frac{\partial (\Phi_{ib}^B(t+1) - F_{ib}^B(t+1))}{\partial K_{ib}^B(t+1)} + \left[ E_t \lambda_{ib}^B(t+1) - \right.
\]

In the middle layer, a large number of perfectly competitive domestic final banks not only combine differentiated intermediate mortgage loans supplied by domestic intermediate banks \( B_{ib}^{CD,t+1} \) to produce final mortgage loans \( B_{ib}^{CD,B,t+1} \) that are needed by domestic real estate intermediate developers, according to the following constant elasticity of substitution portfolio aggregator:
\[ B_i^{CD}(t + 1) = \left[ \int_{b=0}^{1} \left( B_i^{CD}(t + 1) \right)^{\theta_i^{CD}(t+1)}_{\theta_i^{CD}(t+1)-1} \right]^{\frac{\theta_i^{CD}(t+1)}{\theta_i^{CD}(t+1)-1}} \]

but also combine differentiated intermediate corporate loans supplied by domestic intermediate banks \( B_i^{CF}(t + 1) \) to produce final corporate loans \( B_i^{CF}(t + 1) \) for domestic and foreign global final banks, according to the following constant elasticity of substitution portfolio aggregator:

\[ B_i^{CF}(t + 1) = \left[ \int_{b=0}^{1} \left( B_i^{CF}(t + 1) \right)^{\theta_i^{CF}(t+1)}_{\theta_i^{CF}(t+1)-1} \right]^{\frac{\theta_i^{CF}(t+1)}{\theta_i^{CF}(t+1)-1}} \]

Here the exponent \( \frac{\theta_i^{CD}(t+1)}{\theta_i^{CD}(t+1)-1} \) has the meaning of the mortgage loan rate markup shock

\[ \theta_i^{CD}(t + 1) = \frac{\theta_i^{CD}(t+1)}{\theta_i^{CD}(t+1)-1} \]

and the exponent \( \frac{\theta_i^{CF}(t+1)}{\theta_i^{CF}(t+1)-1} \) has the meaning of the corporate loan rate markup shock \( \theta_i^{CF}(t + 1) = \frac{\theta_i^{CF}(t+1)}{\theta_i^{CF}(t+1)-1} \).

“Perfectly competitive” means in equilibrium the domestic final banks of the i-th economy not only maximize profits derived from intermediation of the final mortgage loan \( B_i^{CD}(t + 1) \) with respect to inputs of intermediate mortgage loans \( B_i^{CD}(t + 1) \), implying the following mortgage loans demand functions:

\[ B_i^{CD}(t + 1) = \left( \frac{1 + i_i^{M}(t)}{1 + i_i^{M}(t)} \right)^{-\theta_i^{CD}(t+1)} B_i^{CD}(t + 1) \]

but also maximize profits derived from intermediation of the economy specific local currency denominated final corporate loan \( B_i^{CF}(t + 1) \) with respect to inputs of intermediate corporate loans \( B_i^{CF}(t + 1) \), implying the following corporate loans demand functions:

\[ B_i^{CF}(t + 1) = \left( \frac{1 + i_i^{C}(t)}{1 + i_i^{C}(t)} \right)^{-\theta_i^{CF}(t+1)} B_i^{CF}(t + 1) \]

Since in equilibrium the domestic final banks of the i-th economy generate zero profits, then not only the nominal mortgage loan rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate intermediate developers \( i_i^{M}(t) \) can be expressed as the integral of the nominal mortgage loan rate for the domestic intermediate mortgage loans \( i_i^{M}(t) \):

\[ 1 + i_i^{M}(t) = \left[ \int_{b=0}^{1} \left( 1 + i_i^{M}(t) \right)^{1-\theta_i^{CD}(t+1)} \right]^{\frac{1}{1-\theta_i^{CD}(t+1)}} \]
but also the nominal corporate loan rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks $i_{i,b}^C(t)$ can be expressed as the integral of the nominal corporate loan rate for the domestic intermediate corporate loans $i_{i,b}^C(t)$:

$$1 + i_{i,b}^C(t) = \left[ \int_{b=0}^{1} \left( 1 + i_{i,b}^C(t) \right)^{1-\theta_i^C(t+1)} \, db \right]^{1-\theta_i^C(t+1)}$$

In the upper layer, a large number of perfectly competitive global final banks combine economy specific local currency denominated final corporate loans from the banking sectors of all economies $\{B_{ij}^{CF}(t)\}_{j=1}^N$ to produce domestic currency denominated final corporate loans $B_{i}^{CF}(t)$ that are needed by domestic industry specific intermediate output good firms, according to the following fixed proportions portfolio aggregator:

$$B_{i}^{CF}(t) = \min \left( \frac{E_{i}(t)B_{i}^{CF}(t)}{\phi_{i,j}^F} \right)_{j=1}^N$$

Here the economy specific local currency denominated final corporate loans from the banking sectors of the j-th economy to global final banks of the i-th economy $B_{ij}^{CF}(t)$ satisfy the following equation:

$$B_{ij}^{CF}(t) = \sum_{i=1}^N B_{i}^{CF}(t)$$

“Perfectly competitive” means in equilibrium the global final banks of the i-th economy maximize profits derived from intermediation of the domestic currency denominated final corporate loan $B_{i}^{CF}(t)$ with respect to inputs of economy specific local currency denominated final corporate loans $\{B_{ij}^{CF}(t)\}_{j=1}^N$, implying the following corporate loans demand functions:

$$B_{ij}^{CF}(t) = \phi_{i,j}^F \frac{B_{i}^{CF}(t)}{E_{i}(t)}$$

Since in equilibrium the global final banks of the i-th economy generate zero profits, then the aggregate effective corporate loan rate index (or the weighted average of domestic and foreign nominal corporate loan rate) for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy $i_{i}^{CE}(t)$ can be expressed as the weighted average of the nominal corporate loan rate for economy specific local currency denominated final corporate loans provided by domestic final banks of any specific economy to both domestic and foreign global final banks $i_{i}^C(t−1)$, adjusted by the weighted average of the intertemporal change in the nominal bilateral exchange rate of the currency issued by the i-th economy to the currency issued by any specific economy $\frac{E_{i}(t)}{E_{i}(t−1)}$.

$$1 + i_{i}^{CE}(t) = \sum_{j=1}^N \phi_{i,j}^F \left( 1 + i_{i}^C(t−1) \right) \frac{E_{i}(t)}{E_{i}(t−1)}$$
In each period $t$, a randomly selected fixed fraction $\omega^C$ of domestic intermediate banks do not adjust their mortgage loan rate $i_{1,i}^{M}(t)$ and corporate loan rate $i_{1,i}^{C}(t)$:

\[
\begin{align*}
1 + i_{1,i}^{M}(t) &= 1 + i_{1,i}^{M}(t-1) \\
1 + i_{1,i}^{C}(t) &= 1 + i_{1,i}^{C}(t-1)
\end{align*}
\]

The remaining fraction $(1 - \omega^C)$ of domestic intermediate banks adjust their mortgage loan rate $i_{1,i}^{M}(t)$ and corporate loan rate $i_{1,i}^{C}(t)$ optimally, in equilibrium they all choose a common mortgage loan rate $i_{1,i}^{M,*}(t)$ and a common corporate loan rate $i_{1,i}^{C,*}(t)$, given by the following two necessary first order conditions:

\[
\begin{align*}
\frac{1 + i_{1,i}^{M}(t)}{1 + i_{1,i}^{M}(t-1)} &= E_t \sum_{s=t}^{\infty} (\omega^C)^{s-t} \beta^s \frac{B_i^D(s)}{\lambda_i(t)} \left(1 + i_{1,i}^{M}(s-1)\right) \left(\frac{\partial \Phi_i^B(s)}{\partial i_{1,i}^{M}(s)}\right) \left(\frac{\partial \Phi_i^{DB}(s)}{\partial i_{1,i}^{M}(s)}\right) \left(1 + i_{1,i}^{M}(s)\right) \left(\frac{\partial \Phi_i^D(s)}{\partial i_{1,i}^{M}(s)}\right) - \left(1 + i_{1,i}^{M}(s-1)\right) B_i^D(s) \\
\frac{1 + i_{1,i}^{C}(t)}{1 + i_{1,i}^{C}(t-1)} &= E_t \sum_{s=t}^{\infty} (\omega^C)^{s-t} \beta^s \frac{B_i^F(s)}{\lambda_i(t)} \left(1 + i_{1,i}^{C}(s-1)\right) \left(\frac{\partial \Phi_i^B(s)}{\partial i_{1,i}^{C}(s)}\right) \left(\frac{\partial \Phi_i^{FB}(s)}{\partial i_{1,i}^{C}(s)}\right) \left(1 + i_{1,i}^{C}(s)\right) \left(\frac{\partial \Phi_i^F(s)}{\partial i_{1,i}^{C}(s)}\right) - \left(1 + i_{1,i}^{C}(s-1)\right) B_i^F(s) \\
\end{align*}
\]

Under this financial friction, domestic intermediate banks infrequently adjust their nominal mortgage and corporate loan rates, mimicking the effect of maturity transformation on the spreads between the loan and deposit rates. Then not only the aggregate nominal mortgage loan rate index (or the nominal mortgage loan rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the $i$-th economy to domestic real estate intermediate developers) $i_{1,i}^{M}(t)$ equals an average of the nominal mortgage loan rate set by the randomly selected fixed fraction $(1 - \omega^C)$ of domestic intermediate banks that adjust their mortgage loan rate optimally, and the average of the nominal mortgage loan rates set by the remaining fraction $\omega^C$ of domestic intermediate banks that do not adjust their mortgage loan rates:

\[
1 + i_{1,i}^{M}(t) = \left(1 - \omega^C\right) \left(1 + i_{1,i}^{M,*}(t)\right)^{1-\delta_{t+1}^D} + \omega^C \left(1 + i_{1,i}^{M}(t-1)\right)^{1-\delta_{t+1}^D} + \omega^C \left(1 + i_{1,i}^{M}(t-1)\right)
\]

but also the aggregate nominal corporate loan rate index (or the nominal corporate loan rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the $i$-th economy to both domestic and foreign global final banks) $i_{1,i}^{C}(t)$ equals an average of the nominal corporate loan rate set by the randomly selected fixed fraction $(1 - \omega^C)$ of domestic intermediate banks that adjust their corporate loan rate
optimally, and the average of the nominal corporate loan rates set by the remaining fraction \( \omega_C \) of domestic intermediate banks that do not adjust their corporate loan rates:

\[
1 + i_C(t) = \left(1 - \omega_C \right) \left(1 + i_C^*(t) \right)^{1 - \theta_i^F(t+1)} + \omega_C \left(1 + i_C(t-1) \right)^{1 - \theta_i^F(t+1)}
\]

All the above intertemporal adjustment mechanism of mortgage and corporate loan rates cause **nominal mortgage loan rate rigidity** and **nominal corporate loan rate rigidity** in the lending market.

The mortgage loan default rate applicable to borrowing by domestic real estate intermediate developers from domestic final banks \( \delta_i^M(t) \) satisfies the following mortgage loan default rate relationship.

The linear deviation of the domestic mortgage loan default rate \( \delta_i^M(t) \) depends on a weighted average of its past value \( \delta_i^M(t-1) \) and its attractor value

\[
\left[\xi_{\delta_i^M,Y} \left(\ln Y_i(t) - \ln \tilde{Y}_i(t)\right) + \xi_{\delta_i^M,V} \left(\ln \tilde{V}_i^H(t) - \ln \tilde{V}_i^S(t-1)\right)\right],
\]

and is also influenced by the linear deviation of the contemporaneous mortgage loan default shock \( \nu_i^\delta(t) \), while the linear deviation of the attractor mortgage loan default rate depends on the logarithmic deviation of the contemporaneous output gap \( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \), as well as the intertemporal change in the logarithmic deviation of the price of housing \( \ln \tilde{Y}_i^H(t) - \ln \tilde{Y}_i^H(t-1) \), according to the following mortgage loan default rate relationship:

\[
\delta_i^M(t) = \rho_\delta \delta_i^M(t-1) - (1 - \rho_\delta) \left[\xi_{\delta_i^M,Y} \left(\ln Y_i(t) - \ln \tilde{Y}_i(t)\right) + \xi_{\delta_i^M,V} \left(\ln \tilde{V}_i^H(t) - \ln \tilde{V}_i^H(t-1)\right)\right] + \phi_i^\delta(t)
\]

The corporate loan default rate applicable to borrowing by domestic industry specific intermediate output good firms from domestic global final banks \( \delta_i^C(t) \) satisfies the following corporate loan default rate relationship.

The linear deviation of the domestic corporate loan default rate \( \delta_i^C(t) \) depends on global final banks of the i-th economy to firms of the i-th economy \( \delta_i^C(t) \) depends on a weighted average of its past value \( \delta_i^C(t-1) \) and its attractor value

\[
\left[\xi_{\delta_i^C,Y} \left(\ln Y_i(t) - \ln \tilde{Y}_i(t)\right) + \xi_{\delta_i^C,V} \left(\ln \tilde{V}_i^S(t) - \ln \tilde{V}_i^S(t-1)\right)\right],
\]

and is also influenced by the linear deviation of the contemporaneous corporate loan default shock \( \nu_i^\delta(t) \), while the linear deviation of the attractor corporate loan default rate depends on the logarithmic deviation of the contemporaneous output gap \( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \), as well as the intertemporal change in the logarithmic deviation of the price of corporate equity \( \ln \tilde{V}_i^S(t) - \ln \tilde{V}_i^S(t-1) \), according to the following corporate loan default rate relationship:

\[
\delta_i^C(t) = \rho_\delta \delta_i^C(t-1) - (1 - \rho_\delta) \left[\xi_{\delta_i^C,Y} \left(\ln Y_i(t) - \ln \tilde{Y}_i(t)\right) + \xi_{\delta_i^C,V} \left(\ln \tilde{V}_i^S(t) - \ln \tilde{V}_i^S(t-1)\right)\right] + \phi_i^\delta(t)
\]
In fact, the above \( \delta^C_1(t) \) can be further expressed as the industry-level output weighted average of the linear deviation of the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy \( \sum_{k=1}^{45} \text{Contribution}_{Y_{i,k}} \delta^C_{i,k}(t) \):

\[
\delta^C_1(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_{i,k}} \delta^C_{i,k}(t)
\]

Here \( \text{Contribution}_{Y_{i,k}} \) represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

And the industry-level corporate loan default rate \( \delta^C_{i,k}(t) \) has an expression with the same structure as the above whole-economy-level corporate loan default rate \( \delta^C_1(t) \):

\[
\delta^C_{i,k}(t) = \rho \delta^C_{i,k}(t - 1) - (1 - \rho) \left[ \delta^C_{i,Y}(t) \ln \bar{Y}_{i,k}(t) - \ln \bar{S}_{i,k}(t) \right] + \delta^C_{V,\bar{V}}(t - 1) \delta^C_{i,k}(t)
\]

Based on \( \delta^C_1(t) \) defined above, the linear deviation of the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks \( \delta^C_{i,E}(t) \) can be expressed as the nonfinancial corporate lending weighted average of \( \delta^C_1(t) \) for the domestic currency denominated final corporate loans issued by global final banks of any specific economy to firms of that specific economy, according to the following corporate credit loss rate function:

\[
\delta^C_{i,E}(t) = \sum_{j=1}^{N} \omega_{i,j} \delta^C_{j}(t)
\]

F. Foreign Exchange Sector

The foreign exchange sector can help form the nominal bilateral exchange rate \( E_{i,1}(t) \), the real bilateral exchange rate \( \Omega_{i,j}(t) \), the nominal effective exchange rate \( E_i(t) \), and the real effective exchange rate \( \Omega_i(t) \).

The formation of the nominal bilateral exchange rate of the currency issued by the i-th economy to US dollar (which is the quotation currency for transactions in the foreign exchange market issued by the 1-st economy ('United States')) \( E_{i,1}(t) \) under different exchange rate and inflation targeting arrangements of different countries are different.

Under a free floating exchange rate and flexible inflation targeting arrangement or under a managed exchange rate arrangement, the nominal bilateral exchange rate of the currency issued by the i-th economy to US dollar \( E_{i,1}(t) \) depends on its expected future value \( E_iE_{i,1}(t + 1) \), driven by the nominal short term bond yield differential between the i-th economy and the 1-st economy \( \delta^S(t) - \delta^V(t) \), and is also influenced by the currency risk premium shock differential between the i-th economy and the 1-st economy \( \ln \bar{V}^S(t) - \ln \bar{V}^F(t) \), according to the following foreign exchange market relationship:

\[
\ln E_{i,1}(t) = E_i \ln E_{i,1}(t + 1) - \left[ \left( \delta^S(t) - \ln \bar{V}^S(t) \right) - \left( \delta^V(t) - \ln \bar{V}^F(t) \right) \right]
\]
Under a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union), if the $i^*$-th economy issues the anchor currency for the currency of the $i$-th economy, the nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $E_{i,1}(t)$ equals the nominal bilateral exchange rate of the currency issued by the $i^*$-th economy to US dollar $E_{i^*,1}(t)$, according to the following foreign exchange market relationship:

$$\ln E_{i,1}(t) = \ln E_{i^*,1}(t)$$

While the nominal bilateral exchange rate of the currency issued by the 1-st economy United States to US dollar $E_{1,1}(t)$ should always be equal to 1 (or its logarithm $\ln E_{1,1}(t)$ be equal to 0):

$$\ln E_{1,1}(t) = 0$$

The nominal bilateral exchange rate of the currency issued by the $i$-th economy to the currency issued by the $j$-th economy (here $j \neq 1$) $E_{ij}(t)$ equals to the ratio of the nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $E_{i,1}(t)$ to the nominal bilateral exchange rate of the currency issued by the $j$-th economy to US dollar $E_{j,1}(t)$:

$$E_{ij}(t) = \frac{E_{i,1}(t)}{E_{j,1}(t)}$$

The real bilateral exchange rate of the currency issued by the $i$-th economy relative to the currency issued by the $j$-th economy (which is the relative price of foreign output of the $j$-th economy in terms of domestic output of the $i$-th economy) $\Omega_{ij}(t)$ depends on the nominal bilateral exchange rate of the currency issued by the $i$-th economy to the currency issued by the $j$-th economy $E_{ij}(t)$, adjusted by the ratio of the output price level of the $j$-th economy to the output price level of the $i$-th economy $p^*_j(t)/p^*_i(t)$:

$$\Omega_{ij}(t) = E_{ij}(t) \frac{p^*_j(t)}{p^*_i(t)}$$

The nominal effective exchange rate of the currency issued by the $i$-th economy $E_i(t)$ is defined as the trade weighted average of the nominal bilateral exchange rate of the currency issued by the $i$-th economy to the currency issued by any specific economy $E_{ij}(t)$:

$$E_i(t) = \prod_{j=1}^N (E_{ij}(t))^{\omega_{ij}}$$

The real effective exchange rate of the currency issued by the $i$-th economy $\Omega_i(t)$ is defined as the trade weighted average of the real bilateral exchange rate of the currency issued by the $i$-th economy relative to the currency issued by any specific economy $\Omega_{ij}(t)$:

$$\Omega_i(t) = \prod_{j=1}^N (\Omega_{ij}(t))^{\omega_{ij}}$$
G. Export Sector

The structurally more complex export sector is innovatively constructed based on the structurally simple export sector of the original GFM introduced in Vitek (2015, 2018), which further incorporates the intra-industry and inter-industry structure, and fully characterizes the inter-country cross-industry input-output relations provided by OECD ICIO tables. Among all sectors it is the third most complex sector that has its layer structure corresponding to 45 industries of the structurally more complex production sector.

The structurally more complex export sector consists of two layers of industry specific agents. Variable subscript “n” is for industry specific intermediate export good firms in the lower layer.

In the lower layer, continuums of monopolistically competitive industry specific intermediate export good firms choose state contingent sequences in the current period s = t and all future periods s = t + 1, t + 2, ... for industry specific differentiated intermediate export goods or services $X_{i,k,n}(s)$, in order to maximize the following pre-dividend stock market value:

$$\Pi_{i,k,n}^X(t) + V_{i,k,n}^X(t) = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_t^A(s)}{\lambda_t(t)} \Pi_{i,k,n}^X(s)$$

Here, $\Pi_{i,k,n}^X(t)$ and $V_{i,k,n}^X(t)$ are the dividend payment and price per specific share of domestic industry specific intermediate export good firm; $\lambda_t^A(s)$ is the Lagrange multiplier associated with the dynamic budget constraint for the capital market intermediated household in period s in the above Subsection A.

For industry specific intermediate export goods or services $X_{i,k,n}(s)$ that come from different types of industries of the structurally more complex production sector, the dividend payment per specific share of domestic industry specific intermediate export good firm $\Pi_{i,k,n}^X(s)$ is defined differently as follows.

For industry specific intermediate export goods or services from the domestic industry that produces internationally homogeneous energy or nonenergy commodities to be taken as intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector, $\Pi_{i,k,n}^X(s)$ is defined as the revenue derived from sales of intermediate export goods or services (here they’re internationally homogeneous energy or nonenergy commodities) $X_{i,k,n}(s)$ at the export price for industry specific intermediate export goods or services $P_{i,k,n}^X(s)$, less the expenditure on purchasing internationally homogeneous energy or nonenergy commodities either from the corresponding industry of the domestic production sector or from the international commodity market, and either purchase is at the domestic currency denominated uniform price of internationally homogeneous energy or nonenergy commodities $E_{i,1}(s)P_{eY}(s)$ or $E_{i,1}(s)P_{neY}(s)$ (the uniform price $P_{eY}(s)$ or $P_{neY}(s)$ is denominated in US dollars in the international commodity market) due to the law of one price, and less the industry specific fixed cost $F_{i,k}(s)$ that is adopted to ensure that $\Pi_{i,k,n}^X(s) = \int_{n=0}^{1} \Pi_{i,k,n}^X(s)dn = 0$:

$$\Pi_{i,k,n}^X(s) = P_{i,k,n}^X(s)X_{i,k,n}(s) - E_{i,1}(s)(P_{eY}(s) or P_{neY}(s))X_{i,k,n}(s) - F_{i,k}(s)$$
For industry specific intermediate export goods or services from the domestic industry that produces internationally heterogeneous goods or services to be taken as intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector, $\Pi_{ik,n}^X(s)$ is defined as the revenue derived from sales of intermediate export goods or services (here they’re internationally heterogeneous goods or services) $X_{ik,n}(s)$ at the export price for industry specific intermediate export goods or services $P_{ik,n}^X(s)$, less the expenditure on purchasing internationally heterogeneous goods or services from the corresponding industry of the domestic production sector at the industry specific output price for industry specific final output goods or services $P_{ik}^Y(s)$, and less the industry specific fixed cost $F_{ik}^X(s)$ that is adopted to ensure that $\Pi_{ik,n}^X(s) = \int_{n=0}^{1} \Pi_{ik,n}^X(s) dn = 0$:

$$\Pi_{ik,n}^X(s) = P_{ik,n}^X(s)X_{ik,n}(s) - P_{ik}^Y(s)X_{ik}(s) - F_{ik}^X(s)$$

In the upper layer, a large number of perfectly competitive industry specific final export good firms combine industry specific differentiated intermediate export goods or services supplied by domestic industry specific intermediate export good firms $X_{ik,n}(t)$ to produce industry specific final export goods or services $X_{ik}(t)$, which are needed by economy specific and industry specific intermediate import good firms of other economies, according to the following constant elasticity of substitution production function:

$$X_{ik}(t) = \left[ \int_{n=0}^{1} \left( X_{ik,n}(t) \right)^{\theta_i^X(t)-1} \theta_i^X(t)^{\frac{1}{1-\theta_i^X(t)}} dn \right]^{\frac{1}{\theta_i^X(t)-1}}$$

Here the exponent $\frac{\theta_i^X(t)}{\theta_i^X(t)-1}$ has the meaning of the export price markup shock $\psi_i^X(t) = \frac{\theta_i^X(t)}{\theta_i^X(t)-1}$.

“Perfectly competitive” means in equilibrium the industry specific final export good firms maximize profits derived from production of the industry specific final export goods or services $X_{ik}(t)$ with respect to inputs of industry specific differentiated intermediate export goods or services $X_{ik,n}(t)$, implying the following export goods or services demand functions:

$$X_{ik,n}(t) = \left( \frac{P_{ik,n}^X(t)}{P_{ik}^X(t)} \right)^{-\theta_i^X(t)} X_{ik}(t)$$

Since in equilibrium the industry specific final export good firms generate zero profits, then the industry specific aggregate export price index $P_{ik}^X(t)$ can be expressed as the integral of the export price for industry specific intermediate export goods or services $P_{ik,n}^X(t)$:

$$P_{ik}^X(t) = \left[ \int_{n=0}^{1} \left( P_{ik,n}^X(t) \right)^{1-\theta_i^X(t)} dn \right]^{\frac{1}{1-\theta_i^X(t)}}$$

Corresponding to different industries of the structurally more complex production sector, the formation mechanism of the industry specific export price $P_{ik}^X(t)$ is different.

Specifically speaking, the original GFM introduced in Vitek (2015, 2018) sets the export price of the industry specific final export goods or services from the domestic industry which
produces internationally homogeneous energy or nonenergy commodities (that would become intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector) \( P_{i,k}^X(t) \) equal to the domestic currency denominated uniform price of internationally homogeneous energy or nonenergy commodities \( E_{i,1}(t)P_e^Y(t) \) or \( E_{i,1}(t)P_{ne}^Y(t) \) (the uniform price \( P_e^Y(s) \) or \( P_{ne}^Y(s) \) is denominated in US dollars in the international commodity market):

\[
P_{i,k}^X(t) = E_{i,1}(t)(P_e^Y(t) \text{ or } P_{ne}^Y(t))
\]

Here the uniform price of internationally homogeneous energy or nonenergy commodities \( P_e^Y(t) \) or \( P_{ne}^Y(t) \) (denominated in US dollars) is defined as the world output weighted average of the US dollar denominated economy specific output price of internationally homogeneous energy or nonenergy commodities from any specific economy \( \frac{P_j^Y(t)}{E_{j,1}(t)} \) or \( \frac{P_j^{neY}(t)}{E_{j,1}(t)} \):

\[
P_e^Y(t) = \sum_{j=1}^{N} \omega_j \frac{P_j^Y(t)}{E_{j,1}(t)}
\]

\[
P_{ne}^Y(t) = \sum_{j=1}^{N} \omega_j \frac{P_j^{neY}(t)}{E_{j,1}(t)}
\]

The original GFM introduced in Vitek (2015, 2018) sets the export price of the industry specific final export goods or services from the domestic industry which produces internationally heterogeneous goods or services (that would become intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector) \( P_{i,k}^X(t) \) equal to the domestic currency denominated output price of industry specific final output goods or services from that domestic industry:

\[
P_{i,k}^X(t) = P_{i,k}^Y(t)
\]

Since we want to further include the transmission mechanism between upstream and downstream quantities and prices in the global supply chain in the AGMFM, the export quantity and price of the industry specific final export goods or services should also be modified accordingly, after taking the upstream and downstream quantity and price transmission mechanism similar to that for the output quantity and price of the industry specific final output goods or services introduced in the previous Subsection D into consideration. As these complex settings lead to the expression of industry-level export quantity and price being too lengthy and complex, we will only present the derived linearized relationship equations in Section IV for further introduction.

In addition, over the above-mentioned two layers of industry specific agents, a large number of perfectly competitive final export good firms that exist in the GFM introduced in Vitek (2015, 2018) are still retained in this new model, in order to facilitate the macroeconomic activities and maintain the macroeconomic relations between this export sector and other sectors or markets that do not distinguish between industries.

In this additionally added layer, a large number of perfectly competitive final export good firms combine industry specific final export goods or services from the industry specific final export good firms \( \{X_{i,k}(t)\}_{k=1}^{45} \) to produce final export goods or services \( X_i(t) \) that do not distinguish between industries, according to the following fixed proportions production function:
\[ X_i(t) = \min \left\{ \frac{X_{ik}(t)}{\phi_{ik}^X(1)} \right\}_{k=1}^{45} \]

“Perfectly competitive” means in equilibrium the final export good firms maximize profits derived from production of the final export goods or services \( X_i(t) \) with respect to inputs of industry specific final export goods or services \( \{X_{ik}(t)\}_{k=1}^{45} \), implying the following export goods or services demand functions:

\[ X_{ik}(t) = \phi_{ik}^X X_i(t) \]

Since in equilibrium the final export good firms generate zero profits, then the aggregate export price index for the final export goods or services \( P_1^X(t) \) can be expressed as the weighted average of the export price of the industry specific final export goods or services \( P_{ik}^X(t) \):

\[ P_1^X(t) = \sum_{k=1}^{45} \phi_{ik}^X P_{ik}^X(t) \]

In each period \( t \), a randomly selected fixed fraction \( \omega^X \) of industry specific intermediate export good firms adjust their export price to account for the past industry specific export price inflation \( \frac{P_{ik}(t-1)}{P_{ik}(t-2)} \), for those that produce industry specific intermediate export goods or services coming from the domestic industry that provides internationally homogeneous energy or nonenergy commodities, they additionally need to adjust their export price to account for the contemporaneous change in the domestic currency denominated uniform price of internationally homogeneous energy or nonenergy commodities \( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \) or \( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \)

\[ \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \]

according to the following partial indexation rule:

\[ P_{ik}^X(t) = \left( \frac{P_{ik}(t-1)}{P_{ik}(t-2)} \right)^{1-\mu^X} \left( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \right)^{1-\gamma^X} P_{ik}^X(t-1) \]

\[ P_{ik}^X(t) = \left( \frac{P_{ik}(t-1)}{P_{ik}(t-2)} \right)^{1-\mu^X} \left( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \right)^{1-\gamma^X} P_{ik}^X(t-1) \]

Here \( \frac{P_{ik}(t-1)}{P_{ik}(t-2)} \) and \( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \) are the steady state equilibrium values of \( \frac{P_{ik}(t-1)}{P_{ik}(t-2)} \) and \( \frac{E_{i1}(t)P_{ik}(t)}{E_{i1}(t-1)P_{ik}(t-1)} \). Correspondingly, the remaining fraction \( (1-\omega^X) \) of industry specific intermediate export good firms adjust their export price optimally, in equilibrium they all choose a common industry specific export price \( \phi_{ik}^X(1) \) given by the following necessary first order condition:
Under this specification, the probability that an industry specific intermediate export good firm has adjusted its price optimally is time dependent but state independent. Then the aggregate industry specific export price index $P^X_{ik}(t)$ equals an average of the price set by the randomly selected fixed fraction $(1 - \omega^X)$ of industry specific intermediate export good firms that adjust their export price optimally, and the average of the export prices set by the remaining fraction $\omega^X$ of industry specific intermediate export good firms that adjust their export prices according to the above partial indexation rule:

$$P^X_{ik}(t) = \left\{ \begin{array}{ll} (1 - \omega^X) \left( P^X_{ik}(t) \right)^{1 - \theta_i^X} & + \omega^X \left[ (P^X_{ik}(t-1))^{1 - \theta_i^X} \right] \end{array} \right\}$$

$$P^X_{ik}(t) = \left\{ \begin{array}{ll} (1 - \omega^X) \left( P^X_{ik}(t) \right)^{1 - \theta_i^X} & + \omega^X \left[ (P^X_{ik}(t-1))^{1 - \theta_i^X} \right] \end{array} \right\}$$

$$\omega^X \left\{ \begin{array}{ll} (P^X_{ik}(t-1))^{1 - \mu^X} \left( E_{i1}(t)P^Y_{ik}(t-1) \right)^{\mu^X} & + \omega^X \left[ (P^X_{ik}(t-1))^{1 - \mu^X} \left( E_{i1}(t)P^Y_{ik}(t-1) \right)^{\mu^X} \right] \end{array} \right\}$$

$$P^X_{ik}(t) = \left\{ \begin{array}{ll} (1 - \omega^X) \left( P^X_{ik}(t) \right)^{1 - \theta_i^X} & + \omega^X \left[ (P^X_{ik}(t-1))^{1 - \theta_i^X} \right] \end{array} \right\}$$

$$\omega^X \left\{ \begin{array}{ll} (P^X_{ik}(t-1))^{1 - \mu^X} \left( E_{i1}(t)P^Y_{ik}(t-1) \right)^{\mu^X} & + \omega^X \left[ (P^X_{ik}(t-1))^{1 - \mu^X} \left( E_{i1}(t)P^Y_{ik}(t-1) \right)^{\mu^X} \right] \end{array} \right\}$$

All the above intertemporal adjustment mechanism of export prices cause industry specific nominal export price rigidity in the industry specific export market.
H. Import Sector

The structurally more complex import sector is innovatively constructed based on the structurally simple import sector of the GFM introduced in Vitek (2015, 2018), which also further incorporates the intra-industry and inter-industry structure, and characterizes the inter-country cross-industry input-output relations provided by OECD ICIO tables as much as possible. It is the second most complex sector among all sectors in this model, which also has its layer structure corresponding to 45 industries of the structurally more complex production sector.

The structurally more complex import sector consists of three layers of industry specific agents. Variable subscript “n” is still adopted for economy specific and industry specific intermediate import good firms in the lower layer.

Since we further incorporate the structure of import tariff into the AGMFM relative to the GFM introduced in Vitek (2015, 2018), all import prices in this sector should distinguish between the pre-tariff import price and the post-tariff import price. For simplicity, the following price variable with the superscript M,T is the import price of pre-tariff type, while price variable with the superscript M is the import price of post-tariff type, and there is the following relationship between the post-tariff import price and the corresponding pre-tariff import price:

\[
P_{l,k,j,n}^M(t) = (1 + \tau_{l,k}^M(t))P_{l,k,j,n}^{M,T}(t)
\]

Here \( \tau_{l,k}^M(t) \) is the corresponding import tariff rate for the i-th economy’s imports from foreign k-th industry in period t.

In the lower layer, continuums of monopolistically competitive economy specific and industry specific intermediate import good firms choose state contingent sequences in the current period \( s = t \) and all future periods \( s = t + 1, t + 2, ... \) for economy specific and industry specific differentiated intermediate import goods or services \( M_{l,k,j,n}(s) \), in order to maximize the following pre-dividend stock market value:

\[
\Pi_{l,k,j,n}^M(t) + v_{l,k,j,n}^M(t) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \lambda^A_i(s) \Pi_{l,k,j,n}^M(s)
\]

Here, \( \Pi_{l,k,j,n}^M(t) \) and \( v_{l,k,j,n}^M(t) \) are the dividend payment and price per specific share of domestic economy specific and industry specific intermediate import good firm; \( \lambda^A_i(s) \) is the Lagrange multiplier associated with the dynamic budget constraint for the capital market intermediated household in period s in the above Subsection A.

The dividend payment per specific share of domestic economy specific and industry specific intermediate import good firm \( \Pi_{l,k,j,n}^M(s) \) equals to the total net profits of domestic economy specific and industry specific intermediate import good firm:

\[
\Pi_{l,k,j,n}^M(s) = P_{l,k,j,n}^{M,T}(s)M_{l,k,j,n}(s) - E_{i,l}(s)P_{l,k,j,n}(s)M_{l,k,j,n}(s) - F_{l,k,j,n}(s)
\]

Here the total net profits of domestic economy specific and industry specific intermediate import good firm is defined as the revenue derived from sales of intermediate import goods or services \( M_{l,k,j,n}(s) \) at the pre-tariff import price for economy specific and industry specific
intermediate import goods or services \( p_{i,j,k,n}(s) \), less the expenditure on purchasing industry specific final export goods or services from the export sector of the j-th economy at the i-th economy’s currency denominated industry specific export price of the j-th economy \( E_{ij}(s)p_{j,k}(s) \), and less the economy specific and industry specific fixed cost \( f_{i,j,k}^M(s) \) that is adopted to ensure that \( \Pi_{i,k,j}^M(s) = \int_0^1 \Pi_{i,k,j}^M(s)dn = 0 \).

In the middle layer, a large number of perfectly competitive economy specific and industry specific final import good firms combine economy specific and industry specific differentiated intermediate import goods or services supplied by domestic economy specific and industry specific intermediate import good firms \( M_{i,k,j,n}(t) \) to produce economy specific and industry specific final import goods or services \( M_{i,j,k}(t) \), which are needed by domestic industry specific final import good firms, according to the following constant elasticity of substitution production function:

\[
M_{i,j,k}(t) = \left[ \int_{n=0}^1 \left( M_{i,k,j,n}(t) \right)^{\theta_i^M(t)-1} dn \right]^{\theta_i^M(t)}
\]

Here the exponent \( \frac{\theta_i^M(t)}{\theta_i^M(t)-1} \) has the meaning of the pre-tariff import price markup shock \( \theta_i^M(t) = \frac{\theta_i^M(t)}{\theta_i^M(t)-1} \).

Since the industry specific final export goods or services from the k-th industry of the j-th economy \( X_{j,k}(t) \) equals to the sum of the economy specific and industry specific final import goods or services from the k-th industry of the j-th economy to the import sector of all the other economies except the j-th economy, the economy specific and industry specific final import goods or services \( \{ M_{i,k,j}(t) \}_{i=1}^N \) (here \( M_{i,k,j}(t) \) equals 0 by default) satisfy the following equation:

\[
X_{j,k}(t) = \sum_{i=1}^N M_{i,k,j}(t)
\]

“Perfectly competitive” means in equilibrium the economy specific and industry specific final import good firms maximize profits derived from production of the economy specific and industry specific final import goods or services \( M_{i,k,j}(t) \) with respect to inputs of economy specific and industry specific differentiated intermediate import goods or services \( M_{i,k,j,n}(t) \), implying the following import goods or services demand functions:

\[
M_{i,k,j,n}(t) = \left( \frac{p_{i,k,j,n}(t)}{p_{i,k,j}^M(t)} \right)^{-\theta_i^M(t)} M_{i,k,j}(t)
\]

Since in equilibrium the economy specific and industry specific final import good firms generate zero profits, then the economy specific and industry specific aggregate pre-tariff import price index \( p_{i,k,j}^M(t) \) can be expressed as the integral of the pre-tariff import price of economy specific and industry specific intermediate import goods or services \( p_{i,k,j,n}(t) \):

\[
p_{i,k,j}^M(t) = \int_0^1 \Pi_{i,k,j,n}(s)dn
\]
In the upper layer, a large number of perfectly competitive industry specific final import good firms combine economy specific and industry specific final import goods or services supplied by domestic economy specific and industry specific final import good firms \( \{M_{i,k,(t)}\}_{j=1}^{N} \) to produce industry specific final import goods or services \( M_{i,k,(t)} \), which are either needed by domestic industry specific intermediate output good firms in various industries to serve as intermediate inputs for production, or needed by domestic industry specific final absorption good firms in the additionally added virtual “absorption sector”, according to the following fixed proportions production function:

\[
M_{i,k,(t)} = \min \left\{ \nu_{j,k}^{X}(t) \cdot \frac{M_{i,k,(t)}}{\phi_{i,k}(t)} \right\}_{j=1}^{N}
\]

Here \( \nu_{j,k}^{X}(t) \) is the export demand shock for industry specific final export goods or services from the k-th industry of the j-th economy.

“Perfectly competitive” means in equilibrium the industry specific final import good firms maximize profits derived from production of the industry specific final import goods or services \( M_{i,k,(t)} \) with respect to inputs of economy specific and industry specific final import goods or services \( \{M_{i,k,(t)}\}_{j=1}^{N} \), implying the following import goods or services demand functions:

\[
M_{i,k,(t)}^{j} = \frac{\phi_{i,k}^{M}}{\nu_{j,k}^{X}(t)}
\]

Since in equilibrium the industry specific final import good firms generate zero profits, then the aggregate industry specific pre-tariff import price index for the industry specific final import goods or services \( p_{i,k}^{M,T}(t) \) can be expressed as the weighted average of the pre-tariff import price of the economy specific and industry specific final import goods or services \( p_{i,k}^{M,T}(t) \):

\[
p_{i,k}^{M,T}(t) = \sum_{j=1}^{N} \phi_{i,k}^{M} \cdot \frac{p_{i,k}^{M,T}(t)}{\nu_{j,k}^{X}(t)}
\]

In addition, over the above-mentioned three layers of industry specific agents, a large number of perfectly competitive final import good firms that exist in the GFM introduced in Vitek (2015, 2018) are still retained in this new model, in order to facilitate the macroeconomic activities and maintain the macroeconomic relations between this import sector and other sectors or markets that do not distinguish between industries.

In this additionally added layer, a large number of perfectly competitive final import good firms combine industry specific final import goods or services from the industry specific final import good firms \( \{M_{i,k,(t)}\}_{k=1}^{45} \) to produce final import goods or services \( M_{i}(t) \) that do not distinguish between industries, according to the following fixed proportions production function:
$$M_i(t) = \min \left\{ \frac{M_{i;k}(t)}{\phi_{i;k}^M(t)} \right\}_{k=1}^{45}$$

“Perfectly competitive” means in equilibrium the final import good firms maximize profits derived from production of the final import goods or services $M_i(t)$ with respect to inputs of industry specific final import goods or services $(M_{i;k}(t))_{k=1}^{45}$, implying the following import goods or services demand functions:

$$M_{i;k}(t) = \phi_{i;k}^M(t)M_i(t)$$

Since in equilibrium the final import good firms generate zero profits, then the aggregate pre-tariff import price index for the final import goods or services $P^M_i(T_i)(t)$ can be expressed as the weighted average of the pre-tariff import price of the industry specific final import goods or services $P^M_{i;k}(t)$:

$$P^M_i(T_i)(t) = \sum_{k=1}^{45} \phi_{i;k}^MP^M_{i;k}(t)$$

In each period $t$, a randomly selected fixed fraction $\omega^M$ of economy specific and industry specific intermediate import good firms adjust their pre-tariff import price to account for the past economy specific and industry specific pre-tariff import price inflation $\frac{P^M_{i;k}(t-1)}{P^M_{i;k}(t-2)}$, for those that produce economy specific and industry specific intermediate import goods or services coming from the foreign industry that provides internationally homogeneous energy or nonenergy commodities, they additionally need to adjust their pre-tariff import price to account for the contemporaneous change in the domestic currency denominated uniform price of internationally homogeneous energy or nonenergy commodities $\frac{E_{i,11}(t)P^M_{i,11}(t)}{E_{i,11}(t-1)P^M_{i,11}(t-1)}$ or

$$\frac{E_{i,12}(t)P^M_{i,12}(t)}{E_{i,12}(t-1)P^M_{i,12}(t-1)}$$

according to the following partial indexation rule:

$$P^M_{i;k,l,j,n}(t) = \left[ \frac{P^M_{i;k,l,j,n}(t-1)}{P^M_{i;k,l,j,n}(t-2)} \right]^{1-\mu^M} \left[ \frac{E_{i,11}(t)P^M_{i,11}(t)}{E_{i,11}(t-1)P^M_{i,11}(t-1)} \right]^{1-\gamma^M}$$

$$\left[ \frac{P^M_{i;k,l,j,n}(t-1)}{P^M_{i;k,l,j,n}(t-2)} \right]^{1-\mu^M} \left[ \frac{E_{i,12}(t)P^M_{i,12}(t)}{E_{i,12}(t-1)P^M_{i,12}(t-1)} \right]^{1-\gamma^M}$$

Here $\frac{P^M_{i;k,l,j,n}(t-1)}{P^M_{i;k,l,j,n}(t-2)}$ and $\frac{E_{i,11}(t)P^M_{i,11}(t)}{E_{i,11}(t-1)P^M_{i,11}(t-1)}$ are the steady state equilibrium values of $\frac{P^M_{i;k,l,j,n}(t-1)}{E_{i,11}(t-1)P^M_{i,11}(t-1)}$ and $\frac{E_{i,12}(t)P^M_{i,12}(t)}{E_{i,12}(t-1)P^M_{i,12}(t-1)}$.
Correspondingly, the remaining fraction \((1 - \omega^X)\) of economy specific and industry specific intermediate import good firms adjust their pre-tariff import price optimally, in equilibrium they all choose a common economy specific and industry specific pre-tariff import price \(p_{i k j}^{MT, *}(t)\) given by the following necessary first order condition:

\[
p_{i k j}^{MT, *}(t) = \frac{E_t \sum_{s=t}^{\infty} (\omega^M)^s \Delta E \left( \frac{\Delta N(s)}{N(s)} \right) \cdot \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t)} \right)^{\gamma M} \cdot \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t)} \right)^{1-\gamma M} \cdot \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t)} \right)^{\theta M} \cdot \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t)} \right)^{\beta M} \cdot \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t)} \right)^{\lambda M} \cdot p_{i k j}^{MT}(s) \cdot M_{i k j}(s) }{p_{i k j}^{MT}(s) \cdot M_{i k j}(s)}
\]

Under this specification, the probability that an economy specific and industry specific intermediate import good firm has adjusted its price optimally is time dependent but state independent. Then the aggregate economy specific and industry specific pre-tariff import price index \(p_{i k j}^{MT}(t)\) equals an average of the price set by the randomly selected fixed fraction \((1 - \omega^M)\) of economy specific and industry specific intermediate import good firms that adjust their pre-tariff import price optimally, and the average of the import prices set by the remaining fraction \(\omega^M\) of economy specific and industry specific intermediate import good firms that adjust their pre-tariff import prices according to the above partial indexation rule:

\[
p_{i k j}^{MT}(t) = \left\{ \begin{array}{ll}
(1 - \omega^M) \left( \frac{p_{i k j}^{MT, *}(t)}{p_{i k j}^{MT}(t)} \right)^{1-\theta M} + \omega^M \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t-2)} \right)^{\mu M} \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t-2)} \right)^{1-\mu M} \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t-1)} \right)^{\theta M} \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t-1)} \right)^{1-\theta M} \\
(1 - \omega^M) \left( \frac{p_{i k j}^{MT, *}(t)}{p_{i k j}^{MT}(t)} \right)^{1-\theta M} + \omega^M \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t-2)} \right)^{\mu M} \left( \frac{p_{i k j}^{MT}(t-1)}{p_{i k j}^{MT}(t-2)} \right)^{1-\mu M} \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t-1)} \right)^{\theta M} \left( \frac{p_{i k j}^{MT}(t)}{p_{i k j}^{MT}(t-1)} \right)^{1-\theta M} 
\end{array} \right\}^{\frac{1}{1-\theta M}}
\]
\[
\begin{align*}
P_{ikj}^{MT}(t) &= \left(1 - \omega^{M}\right) \left(P_{ikj}^{MT}(t)\right)^{1 - \theta^{M}(t)} + \\
&\omega^{M} \left\{ \left(\frac{P_{ikj}^{MT}(t-1)}{P_{ikj}^{MT}(t-2)} \right)^{1 - \mu^{M}} \frac{E_{k}(t)P_{ik}^{E}(t)}{E_{k}(t-1)P_{ik}^{E}(t-1)} \right\}^{1 - \gamma^{M}} \left\{ \left(\frac{P_{ikj}^{MT}(t-1)}{P_{ikj}^{MT}(t-2)} \right)^{1 - \mu^{M}} \frac{E_{k}(t)P_{ik}^{E}(t)}{E_{k}(t-1)P_{ik}^{E}(t-1)} \right\}^{1 - \gamma^{M}} P_{ikj}^{MT}(t-1) \right\}^{1 - \theta^{M}(t)}
\end{align*}
\]

All the above intertemporal adjustment mechanism of pre-tariff import prices cause economy specific and industry specific nominal import price rigidity in the economy specific and industry specific import market.

Since we want to further include the transmission mechanism between upstream and downstream quantities and prices in the global supply chain in the AGMFM, after taking the upstream and downstream quantity and price transmission mechanism similar to that for the output quantity and price of the industry specific final output goods or services introduced in the previous Subsection D into consideration, the import quantity and price of the industry specific final import goods or services should also be modified accordingly. As these complex settings lead to the expression of industry-level import quantity and price being too lengthy and complex, we will only present the derived linearized relationship equations in Section IV for further introduction.

I. Balance of Payments Sector

Since the net foreign asset position is always the core of defining other balance of payments related variables, we first discuss this variable as follows.

Distribution of financial wealth across various sectors of the i-th economy requires that:

\[A_{i}(t) = A_{i}^{H}(t) + A_{i}^{D}(t) + A_{i}^{F}(t) + A_{i}^{B}(t) + A_{i}^{M}(t) + A_{i}^{G}(t)\]

Here, 
\(A_{i}(t)\) is the net foreign asset position of the whole i-th economy;
\(A_{i}^{H}(t), A_{i}^{D}(t), A_{i}^{F}(t), A_{i}^{B}(t), A_{i}^{M}(t)\) and \(A_{i}^{G}(t)\) are the financial wealth of the household sector, the construction sector, the production sector, the banking sector, the export sector, the import sector, and the government sector of the i-th economy respectively.

Since nondiscretionary and discretionary lump sum transfer payments only redistribute financial wealth of households instead of substantially increasing or decreasing the total financial wealth of the household sector, then distribution of financial wealth of the household sector requires that:

\[A_{i}^{H}(t) = A_{i}^{RH}(t) + A_{i}^{AH}(t) + V_{i}^{C}(t-1) + V_{i}^{X}(t-1) + V_{i}^{M}(t-1)\]

Here, 
\(A_{i}^{RH}(t)\) is the total nominal property balances of the domestic household sector;
\(A_{i}^{AH}(t)\) is the total nominal portfolio balances of the domestic household sector;
\(V_{i}^{C}(t-1)\) is the total past value of shares of domestic intermediate banks held by the domestic household sector;
\(V_{i}^{X}(t-1)\) is the total past value of shares of domestic industry specific intermediate export...
good firms held by the domestic household sector;
\( V_i^M(t - 1) \) is the total past value of shares of domestic economy specific and industry specific intermediate import good firms held by the domestic household sector.

Distribution of financial wealth of the construction sector requires that:
\[
A_{FD}(t) = -B_{CD}^i(t) - V_i^H(t - 1)
\]

Here,
\( B_{CD}^i(t) \) is the mortgage loans issued by the banking sector of the i-th economy to all real estate intermediate developers of the i-th economy;
\( V_i^H(t - 1) \) is the total past value of shares of domestic real estate intermediate developers held by the domestic household sector.

Distribution of financial wealth of the production sector requires that:
\[
A_{FD}(t) = -\sum_{j=1}^{N} B_{CF}^i(t) - V_i^S(t - 1)
\]

Here,
\( \sum_{j=1}^{N} B_{CF}^i(t) \) is the sum of corporate loans from the banking sector of all economies to all industry specific intermediate output good firms of the i-th economy;
\( V_i^S(t - 1) \) is the total past value of shares of domestic industry specific intermediate output good firms held by the domestic and foreign household sectors.

Distribution of financial wealth of the banking sector requires that:
\[
A_{FB}^i(t) = K_{FB}^i(t) - V_i^C(t - 1)
\]

Here \( K_{FB}^i(t) \) is the aggregate bank capital of the domestic banking sector.

Distribution of financial wealth of the export sector requires that:
\[
A_{FX}^i(t) = -V_i^X(t - 1)
\]

Distribution of financial wealth of the import sector requires that:
\[
A_{FM}^i(t) = -V_i^M(t - 1)
\]

In addition, the US dollar denominated current account balance of the i-th economy \( CA_i(t) \) is defined as the intertemporal change in the US dollar denominated net foreign asset position of the i-th economy \( E_{i,j}(t - 1)A_{ij}(t) \):
\[
CA_i(t) = E_{i,j}(t)A_{i}(t + 1) - E_{i,j}(t - 1)A_{i}(t)
\]

The US dollar denominated trade balance of the i-th economy \( TB_i(t) \) is defined as the US dollar denominated total value of exports of the i-th economy \( E_{i,j}(t)P_{i}^X(t)X_i(t) \), less the US dollar denominated total value of imports of the i-th economy \( E_{i,j}(t)P_{i}^M(t)M_i(t) \):
\[
TB_i(t) = E_{i,j}(t)P_{i}^X(t)X_i(t) - E_{i,j}(t)P_{i}^M(t)M_i(t)
\]
If we impose restrictions that the ratio of the i-th economy’s currency denominated value of the j-th economy’s short term bonds held by all sectors (mainly refers to the household sector, if j = i, also refers to the government sector) of the i-th economy to the net foreign asset position of the i-th economy \( \frac{E_{ij}(t-1)B^{S}_{i}(t)}{A_{i}(t)} \) equals \( \omega^{A}_{j} \) (also called the capital market capitalization weight of the j-th economy among all economies) that doesn’t change with the period t, then a combination of constraints of various sectors, various dividend payment definition for share holding, and the output expenditure decomposition reveals the following relationship between \( CA_{i}(t) \) and \( TB_{i}(t) \):

\[
CA_{i}(t) = \left\{ \sum_{j=1}^{N} \omega^{A}_{j} \left[ \left( 1 + i^{S}_{j}(t-1) \right) \frac{E_{ij}(t)}{E_{ij}(t-1)} - 1 \right] \right\} E_{1,i}(t-1)A_{i}(t) + TB_{i}(t)
\]

Here \( \sum_{j=1}^{N} \omega^{A}_{j} \left[ \left( 1 + i^{S}_{j}(t-1) \right) \frac{E_{ij}(t)}{E_{ij}(t-1)} - 1 \right] \) is the sum of the US dollar denominated net international investment income of the i-th economy from holding short term bonds, long term bonds and corporate equities of all economies (including the i-th economy itself) around the world, the reason why only the nominal short term bond yield \( i^{S}_{i}(t) \) appears is because the nominal portfolio return (portfolio balances of capital market intermediated households are allocated across the values of internationally diversified short term bonds, internationally diversified and vintage diversified long term bonds, and internationally diversified and industry diversified and firm diversified stocks) \( i^{A,H}_{i}(t) \) can ultimately be attributed to the nominal short term bond yield \( i^{S}_{i}(t) \).

Clearing of the international commodity markets or multilateral consistency in nominal trade flows requires that the sum of the US dollar denominated trade balance of any specific economy \( TB_{j}(t) \) equals zero:

\[
\sum_{j=1}^{N} TB_{j}(t) = 0
\]

Since we further incorporate the structure of international direct investment into this AGMFM relative to the GFM introduced in Vitek (2015, 2018), the innovatively added domestic and foreign business investment \( I^{K,_{in}}_{i}(t) \) for the production sector of the i-th economy is defined as the sum of the business investment inflow from the j-th economy to the production sector of the i-th economy (including from the i-th economy itself) \( I^{K}_{j \rightarrow i}(t) \), while \( I^{K}_{i \rightarrow j}(t) \) is always simply assumed to be proportional to the total business investment outflow from the j-th economy to the production sector of all economies including the j-th economy itself \( I^{K}_{j}(t) \):

\[
I^{K,_{in}}_{i}(t) = \sum_{j=1}^{N} I^{K}_{j \rightarrow i}(t)
\]

Based on the above definition of international investment flow \( I^{K,_{in}}_{i}(t) \), the direct investment balance (denominated in US dollars) can be defined as the sum of the US dollar denominated value of business investment inflow from the j-th economy to the production sector of the i-th economy (including from the i-th economy itself) \( E_{ij}(t)P^{C}_{j}(t)I^{K}_{j \rightarrow i}(t) \), minus the US dollar denominated value of the total business investment outflow from the i-th economy to the production sector of all economies including the i-th economy itself \( E_{1,i}(t)P^{C}_{i}(t)I^{K}_{i}(t) \):
In order to form a closed loop of national accounting, industry specific final public investment goods or services produced industry specific public consumption goods or services produced industry specific private consumption goods or services produced industry specific business investment goods or services produced industry specific private consumption goods or services come from domestic industry specific final output good firms, including domestically absorbed good firms.

In the lower layer, a large number of perfectly competitive industry specific final absorption good firms combine the industry specific final output goods or services that come from domestic industry specific final output good firms, including domestically produced industry specific private consumption goods or services $G_i^c(t)$, domestically produced industry specific residential investment goods or services $I_i^{Hh}(t)$, domestically produced industry specific business investment goods or services $I_i^{Kh}(t)$, domestically produced industry specific public consumption goods or services $G_i^{Ch}(t)$, and domestically produced industry specific public investment goods or services $G_i^{If}(t)$, with the industry specific final import goods or services that come from domestic industry specific final import good firms, including foreign produced industry specific private consumption goods or services $C_i^f(t)$, foreign produced industry specific residential investment goods or services $I_i^{If}(t)$, foreign produced industry specific business investment goods or services $I_i^{Kf}(t)$, foreign produced industry specific public consumption goods or services $C_i^{Ch}(t)$, and foreign produced industry specific public investment goods or services $C_i^{If}(t)$, in order to produce industry specific final private consumption goods or services $C_i(t)$, industry specific final residential investment goods or services $I_i^{Hf}(t)$, industry specific final business investment goods or services $I_i^{Kf}(t)$, industry specific final public consumption goods or services $G_i^{Ch}(t)$, and industry specific final public investment goods or services $G_i^{If}(t)$, according to the following constant elasticity of substitution production functions:

$$C_i(t) = \left(1 - \phi_i^c \right) \left( -1 \right) \frac{1}{\psi_i^{Ch}(t)} \left( C_i^{Ch}(t) \right)^{\psi_i^{Ch}(t)-1} + \left( \phi_i^c \right) \frac{1}{\psi_i^{Ch}(t)} \left( \psi_i^{Ch}(t) C_i^{Ch}(t) \right)^{\psi_i^{Ch}(t)-1}$$

$$I_i^{Hf}(t) = \left(1 - \phi_i^f \right) \frac{1}{\psi_i^{Hf}(t)} \left( I_i^{Hf}(t) \right)^{\psi_i^{Hf}(t)-1} + \left( \phi_i^f \right) \frac{1}{\psi_i^{Hf}(t)} \left( \psi_i^{Hf}(t) I_i^{Hf}(t) \right)^{\psi_i^{Hf}(t)-1}$$

$$I_i^{Kf}(t) = \left(1 - \phi_i^f \right) \frac{1}{\psi_i^{Kf}(t)} \left( I_i^{Kf}(t) \right)^{\psi_i^{Kf}(t)-1} + \left( \phi_i^f \right) \frac{1}{\psi_i^{Kf}(t)} \left( \psi_i^{Kf}(t) I_i^{Kf}(t) \right)^{\psi_i^{Kf}(t)-1}$$

J. Virtual “Absorption Sector”

In order to form a closed loop of national accounting, the virtual “absorption sector” is additionally added, and this sector consists of two layers of virtual final absorption good firms.
industry of the i-th economy.

Let the demand functions:

\[ G^C_{ik}(t) = \left(1 - \phi_i^M \right) \frac{\psi_{i,k}^M \left( G^{Ch}_{ik}(t) \right)}{\psi_{i,k}^M \left( C_{ik}(t) \right)} \frac{\psi_{i,k}^M (t)}{\psi_{i,k}^M (t-1)} + \phi_i^M \frac{\psi_{i,k}^M \left( G^{Cf}_{ik}(t) \right)}{\psi_{i,k}^M \left( G_{ik}(t) \right)} \frac{\psi_{i,k}^M (t)}{\psi_{i,k}^M (t-1)} \]

\[ G^I_{ik}(t) = \left(1 - \phi_i^M \right) \frac{\psi_{i,k}^M \left( G^{If}_{ik}(t) \right)}{\psi_{i,k}^M \left( I_{ik}(t) \right)} \frac{\psi_{i,k}^M (t)}{\psi_{i,k}^M (t-1)} + \phi_i^M \frac{\psi_{i,k}^M \left( G^{If}_{ik}(t) \right)}{\psi_{i,k}^M \left( G_{ik}(t) \right)} \frac{\psi_{i,k}^M (t)}{\psi_{i,k}^M (t-1)} \]

Here,

\[ \nu_{i,k}(t) \] is the import demand shock for the i-th economy’s imports from the k-th industry of all other economies except the i-th economy;

the economy specific and industry specific parameter \( \psi_{i,k}^M \) equals \( \psi^M \left( 1 - \frac{\theta_i^M(t)}{\theta_i^M(t-1) \nu_{i,k}(t)} \right) \),

\( \theta_i^M(t) = \frac{\theta_i^M(t)}{\theta_i^M(t-1)} \) has the meaning of the import price markup shock, and \( \frac{\theta_i^M(t)}{\nu_{i,k}(t)} \) is the steady state equilibrium value of \( \frac{M_{i,k}(t)}{I_{i,k}(t)} \), which is the ratio of total imports of the i-th economy from the k-th industry of all other economies except the i-th economy to the total output of the k-th industry of the i-th economy.

“Perfectly competitive” means in equilibrium the industry specific final absorption good firms maximize profits derived from production of the industry specific final private consumption goods or services \( C_{ik}(t) \), the industry specific final residential investment goods or services \( I^R_{ik}(t) \), the industry specific final business investment goods or services \( I^K_{ik}(t) \), the industry specific final public consumption goods or services \( G^C_{ik}(t) \), and the industry specific final public investment goods or services \( G^I_{ik}(t) \), with respect to inputs of the domestically produced industry specific final output goods or services \( \{ C^h_{ik}(t), I^H_{ik}(t), I^K_{ik}(t), G^C_{ik}(t), G^I_{ik}(t) \} \) and the foreign produced industry specific final import goods or services \( \{ C^f_{ik}(t), I^{Hf}_{ik}(t), I^{Kf}_{ik}(t), G^{Ch}_{ik}(t), G^{If}_{ik}(t) \} \), implying the following domestically produced industry specific final output goods or services demand functions and foreign produced industry specific final import goods or services demand functions:

\[ C^h_{ik}(t) = (1 - \phi_i^M) \frac{P^h_{i,k}(t)}{P^h_{i,k}(t)} \nu_{i,k}(t) C_{ik}(t) \]

\[ I^H_{ik}(t) = (1 - \phi_i^M) \frac{P^H_{i,k}(t)}{P^H_{i,k}(t)} \nu_{i,k}(t) I^H_{ik}(t) \]

\[ I^K_{ik}(t) = (1 - \phi_i^M) \frac{P^K_{i,k}(t)}{P^K_{i,k}(t)} \nu_{i,k}(t) I^K_{ik}(t) \]

\[ G^C_{ik}(t) = (1 - \phi_i^M) \frac{P^C_{i,k}(t)}{P^C_{i,k}(t)} \nu_{i,k}(t) G^C_{ik}(t) \]

\[ G^I_{ik}(t) = (1 - \phi_i^M) \frac{P^I_{i,k}(t)}{P^I_{i,k}(t)} \nu_{i,k}(t) G^I_{ik}(t) \]

\[ C_{ik}(t) = \phi_i^M \frac{1}{\nu_{i,k}(t)} \frac{P^M_{i,k}(t)}{P^M_{i,k}(t)} \nu_{i,k}(t) C_{ik}(t) \]
Then the domestically produced industry specific final output goods or services ensure multilateral consistency in nominal trade flows.

Since in equilibrium the industry specific final absorption good firms generate zero profits, then the industry specific post-tariff import price $P^{hf}_{i,k}(t)$ can be expressed as the weighted average of the core price level $P_i(t)$ and the industry specific post-tariff import price $P^{hf}_{i,k}(t)$:

$$P^{hf}_{i,k}(t) = \left[ (1 - \Phi^M_i(t)) \left( R_i(t) \right)^{1-\Psi^M_{i,k}(t)} + \Phi^M_i \left( \frac{P^{M}_{i,k}(t)}{\nu^M_{i,k}(t)} \right)^{1-\Psi^M_{i,k}(t)} \right]^{1-\Psi^M_{i,k}(t)}$$

The industry specific internal terms of trade $T^i_{i,k}(t)$, the industry specific external terms of trade $T^f_{i,k}(t)$, the internal terms of trade $T^i_{i,k}(t)$, the external terms of trade $T^f_{i,k}(t)$, and the terms of trade $T_i(t)$ are additionally defined as follows:

$$T^i_{i,k}(t) = \frac{P^{i}_{i,k}(t)}{P_i(t)}$$
$$T^f_{i,k}(t) = \frac{P^{f}_{i,k}(t)}{P_i(t)}$$
$$T^i(t) = \frac{P^{i}_{i,k}(t)}{P_i(t)}$$
$$T^f(t) = \frac{P^{f}_{i,k}(t)}{P_i(t)}$$
$$T_i(t) = v^T(t) \frac{T^i_{i,k}(t)}{T^f_{i,k}(t)}$$

Here the global terms of trade shifter $v^T(t)$ is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows.

Then the domestically produced industry specific final output goods or services $\{C^h_{i,k}(t), I^{kh}_{i,k}(t), I^{kh}_{i,k}(t), G^c_{i,k}(t), G^c_{i,k}(t)\}$ and the foreign produced industry specific final import goods or services $\{C^f_{i,k}(t), I^{hf}_{i,k}(t), I^{hf}_{i,k}(t), G^c_{i,k}(t), G^c_{i,k}(t)\}$ can also be expressed as follows:
In the upper layer, a large number of perfectly competitive final absorption good firms combine the industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services from domestic industry specific final absorption good firms \{C_{ijkl}(t), I_{ijkl}^H(t), I_{ijkl}^K(t), G_{ijkl}^C(t), G_{ijkl}^l(t)\}_{k=1}^{45} to produce final private consumption, residential investment, business investment, public consumption and public investment goods or services that do not distinguish between industries \{C_i(t), I_i^H(t), I_i^K(t), G_i^C(t), G_i^l(t)\} , according to the following fixed proportions production functions:

\[ C_i(t) = \min \left( \frac{C_{ijkl}(t)}{\phi_{ijkl}^H} \right)_{k=1}^{45} \]
\[ I_i^H(t) = \min \left( \frac{I_{ijkl}^H(t)}{\phi_{ijkl}^H} \right)_{k=1}^{45} \]
I_i^K(t) = \min \left\{ \frac{f_i^K(t)}{\phi_i^K} \right\}_{k=1}^{45} \\
G_i^C(t) = \min \left\{ \frac{c_i^C(t)}{\phi_i^C} \right\}_{k=1}^{45} \\
G_i^I(t) = \min \left\{ \frac{g_i^I(t)}{\phi_i^I} \right\}_{k=1}^{45}

“Perfectly competitive” means in equilibrium the final absorption good firms maximize profits derived from production of the final private consumption, residential investment, business investment, public consumption and public investment goods or services that do not distinguish between industries \{C_i(t), I_i^H(t), I_i^K(t), G_i^C(t), G_i^I(t)\}, with respect to inputs of the industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services \{C_{i,k}(t), I_{i,k}^H(t), I_{i,k}^K(t), G_{i,k}^C(t), G_{i,k}^I(t)\}_{k=1}^{45}, implying the following industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services demand functions:

\[ C_{i,k}(t) = \phi_{i,k}^f C_i(t) \]
\[ I_{i,k}^H(t) = \phi_{i,k}^f I_{i}^H(t) \]
\[ I_{i,k}^K(t) = \phi_{i,k}^f I_{i}^K(t) \]
\[ G_{i,k}^C(t) = \phi_{i,k}^f G_{i}^C(t) \]
\[ G_{i,k}^I(t) = \phi_{i,k}^f G_{i}^I(t) \]

Since in equilibrium the final absorption good firms generate zero profits, then the aggregate absorption price index \( p_i^{hf}(t) \) for the final private consumption, residential investment, business investment, public consumption and public investment goods or services \{C_i(t), I_i^H(t), I_i^K(t), G_i^C(t), G_i^I(t)\} can be expressed as the weighted average of the aggregate industry specific absorption price \( p_{i,k}^{hf}(t) \):

\[ p_i^{hf}(t) = \sum_{k=1}^{45} \phi_{i,k} f_{i,k} p_{i,k}^{hf}(t) \]

In fact, the consumption price \( p_i^C(s) \) in the above Subsection A, the price of residential investment \( p_i^H(s) \) in the above Subsection C, the price of business investment \( p_i^K(s) \) in the above Subsection D, the price of public consumption \( p_i^C(t) \) in the following Subsection K all equal to the aggregate absorption price index \( p_i^{hf}(t) \), while the price of industry specific public investment \( p_{i,k}^C(t) \) in the following Subsection K equals to the aggregate industry specific absorption price \( p_{i,k}^{hf}(t) \).

K. Government Sector

The government sector consists of a monetary authority which conducts monetary policies, a fiscal authority which conducts fiscal policies, and a macroprudential authority which conducts macroprudential policies.
For the convenience of writing all the following equations, \( \text{Variable}_i(t) \) is used to represent the linear deviation of \( \text{Variable}_i(t) \) from its steady state equilibrium value \( \text{Variable}_i(t) \):

\[
\text{Variable}_i(t) = \text{Variable}_i(t) - \overline{\text{Variable}_i(t)}
\]

\( \ln \text{Variable}_i(t) \) is used to represent the logarithmic deviation of \( \text{Variable}_i(t) \) from its steady state equilibrium value \( \text{Variable}_i(t) \):

\[
\ln \text{Variable}_i(t) = \ln \text{Variable}_i(t) - \ln \overline{\text{Variable}_i(t)} = \frac{\text{Variable}_i(t) - \text{Variable}_i(t)}{\text{Variable}_i(t)}
\]

**The monetary authority** implements monetary policies through control of the nominal policy interest rate, and the specific adjustment formula of nominal policy interest rate depends on the specific exchange rate and inflation targeting arrangement of a specific economy.

Under a free floating exchange rate and flexible inflation targeting arrangement, the linear deviation of the nominal policy interest rate \( \varpi_i^p(t) \) depends on a weighted average of its past value \( \varpi_i^p(t - 1) \) and its desired value \( \xi^p \text{E}_i \tilde{\pi}_i^c(t + 1) + \xi^\upsilon \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) \), and is also influenced by the linear deviation of the contemporaneous monetary policy shock \( \varpi_i^p(t) \), while the linear deviation of the desired nominal policy interest rate responds to the linear deviation of the expected future consumption price inflation \( \text{E}_i \tilde{\pi}_i^c(t + 1) \) and the logarithmic deviation of the contemporaneous output gap \( \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) \), according to the following monetary policy rule:

\[
\varpi_i^p(t) = \rho \varpi_i^p(t - 1) + (1 - \rho^i) \left[ \xi^p \text{E}_i \tilde{\pi}_i^c(t + 1) + \xi^\upsilon \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) \right] + \varpi_i^p(t)
\]

Under a managed exchange rate arrangement, the linear deviation of the nominal policy interest rate \( \varpi_i^p(t) \) depends on a weighted average of its past value \( \varpi_i^p(t - 1) \) and its desired value \( \xi^p \text{E}_i \tilde{\pi}_i^c(t + 1) + \xi^\upsilon \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) + \xi^\epsilon \left( \ln \tilde{E}_i(t) - \ln \tilde{E}_i(t - 1) \right) \), and is also influenced by the linear deviation of the contemporaneous monetary policy shock \( \varpi_i^p(t) \), while the linear deviation of the desired nominal policy interest rate responds to the linear deviation of the expected future consumption price inflation \( \text{E}_i \tilde{\pi}_i^c(t + 1) \), the logarithmic deviation of the contemporaneous output gap \( \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) \), and the intertemporal change in the logarithmic deviation of the nominal effective exchange rate of the currency issued by the i-th economy \( \left( \ln \tilde{E}_i(t) - \ln \tilde{E}_i(t - 1) \right) \), according to the following monetary policy rule:

\[
\varpi_i^p(t) = \rho \varpi_i^p(t - 1) + (1 - \rho^i) \left[ \xi^p \text{E}_i \tilde{\pi}_i^c(t + 1) + \xi^\upsilon \left( \ln \tilde{Y}_i(t) - \ln \tilde{Y}_i(t) \right) + \xi^\epsilon \left( \ln \tilde{E}_i(t) - \ln \tilde{E}_i(t - 1) \right) \right] + \varpi_i^p(t)
\]

Under a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union), the linear deviation of the nominal policy interest rate \( \varpi_i^p(t) \) tracks the linear deviation of the contemporaneous nominal policy interest rate of the i*-th economy that issues the anchor
currency for the currency issued by the i-th economy $\hat{i}_i^p(t)$, and also responds to the intertemporal change in the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to the anchor currency issued by the $i^*$-th economy $(\ln \hat{e}_{i,i^*}(t) - \ln \hat{e}_{i,i^*}(t-1))$, according to the following monetary policy rule:

$$
\hat{i}_i^p(t) = \hat{i}_i^p(t) + \xi_i^e \left( \ln \hat{e}_{i,i^*}(t) - \ln \hat{e}_{i,i^*}(t-1) \right)
$$

The fiscal authority implements fiscal policies through control of public consumption and industry specific public investment, adjustments of tax rates applicable to corporate earnings, household labor income and imports, the operation of a budget neutral nondiscretionary lump sum transfer program that redistributes national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households, the operation of a discretionary lump sum transfer program that provides income support only to credit constrained households, and the accumulation of industry differentiated public investment. Here the budgetary resources of the fiscal authority can be transferred intertemporally through transactions in the domestic money and bond markets.

The control of public consumption $G_{iC_i}(t)$ and industry specific public investment $G_{i,kI_i}(t)$ (final public consumption goods or services and industry specific final public investment goods or services come from the additionally added virtual “absorption sector”) satisfy countercyclical fiscal expenditure rules, in order to achieve public wealth stabilization objectives.

Specifically, the logarithmic deviation of the public consumption $\ln G_{iC_i}(t)$ depends on a weighted average of its past value $\ln G_{iC_i}(t-1)$ and its desired value $\ln \hat{Y}_i(t)$, and is also influenced by the linear deviation of the contemporaneous public consumption shock $\nu_{iC_i}^C(t)$, while the logarithmic deviation of the desired public consumption equals to the logarithmic deviation of the contemporaneous potential output (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) $\ln \hat{Y}_i(t)$, according to the following fiscal expenditure rule:

$$
\ln G_{iC_i}(t) = \rho_G \ln G_{iC_i}(t-1) + (1 - \rho_G) \ln \hat{Y}_i(t) + \nu_{iC_i}^C(t)
$$

Since we further incorporate the intra-industry and inter-industry structure into the AGMFM and we want to consider public investment with industry differences, the logarithmic deviation of the public investment for the k-th industry of the i-th economy $\ln G_{i,kI_i}(t)$ depends on a weighted average of its past value $\ln G_{i,kI_i}(t-1)$ and its desired value $\ln \hat{Y}_i(t)$, and is also influenced by the linear deviation of the contemporaneous public investment shock for the k-th industry of the i-th economy $\nu_{i,kI_i}^I(t)$, while the logarithmic deviation of the desired public investment for the k-th industry of the i-th economy equals to the logarithmic deviation of the contemporaneous potential output of the i-th economy $\ln \hat{Y}_i(t)$, according to the following industry specific fiscal expenditure rule:

$$
\ln G_{i,kI_i}(t) = \rho_G \ln G_{i,kI_i}(t-1) + (1 - \rho_G) \ln \hat{Y}_i(t) + \nu_{i,kI_i}^I(t)
$$
The adjustments of corporate income tax rate \( \tau^K_i(t) \), labor income tax rate \( \tau^L_i(t) \), and industry-level import tariff rate \( \tau^{M}_{i,k}(t) \) applicable to corporate earnings, household labor income, and imports from foreign k-th industry satisfy acyclical fiscal revenue rules.

Specifically, the linear deviation of the corporate income tax rate \( \delta \tau^K_i(t) \) depends on its past value \( \delta \tau^K_i(t-1) \), and is also influenced by the linear deviation of the contemporaneous corporate income tax rate shock \( \nu^K_i(t) \), according to the following fiscal revenue rule:

\[
\delta \tau^K_i(t) = \rho_{i,K} \delta \tau^K_i(t-1) + \nu^K_i(t)
\]

Specifically, the linear deviation of the labor income tax rate \( \delta \tau^L_i(t) \) depends on its past value \( \delta \tau^L_i(t-1) \), and is also influenced by the linear deviation of the contemporaneous labor income tax rate shock \( \nu^L_i(t) \), according to the following fiscal revenue rule:

\[
\delta \tau^L_i(t) = \rho_{i,L} \delta \tau^L_i(t-1) + \nu^L_i(t)
\]

Since we further incorporate the structure of import tariff into the AGMFM relative to the GFM introduced in Vitek (2015, 2018), the innovatively added linear deviation of the industry-level import tariff rate \( \delta \tau^{M}_{i,k}(t) \) depends on its past value \( \delta \tau^{M}_{i,k}(t-1) \), and is also influenced by the linear deviation of the contemporaneous industry-level import tariff rate shock \( \nu^{M}_{i,k}(t) \), according to the following fiscal revenue rule:

\[
(\tau^M_{i,k}(t) - \tau^M_{i,k}(t-1)) = \rho^M_{i,k}(\tau^M_{i,k}(t-1) - \tau^M_{i,k}) + \nu^{M}_{i,k}(t)
\]

Here \( \rho^M_{i,k} \) represents the coefficient for the intertemporal change of country differentiated and industry differentiated import tariff rate, which is estimated based on the specific tariff reduction commitments of each country in the tariff terms of for example Regional Comprehensive Economic Partnership (RCEP).

The total scale of nondiscretionary lump sum transfer payment program equals to the scale of nondiscretionary lump sum transfer payments to all domestic credit constrained households \( T^{CN}_i(t) \). The ratio of nondiscretionary lump sum transfer payments to nominal output \( \frac{T^{CN}_i(t)}{P^N_i(t)Y_i(t)} \) satisfies a nondiscretionary transfer payment rule that stabilizes national financial wealth, in order to achieve national financial wealth stabilization objectives.

Specifically, the linear deviation of the ratio of nondiscretionary lump sum transfer payments by the fiscal authority to nominal output \( \delta \frac{T^{CN}_i(t)}{P^N_i(t)Y_i(t)} \) responds to the linear deviation of the ratio of the contemporaneous net foreign asset position to past nominal output \( \frac{\Delta_i(t)}{P^N_i(t-1)Y_i(t-1)} \), according to the following nondiscretionary transfer payment rule:

\[
\delta \frac{T^{CN}_i(t)}{P^N_i(t)Y_i(t)} = \zeta N \frac{\Delta_i(t)}{P^N_i(t-1)Y_i(t-1)}
\]

The total scale of discretionary lump sum transfer program is the scale of discretionary lump sum transfer payments to all domestic credit constrained households \( T^{CD}_i(t) \). The ratio of
discretionary lump sum transfer payments to nominal output \( \frac{T_{C,D}^i(t)}{P_t^i(t)Y_i(t)} \) satisfies a discretionary transfer payment rule that stabilizes public financial wealth, in order to achieve public financial wealth stabilization objectives.

Specifically, the linear deviation of the ratio of discretionary lump sum transfer payments by the fiscal authority to nominal output \( \frac{T_{C,D}^i(t)}{P_t^i(t)Y_i(t)} \) responds to the linear deviation of the ratio of the contemporaneous net government asset of the fiscal authority (also called accumulated public financial wealth) to past nominal output \( \frac{A_{G}^i(t)}{P_t^i(t-1)Y_i(t-1)} \), and is also influenced by the linear deviation of the contemporaneous transfer payment shock \( \nu_{T}^i(t) \), according to the following discretionary transfer payment rule:

\[
\frac{T_{C,D}^i(t)}{P_t^i(t)Y_i(t)} = \zeta_{TD} \frac{A_{G}^i(t)}{P_t^i(t-1)Y_i(t-1)} + \nu_{T}^i(t)
\]

The industry differentiated public investment \( G_{i,k}^i(t) \) is accumulated to form industry differentiated public physical capital stock \( K_{i,k}^i(t) \), which further influences industry differentiated labor productivity \( \Lambda_{i,k}^i(t) \), in order to reflect the government's differentiated industry policies or differentiated support for sunrise and sunset industries.

Specifically, the logarithmic deviation of the past industry specific final public investment goods or services for the \( k \)-th industry of the \( i \)-th economy \( \ln G_{i,k}^i(t-1) \) is accumulated to form the logarithmic deviation of the contemporaneous public physical capital stock for the \( k \)-th industry of the \( i \)-th economy \( \ln K_{i,k}^i(t) \), according to the following perpetual inventory method:

\[
\ln K_{i,k}^i(t) = (1 - \delta^G)\ln K_{i,k}^i(t-1) + \delta^G \ln G_{i,k}^i(t-1)
\]

Since we further incorporate the intra-industry and inter-industry structure into the AGMFM and we want to consider technology diffusion among all industries of all economies, we set that the logarithmic deviation of the labor productivity for the \( k \)-th industry of the \( i \)-th economy \( \ln \Lambda_{i,k}^i(t) \) not only depends on the logarithmic deviation of the contemporaneous cross-border cross-industry productivity shifter \( \sum_{m',=1}^N \sum_{m'=1}^{M} Y_{j',m'-i,k} \ln \nu_{i,m}^A(t) \left( 1 - \lambda^A \sum_{j'=1}^N \sum_{m'=1}^{M} Y_{j',m'-i,k} \right) \) that satisfies dynamic factor process to reflect technology diffusion among all industries of all economies, but also depends on the logarithmic deviation of the contemporaneous industry-level per capita public physical capital stock \( \ln \bar{K}_{i,k}^G(t) - \ln \bar{N}_i(t) \) (here the logarithmic deviation of industry-level labor force is simply assumed to be the same as the logarithmic deviation of total labor force) to reflect the promotion effect of per capita public physical capital stock on industry-level labor productivity improvement, through ways such as infrastructure construction and public expenditure for R & D:
\[
\ln \tilde{A}_{ik}(t) = \phi A \left[ \lambda A \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{,jm-i,k}}{\sum_{j'=1}^{45} \sum_{m'=1}^{45} Y_{,j'm-i,k}} \ln \tilde{A}_{i,m}(t) + \left( 1 - \lambda A \right) \left( \ln \tilde{G}_{ik}(t) - \ln \tilde{N}_{i}(t) \right) \right]
\]

Here, \( \frac{Y_{,jm-i,k}}{\sum_{j'=1}^{45} \sum_{m'=1}^{45} Y_{,j'm-i,k}} \) represents the ratio of goods or services from the m-th industry of the j-th economy that are adopted as intermediate inputs of the k-th industry of the i-th economy to goods or services from all industries of all economies that are adopted as intermediate inputs of the k-th industry of the i-th economy; and \( \frac{Y_{,ik}}{\sum_{j'=1}^{45} \sum_{m'=1}^{45} Y_{,j'm-i,k}} \) represents the ratio of intermediate goods or services from the k-th industry of the i-th economy that continue to be used as intermediate inputs for the k-th industry of the i-th economy to goods or services from all industries of all economies that are adopted as intermediate inputs of the k-th industry of the i-th economy.

All these parameters are used to reflect that the degree of cross-border cross-industry technology diffusion is directly proportional to the scale of input-output relation between different industries in different countries.

The fiscal authority accumulates public financial wealth (also called net government asset) \( A_{i}^{G}(t) \) subject to the dynamic budget constraint function of the government as follows:

\[
A_{i}^{G}(t + 1) = \left( 1 + i_{1}^{AG}(t) \right) A_{i}^{G}(t) + \sum_{k=1}^{45} \sum_{k'=1}^{45} \sum_{k'=1}^{45} P_{i,k,l}(t) Y_{,ik,l}(t) + \sum_{k=1}^{45} \sum_{k'=1}^{45} \sum_{k'=1}^{45} P_{i,k,l}(t) Y_{,ik,l}(t) - T_{,ik,l}(t) \]

Here, \( i_{1}^{AG}(t) \) is the return rate of accumulated public financial wealth \( A_{i}^{G}(t) \).

Since \( A_{i}^{G}(t) \) is distributed across the values of domestic short term bonds \( B_{i}^{S,G}(t) \) and domestic vintage diversified long term bonds \( B_{i}^{L,G}(t) \) that have the return rates \( i_{1}^{S}(t-1) \) and \( i_{1}^{L}(t) \), then \( i_{1}^{G}(t) \) has the following expression:

\[
\left( 1 + i_{1}^{G}(t) \right) A_{i}^{G}(t) = \left( 1 + i_{1}^{S}(t-1) \right) B_{i}^{S,G}(t) + \left( 1 + i_{1}^{L}(t) \right) B_{i}^{L,G}(t)
\]

Since domestic vintage diversified long term bonds are perpetual bonds with coupon payments that decay exponentially at rate \( \omega^{B} \), the total return yielded by domestic vintage diversified long term bonds for the fiscal authority \( i_{1}^{L}(t)B_{i}^{L,G}(t) \) is split into returns yielded by domestic currency denominated values of various domestic vintage specific long term bonds \( \{ V_{i,v}(t)B_{i}^{L,G}(t) \}_{v=1}^{t-1} \) as follows:

\[
\left( 1 + i_{1}^{BL}(t) \right) B_{i}^{L,G}(t) = \sum_{v=1}^{t-1} \left( \Pi_{i,v}^{B}(t) + V_{i,v}(t) \right) B_{i}^{L,G}(t)
\]

Here, \( \Pi_{i,v}^{B}(t) \) is the domestic currency denominated coupon payment per vintage specific long term bond, which has the following expression:
\[ \Pi^B_{L_V}(t) = (1 + i^L_1(v) - \omega^B)(\omega^B)^{1-v}V^B_{L_V}(v) \]

Here \( i^L_1(v) \) is the yield to maturity on domestic vintage specific long term bond at issuance, and \( V^B_{L_V}(v) = 1. \)

If we impose restrictions that the values of any two in the set \( \{ B^L_{i,v}(t)\}_{v=1}^{b-1} \) are equal, then \( i^B_{i,v}(t) \), which could also be written as \( i^L_1(t - 1) \) and called the nominal effective long term market interest rate of the i-th economy, has the following expression:

\[ i^B_{i,v}(t) = i^L_1(t - 1) + \omega^B i^L_1(t - 2) + (1 - \omega^B) \left\{ \omega^B \left[ \left(1 + i^L(t - 1) \right) + (1 - \omega^B) \right] - 1 \right\} \]

\( \sum^{45}_{k=1} \int^{1}_{l=0} i^L(t) \left( P^L_{i,k}(t) Y_{i,k,l}(t) - W_{i}(t)L_{i,k,l}(t) \right) dl \) is the tax revenue from corporate earnings,

\( \int^{1}_{h=0} \int^{1}_{f=0} i^L(t) W_{i}(t)L_{h,f,l}(t) df dh \) is the tax revenue from labor income,

\( \sum^{45}_{k=1} i^L(t) P^M_{i,k}(t) M_{i,k}(t) \) is the total import tariff revenue, the sum of the three is the total tax revenue of the fiscal authority \( T_i(t) \):

\[ T_i(t) = \sum^{45}_{k=1} \int^{1}_{l=0} i^L(t) \left( P^L_{i,k}(t) Y_{i,k,l}(t) - W_{i}(t)L_{i,k,l}(t) \right) dl + \int^{1}_{h=0} \int^{1}_{f=0} i^L(t) W_{i}(t)L_{h,f,l}(t) df dh + \sum^{45}_{k=1} i^L(t) P^M_{i,k}(t) M_{i,k}(t) \]

\( T^C_i(t) \) is the sum of nondiscretionary and discretionary lump sum transfer payments by the fiscal authority for all domestic households.

The fiscal authority purchases final public consumption goods or services \( G^C_i(t) \) at price \( P^C_i(t) = P^h_i(t) \) and industry specific final public investment goods or services \( G^I_{i,k}(t) \) at price \( P^I_{i,k}(t) = P^h_{i,k}(t) \), both come from the additionally added virtual “absorption sector”.

The newly accumulated public financial wealth \( A^C_i(t + 1) \) is again allocated across the values of domestic short term bonds \( B^S_{i,v}(t + 1) \) and domestic vintage diversified long term bonds \( B^L_{i,v}(t + 1) \), and \( B^L_{i,v}(t + 1) \) is allocated across the values of various domestic vintage specific long term bonds as follows:

\[ B^L_{i,v}(t + 1) = \sum^{25}_{v=1} V^E_{i,v}(v) B^L_{i,v}(t + 1) \]

In addition, the fiscal balance of the fiscal authority \( FB_i(t) \) is defined as the intertemporal change in the accumulated public financial wealth \( A^C_i(t) \):

\[ FB_i(t) = A^C_i(t + 1) - A^C_i(t) \]

The primary fiscal balance of the fiscal authority \( PB_i(t) \) is defined as the total tax revenue of the fiscal authority \( T_i(t) \), less the sum of nondiscretionary and discretionary lump sum transfer payments by the fiscal authority \( T^C_i(t) \), and less the total public consumption expenditure \( P^C_i(t) G^C_i(t) \) and the sum of industry specific public investment expenditure \( \sum^{45}_{k=1} P^I_{i,k}(t) G^I_{i,k}(t) \):

\[ PB_i(t) = T_i(t) - T^C_i(t) - P^C_i(t) G^C_i(t) - \sum^{45}_{k=1} P^I_{i,k}(t) G^I_{i,k}(t) \]
Then the relationship between $FB_i(t)$ and $PB_i(t)$ is as follows:

$$FB_i(t) = i^{AG}_i(t)A^{C}_i(t) + PB_i(t)$$

If we impose restrictions that $\frac{B^{SG}_i(t)}{A^*_i(t)}$ equals $\frac{B^{SG}_i(t)}{A^*_i(t)}$ that doesn’t change with the period $t$, and $\frac{B^{LG}_i(t)}{A^*_i(t)}$ equals $\frac{B^{LG}_i(t)}{A^*_i(t)}$ that doesn’t change with the period $t$, then the above equation could also be expressed as follows:

$$FB_i(t) = \left( \frac{B^{SG}_i(t)}{A^*_i(t)} i^{S}_i(t-1) + \frac{B^{LG}_i(t)}{A^*_i(t)} i^{LE}_i(t-1) \right) A^{C}_i(t) + PB_i(t)$$

**The macroprudential authority** implements macroprudential policies through control of a regulatory bank capital ratio requirement, and control of loan to value ratio limits.

The regulatory bank capital ratio requirement applicable to bank capital accumulation of domestic intermediate banks $\kappa^R_i(t)$ satisfies a countercyclical capital buffer rule.

Specifically, the linear deviation of the regulatory bank capital ratio requirement $\kappa^R_i(t)$ depends on a weighted average of its past value $\kappa^R_i(t-1)$ and its desired value:

$$\kappa^R_i = \rho_{\kappa} \kappa^R_i(t-1) + (1 - \rho_{\kappa}) \left[ \xi^{KB}_i \left( \ln B^{C,B}_i(t) - \ln B^{C,B}_i(t-1) \right) - \xi^{KB}_i \left( \ln B^{C,B}_i(t) - \ln B^{C,B}_i(t-1) \right) \right] + \xi^{KH}_i \left( \ln V_i^H(t-1) - \ln V_i^H(t-2) \right) + \xi^{KV,H}_i \left( \ln V_i^H(t-1) - \ln V_i^H(t-2) \right)$$

The regulatory mortgage loan to value ratio limit applicable to borrowing by domestic real estate intermediate developers from domestic final banks $\phi^D_i(t)$ satisfies the mortgage loan to value limit rule.

Specifically, the linear deviation of the regulatory mortgage loan to value ratio limit $\phi^D_i(t)$ depends on a weighted average of its past value $\phi^D_i(t-1)$ and its desired value:

$$\phi^D_i = \rho_{\phi} \phi^D_i(t-1) + (1 - \rho_{\phi}) \left[ \xi^{KB}_i \left( \ln B^{C,D}_i(t) - \ln B^{C,D}_i(t-1) \right) + \xi^{KB}_i \left( \ln V_i^H(t-1) - \ln V_i^H(t-2) \right) \right] + \xi^{KB}_i \left( \ln V_i^H(t-1) - \ln V_i^H(t-2) \right)$$
the logarithmic deviation of the price of housing \( (\ln V_i^H(t) - \ln V_i^H(t-1)) \), according to the following mortgage loan to value limit rule:

\[
\hat{\phi}_i^D(t) = \rho_{\phi} \hat{\phi}_i^D(t-1) - (1 - \rho_{\phi}) \left[ \xi_{\phi}^D \big( \ln B_i^{C,D}(t) - \ln B_i^{C,D}(t-1) \big) + \chi_{\phi}^D \left( \ln V_i^H(t) - \ln V_i^H(t-1) \right) \right] + \psi_i^D(t)
\]

The regulatory corporate loan to value ratio limit applicable to borrowing by domestic intermediate output good firms from domestic global final banks \( \phi_i^F(t) \) satisfies the corporate loan to value limit rule.

Specifically, the linear deviation of the regulatory corporate loan to value ratio limit \( \hat{\phi}_i^F(t) \) depends on a weighted average of its past value \( \hat{\phi}_i^F(t-1) \) and its desired value 

\[
- \left[ \xi_{\phi}^F \big( \ln B_i^{C,F}(t) - \ln B_i^{C,F}(t-1) \big) + \chi_{\phi}^F \left( \ln V_i^S(t) - \ln V_i^S(t-1) \right) \right],
\]

and is also influenced by the linear deviation of the contemporaneous corporate loan to value limit shock \( \psi_i^F(t) \), while the linear deviation of the desired regulatory corporate loan to value ratio limit responds to the intertemporal change in the logarithmic deviation of the domestic currency denominated total final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( (\ln B_i^{C,F}(t) - \ln B_i^{C,F}(t-1)) \), and the intertemporal change in the logarithmic deviation of the price of corporate equity \( (\ln V_i^S(t) - \ln V_i^S(t-1)) \), according to the following corporate loan to value limit rule:

\[
\hat{\phi}_i^F(t) = \rho_{\phi} \hat{\phi}_i^F(t-1) - (1 - \rho_{\phi}) \left[ \xi_{\phi}^F \big( \ln B_i^{C,F}(t) - \ln B_i^{C,F}(t-1) \big) + \chi_{\phi}^F \left( \ln V_i^S(t) - \ln V_i^S(t-1) \right) \right] + \psi_i^F(t)
\]

### L. International Commodity Markets

The formation mechanism between the price of internationally homogeneous energy or nonenergy commodities and the price of internationally heterogeneous goods or services in the international commodity markets are totally different.

Since the price of internationally homogeneous energy or nonenergy commodities satisfies the law of one price, the export price of internationally homogeneous energy or nonenergy commodities as the industry specific final export goods or services that are produced by the k-th industry of the i-th economy \( p_i^{e,k}(t) \) equals to the i-th economy's currency denominated uniform price of internationally homogeneous energy or nonenergy commodities \( \epsilon_i(t) p_e^Y(t) \) or \( \epsilon_i(t) p_{ne}^Y(t) \) (the uniform price \( p_e^Y(t) \) or \( p_{ne}^Y(t) \) is denominated in US dollars in the international commodity markets):

\[
p_i^{e,k}(t) = \epsilon_i(t) (p_e^Y(t) \text{ or } p_{ne}^Y(t))
\]

Then economy specific and industry specific intermediate import good firms of the j-th economy that specialize in importing goods or services from the k-th industry of the i-th economy purchase these internationally homogeneous energy or nonenergy commodities at the j-th economy's currency denominated pre-tariff import price \( p_{i,j,k,ln}^{MT}(t) \):
\[ P_{j,k,i,n}^{M,T}(t) = E_{j,i}(t) P_{i,k}^{X}(t) = E_{j,i}(t)(P_{e}^{Y}(t) \text{ or } P_{ne}^{Y}(t)) \]

Since the uniform price of internationally homogeneous energy or nonenergy commodities \( P_{e}^{Y}(t) \) or \( P_{ne}^{Y}(t) \) (denominated in US dollars) reflects the whole world economy’s minimum cost of producing one unit of internationally homogeneous energy or nonenergy commodities as the industry specific final export goods or services, then \( P_{e}^{Y}(t) \) or \( P_{ne}^{Y}(t) \) equals to the world output weighted average of the US dollar denominated economy specific output price of internationally homogeneous energy or nonenergy commodities produced by any specific economy \( \frac{P_{e}^{Y}(t)}{E_{j,i}(t)} \text{ or } \frac{P_{ne}^{Y}(t)}{E_{j,i}(t)} \).

\[ P_{e}^{Y}(t) = \sum_{j=1}^{N} w_j P_{j}^{Y}(t) \]
\[ P_{ne}^{Y}(t) = \sum_{j=1}^{N} w_j P_{j}^{Y}(t) \]

As a contrast, the export price of internationally heterogeneous goods or services as the industry specific final export goods or services that are produced by the k-th industry of the i-th economy \( P_{i,k}^{X}(t) \) equals to the i-th economy’s currency denominated output price of industry specific final output goods or services produced by the k-th industry of the i-th economy:

\[ P_{i,k}^{X}(t) = P_{i,k}^{Y}(t) \]

Then economy specific and industry specific intermediate import good firms of the j-th economy that specialize in importing goods or services from the k-th industry of the i-th economy purchase these internationally heterogeneous goods or services at the j-th economy’s currency denominated pre-tariff import price \( P_{j,k,i,n}^{M,T}(t) \):

\[ P_{j,k,i,n}^{M,T}(t) = E_{j,i}(t) P_{i,k}^{X}(t) = E_{j,i}(t) P_{i,k}^{Y}(t) \]

However, since we want to further consider the transmission mechanism between upstream and downstream prices in the global supply chain in this AGMFM, both the export price of the industry specific final export goods or services and the import price of the industry specific final import goods or services should be modified accordingly, after taking the upstream and downstream price transmission mechanism similar to that for the output price of the industry specific final output goods or services introduced in the previous Subsection D into consideration. We are only going to present the derived linearized relationship equations of these industry-level export prices, import prices and also the adjusted output prices in Section IV, since the above complex setting leads to the expression of industry-level export, import and output prices being too lengthy and complicated.

\[ P_{i,k,i,n}^{M,T}(t) = E_{j,i}(t) P_{i,k}^{X}(t) = E_{j,i}(t) P_{i,k}^{Y}(t) \]

M. Combination of Domestic Interbank, Money, Bond, and Equity Markets and International Financial Markets

The domestic interbank, money, bond and equity markets and the international financial markets can cooperate to help form the nominal interbank loans rate \( i_{i}^{B}(t) \), the nominal short term bond yield \( i_{i}^{S}(t) \), the nominal long term bond yield \( i_{i}^{L}(t) \), the price of real
estate developer equity $V_i^H(t)$, the price of corporate equity $V_i^S(t)$, and the nominal effective long term market interest rate $i_{i}^{LE}(t)$.

Following the symbol expression in the government sector, $\text{Variable}_i(t)$ is used to represent the linear deviation of $\text{Variable}_i(t)$ from its steady state equilibrium value $\overline{\text{Variable}_i(t)}$:

$$\text{Variable}_i(t) = \text{Variable}_i(t) - \overline{\text{Variable}_i(t)}$$

$\ln \text{Variable}_i(t)$ is used to represent the logarithmic deviation of $\text{Variable}_i(t)$ from its steady state equilibrium value $\overline{\text{Variable}_i(t)}$:

$$\ln \text{Variable}_i(t) = \ln \text{Variable}_i(t) - \ln \overline{\text{Variable}_i(t)} = \frac{\text{Variable}_i(t) - \overline{\text{Variable}_i(t)}}{\overline{\text{Variable}_i(t)}}$$

The nominal interbank loans rate $i_i^B(t)$ depends on the contemporaneous nominal short term bond yield adjusted by the liquidity risk premium.

Specifically, the linear deviation of the nominal interbank loans rate of the i-th economy $\hat{i}_i^B(t)$ depends on the linear deviation of the contemporaneous nominal short term bond yield of the i-th economy $\hat{i}_i^S(t)$, and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign liquidity risk premium for the i-th economy $\ln \nu_i^B(t)$, according to the following interbank market relationship:

$$\hat{i}_i^B(t) = \hat{i}_i^S(t) + \ln \nu_i^B(t)$$

Here the logarithmic deviation of the weighted average of domestic and foreign liquidity risk premium for the i-th economy $\ln \nu_i^B(t)$ is defined as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous liquidity risk premium shock of any specific economy except the i-th economy $\sum_{j=1}^{N} \omega_j^A \ln \nu_j^B(t)$, adds the logarithmic deviation of the contemporaneous liquidity risk premium shock of the i-th economy $\ln \nu_i^B(t)$. $\lambda_i^M$ represents the i-th economy's interbank market contagion level:

$$\ln \nu_i^B(t) = \lambda_i^M \sum_{j=1}^{N} \omega_j^A \ln \nu_j^B(t) + (1 - \lambda_i^M \omega_i^A) \ln \nu_i^B(t)$$

Short term bonds are discount bonds, the nominal short term bond yield $i_i^S(t)$ depends on the contemporaneous nominal policy interest rate adjusted by the credit risk premium.

Specifically, the linear deviation of the nominal short term bond yield of the i-th economy $\hat{i}_i^S(t)$ depends on the linear deviation of the contemporaneous nominal policy interest rate of the i-th economy $\hat{i}_i^P(t)$, and is also influenced by the logarithmic deviation of the contemporaneous weighted average of the domestic and foreign credit risk premium for the i-th economy $\ln \nu_i^S(t)$, according to the following money market relationship:

$$\hat{i}_i^S(t) = \hat{i}_i^P(t) + \ln \nu_i^S(t)$$

Here the logarithmic deviation of the weighted average of domestic and foreign credit risk premium for the i-th economy $\ln \nu_i^S(t)$ is defined as the capital market capitalization weighted...
average of the logarithmic deviation of the contemporaneous credit risk premium shock of any specific economy except the i-th economy \( \sum_{j=1}^{N} \omega_j A \ln \nu_j^B(t) \). adds the logarithmic deviation of the contemporaneous credit risk premium shock of the i-th economy \( \ln \nu_i^B(t) \). \( \lambda_i^B \) represents the i-th economy's capital market contagion level:

\[
\ln \nu_i^B(t) = \lambda_i^B \sum_{j=1}^{N} \omega_j A \ln \nu_j^B(t) + (1 - \lambda_i^B \omega_i A) \ln \nu_i^B(t)
\]

Long term bonds are perpetual bonds with coupon payments that decay exponentially at a fixed rate, the nominal long term bond yield \( \hat{i}_i^L(t) \) depends on its expected future value, driven by the contemporaneous nominal short term bond yield adjusted by the duration risk premium.

Specifically, the linear deviation of the nominal long term bond yield of the i-th economy \( \hat{i}_i^L(t) \) not only depends on its expected future value \( E_t \hat{i}_i^L(t + 1) \), but also depends on the linear deviation of the contemporaneous nominal short term bond yield of the i-th economy \( \hat{i}_i^S(t) \), adjusted by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign duration risk premium for the i-th economy \( \ln \nu_i^B(t) \), according to the following bond market relationship:

\[
\hat{i}_i^L(t) = \omega B \beta E_t \hat{i}_i^L(t + 1) + \left( \frac{1 - \omega B \beta}{\omega B + 1 - \omega B \beta} \right) \left( \hat{i}_i^S(t) + \ln \nu_i^B(t) \right)
\]

Here the logarithmic deviation of the weighted average of domestic and foreign duration risk premium for the i-th economy \( \ln \nu_i^B(t) \) is defined as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous duration risk premium shock of any specific economy except the i-th economy \( \sum_{j=1}^{N} \omega_j A \ln \nu_j^B(t) \), adds the logarithmic deviation of the contemporaneous duration risk premium shock of the i-th economy \( \ln \nu_i^B(t) \). \( \lambda_i^B \) represents the i-th economy's capital market contagion level:

\[
\ln \nu_i^B(t) = \lambda_i^B \sum_{j=1}^{N} \omega_j A \ln \nu_j^B(t) + (1 - \lambda_i^B \omega_i A) \ln \nu_i^B(t)
\]

For the convenience of introduction, we introduce the price of real estate developer equity that should belong to the previous construction sector, and the price of whole-economy-level and industry-level corporate equity that should belong to the previous structurally more complex production sector, together with their corresponding equity risk premium in this subsection.

The price of real estate developer equity (also called the price of housing) \( V_i^H(t) \) depends on its expected future value driven by expected future real estate developer profits, and also depends on the contemporaneous nominal interbank loans rate adjusted by the contemporaneous domestic housing risk premium shock.

Specifically, the logarithmic deviation of the price of housing \( \ln V_i^H(t) \) not only depends on its expected future value \( E_t \ln V_i^H(t + 1) \), driven by the logarithmic deviation of the expected future real estate developer profits \( E_t \ln \Pi_i^H(t + 1) \), but also depends on the linear deviation of the contemporaneous nominal interbank loans rate \( i_i^B(t) \), and is also influenced by the
logarithmic deviation of the contemporaneous domestic housing risk premium shock \( \ln \nu_i^H(t) \), according to the following housing market relationship:

\[
\ln \nu_i^H(t) = \beta E_t \ln \nu_i^H(t + 1) + (1 - \beta) E_t \ln \Pi_i^H(t + 1) - (\hat{i}_i(t) + \ln \nu_i^H(t))
\]

The price of whole-economy-level corporate equity \( V_i^S(t) \) depends on its expected future value driven by expected future whole-economy-level corporate profits, and also depends on the contemporaneous nominal short term bond yield adjusted by the weighted average of domestic and foreign whole-economy-level equity risk premium.

Specifically, the logarithmic deviation of the price of corporate equity \( \ln V_i^S(t) \) not only depends on its expected future value \( E_t \ln V_i^S(t + 1) \), driven by the logarithmic deviation of the expected future corporate profits \( E_t \ln \Pi_i^S(t + 1) \), but also depends on the linear deviation of the contemporaneous nominal short term bond yield \( \hat{i}_i^S(t) \), and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign equity risk premium for the i-th economy \( \ln \nu_i^S(t) \), according to the following corporate equity market relationship:

\[
\ln V_i^S(t) = \beta E_t \ln V_i^S(t + 1) + (1 - \beta) E_t \ln \Pi_i^S(t + 1) - (\hat{i}_i^S(t) + \ln \nu_i^S(t))
\]

Here the logarithmic deviation of the weighted average of domestic and foreign risk premium for the i-th economy \( \ln \nu_i^S(t) \) is defined as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous equity risk premium shock of any specific economy except the i-th economy \( \sum_{j=1}^N \omega_j \ln \nu_j^S(t) \), adds the logarithmic deviation of the contemporaneous equity risk premium shock of the i-th economy \( \ln \nu_i^S(t) \). \( \lambda_i^S \) represents the i-th economy's capital market contagion level:

\[
\ln \nu_i^S(t) = \lambda_i^S \sum_{j=1}^N \omega_j \ln \nu_j^S(t) + (1 - \lambda_i^S \omega_i^A) \ln \nu_i^S(t)
\]

The price of industry-level corporate equity \( V_{ik}^S(t) \) depends on its expected future value driven by expected future industry-level corporate profits, and also depends on the contemporaneous nominal short term bond yield adjusted by the weighted average of domestic and foreign industry-level equity risk premium.

The structure of the following equation defining the industry-level corporate equity price and the above equation defining the whole-economy-level corporate equity price is similar. Specifically, the logarithmic deviation of the price of corporate equity of the k-th industry of the i-th economy \( \ln V_{ik}^S(t) \) not only depends on its expected future value \( E_t \ln V_{ik}^S(t + 1) \), driven by the logarithmic deviation of the expected future corporate profits of the k-th industry of the i-th economy \( E_t \ln \Pi_{ik}^S(t + 1) \), but also depends on the linear deviation of the contemporaneous nominal short term bond yield of the i-th economy \( \hat{i}_i^S(t) \), and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign equity risk premium for the k-th industry of the i-th economy \( \ln \nu_{ik}^S(t) \) defined in the following equation, according to the industry-level equity market relationship adjusted correspondingly:

\[
\ln V_{ik}^S(t) = \beta E_t \ln V_{ik}^S(t + 1) + (1 - \beta) E_t \ln \Pi_{ik}^S(t + 1) - (\hat{i}_i^S(t) + \ln \nu_{ik}^S(t))
\]
We simply assume that the equity market contagion mechanism for equity price of listed companies in the same industry of various countries is similar to the equity market contagion mechanism among various countries for equity price of all listed companies that do not distinguish between industries, then the structure of the following equation defining the industry-level weighted average of domestic and foreign equity risk premium is similar to that of the above equation defining the whole-economy-level weighted average of domestic and foreign equity risk premium. Specifically, the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the k-th industry of the i-th economy \( \ln \tilde{\upsilon}_{i,k}^S(t) \) is defined as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous equity risk premium shock for the k-th industry of any specific economy except the i-th economy \( \left( \sum_{j=1}^{N} \omega_j^A \ln \tilde{\upsilon}_{j,k}^S(t) - \omega_i^A \ln \tilde{\upsilon}_{i,k}^S(t) \right) \), adds the logarithmic deviation of the contemporaneous equity risk premium shock for the k-th industry of the i-th economy:

\[
\ln \tilde{\upsilon}_{i,k}^S(t) = \lambda_i^S \sum_{j=1}^{N} \omega_j^A \ln \tilde{\upsilon}_{j,k}^S(t) + (1 - \lambda_i^S \omega_i^A) \ln \tilde{\upsilon}_{i,k}^S(t)
\]

Here the parameter \( \lambda_i^S \) is the same as that in the previous equation defining the whole-economy-level weighted average of domestic and foreign equity risk premium.

The nominal effective long term market interest rate of the i-th economy \( i_i^L(t) \), which represents the return rate of domestic vintage diversified long term bonds \( B_i^L(t) \) that are held by the fiscal authority as part of accumulated public financial wealth, depends on its past value, driven by the contemporaneous nominal long term bond yield of the i-th economy.

Specifically, the linear deviation of the nominal effective long term market interest rate of the i-th economy \( i_i^L(t) \) depends on its past value \( i_i^L(t - 1) \), driven by the linear deviation of the contemporaneous nominal long term bond yield of the i-th economy:

\[
i_i^L(t) = \omega_i^B i_i^L(t-1) + \omega_i^B (1 - \omega_i^B)(\omega_i^B + \frac{1-\omega_i^B}{\omega_i^B}) i_i^L(t)
\]

**N. Settings for Market Clearing Conditions**

All market clearing conditions not yet discussed by introduction of the above sectors or markets are introduced as follows.

Clearing of the output market for industry specific final output goods or services requires satisfaction of the following equation, which is the constraint among industry specific final output goods or services produced by the k-th industry of the i-th economy \( Y_{i,k}(t) \), a fraction \( \omega_{h,k} \) of the industry specific final output goods or services to serve as intermediate inputs for the production of all domestic industries of the i-th economy, the domestically produced industry specific final output goods or services to serve as inputs for the domestic absorption sector of the i-th economy \( \{ C_{i,k}(t), I_{i,k}(t), G_{i,k}(t), C_{i,k}^L(t), C_{i,k}^G(t) \} \), and the industry specific final export goods or services \( X_{i,k}(t) \) that would become intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector:
\[ Y_{i,k}(t) = \omega^h_{i,k-\text{all domestic industries}} Y_{i,k}(t) + \left( C^h_{i,k}(t) + I^H_{i,k}(t) + I^K_{i,k}(t) + G^C_{i,k}(t) + G^L_{i,k}(t) \right) + X_{i,k}(t) \]

Clearing of the import market for industry specific final import goods or services requires satisfaction of the following equation, which is the constraint among industry specific final import goods or services from the k-th industry of all other economies except the i-th economy \( M_{i,k}(t) \), a fraction \( \omega^f_{\text{all other economies' k-th industry-\text{all domestic industries}} of the industry specific final import goods or services from foreign k-th industry to serve as intermediate inputs for the production of all domestic industries of the i-th economy, and the foreign produced industry specific final import goods or services to serve as inputs for the domestic absorption sector of the i-th economy \( \{ C^f_{i,k}(t), I^H_{i,k}(t), I^K_{i,k}(t), G^C_{i,k}(t), G^L_{i,k}(t) \} \):

\[ M_{i,k}(t) = \omega^f_{\text{all other economies' k-th industry-\text{all domestic industries}}} M_{i,k}(t) + \left( C^f_{i,k}(t) + I^H_{i,k}(t) + I^K_{i,k}(t) + G^C_{i,k}(t) + G^L_{i,k}(t) \right) \]

Combination of the above output market clearing condition and the import market clearing condition yields the following output expenditure decomposition:

\[ P^X_{i,k}(t) \left( 1 - \omega^h_{i,k-\text{all domestic industries}} \right) Y_{i,k}(t) = \left( P^h_{i,k}(t) C_{i,k}(t) + P^h_{i,k}(t) I^H_{i,k}(t) + P^h_{i,k}(t) I^K_{i,k}(t) + P^h_{i,k}(t) G^C_{i,k}(t) + P^h_{i,k}(t) I^K_{i,k}(t) + P^M\text{T}_{i,k}(t) \left( 1 - \omega^f_{\text{all other economies' k-th industry-\text{all domestic industries}}} \right) M_{i,k}(t) \]

Here, \( P^X_{i,k}(t) \) is the industry specific output price;
\( P^h_{i,k}(t) \) is the industry specific absorption price for the industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services \( \{ C_{i,k}(t), I^H_{i,k}(t), I^K_{i,k}(t), G^C_{i,k}(t), G^L_{i,k}(t) \} \); \( P^M\text{T}_{i,k}(t) \) is the industry specific export price;
\( P^M\text{T}_{i,k}(t) \) is the industry specific pre-tariff import price.

Since the industry specific total investment \( I_{i,k}(t) \) equals the sum of the industry specific residential investment \( I^H_{i,k}(t) \) and the industry specific business investment \( I^K_{i,k}(t) \), the industry specific domestic public demand \( G_{i,k}(t) \) equals the sum of the industry specific public consumption \( G^C_{i,k}(t) \) and the industry specific public investment \( G^L_{i,k}(t) \), and the industry specific demand \( D_{i,k}(t) \) equals the sum of the industry specific private consumption \( C_{i,k}(t) \), the industry specific total investment \( I_{i,k}(t) \) and the industry specific demand \( D_{i,k}(t) \), then the above output expenditure decomposition can also be expressed as follows:

\[ P^X_{i,k}(t) \left( 1 - \omega^h_{i,k-\text{all domestic industries}} \right) Y_{i,k}(t) = \left( P^h_{i,k}(t) C_{i,k}(t) + P^h_{i,k}(t) I^H_{i,k}(t) + P^h_{i,k}(t) I^K_{i,k}(t) + P^M\text{T}_{i,k}(t) \left( 1 - \omega^f_{\text{all other economies' k-th industry-\text{all domestic industries}}} \right) M_{i,k}(t) \]

Here, \( P^X_{i,k}(t) \) is the industry specific output price;
\( P^h_{i,k}(t) \) is the industry specific absorption price for the industry specific final private consumption, residential investment, business investment, public consumption and public investment goods or services \( \{ C_{i,k}(t), I^H_{i,k}(t), I^K_{i,k}(t), G^C_{i,k}(t), G^L_{i,k}(t) \} \); \( P^M\text{T}_{i,k}(t) \) is the industry specific export price;
\( P^M\text{T}_{i,k}(t) \) is the industry specific pre-tariff import price.
\[ M_{i,k}(t) = p_{i,k}^h(t)D_{i,k}(t) + p_{i,k}^X(t)X_{i,k}(t) - p_{i,k}^{M,T}(t) \left( 1 - \omega_{\text{all other economies' } k\text{-th industry to i, all domestic industries}} \right) M_{i,k}(t) \]

Clearing of the lending market not only requires that the domestic mortgage loan supply by the banking sector of the i-th economy \( B_{i}^{C,D}(t) \) equals the total borrowing demand of real estate developers of the i-th economy \( B_{i}^{C,D}(t) \):

\[ B_{i}^{C,D}(t) = B_{i}^{C,D}(t) \]

but also requires that the economy specific corporate loan supply by the banking sector of the i-th economy for firms of all economies \( B_{i}^{C,F}(t) \) equals the total borrowing demand for corporate loans from the banking sector of the i-th economy by firms of all economies \( \sum_{j=1}^{N} B_{j,i}^{C,F}(t) \):

\[ B_{i}^{C,F}(t) = \sum_{j=1}^{N} B_{j,i}^{C,F}(t) \]

In other words, if we additionally define \( B_{i,j}^{C,F}(t) \) as the economy specific corporate loan supply by the banking sector of the i-th economy for firms of the j-th economy, then it requires that:

\[ B_{i,j}^{C,F}(t) = B_{i,j}^{C,F}(t) \]

**IV. All Industry-Level Equations of the AGMFM**

This section and Appendix II introduce all linearized relationship equations, which constitute the linear state space representation of an approximate multivariate linear rational expectations representation of the mathematical model introduced in Section III. All the linearized relationship equations are derived by analytically linearizing the equilibrium conditions of the mathematical model around its stationary deterministic steady state equilibrium, and consolidating them by substituting out intermediate variables.

In both this section and Appendix II, we will specify which equations come from the original GFM introduced in Vitek (2015, 2018). However, we have reasonably adjusted their timeline or some of their parameters, since our complete replication of the original model shows that it could be too far to pass the verification of rank condition. We will also specify which equations come from our own innovative work, such as equations around further incorporating the structure of import tariff and the structure of international direct investment into the AGMFM model, and equations around building the intra-industry and inter-industry structure that does not exist in the original model. Other equations not specifically pointed out are consistent with the corresponding equations in the original model.

In this section, we only focus on all the industry-level equations that characterize industries under the whole economy and their inter-country cross-industry input-output relations, and leave the introduction of whole-economy-level or international-market-level equations to Appendix II. The aim of innovatively constructing all these industry-level equations is to incorporate an intra-industry and inter-industry structure for each economy, fully characterize
the massive and accurate inter-country cross-industry input-output relations, and utilize the rich industry-level information provided by the OECD ICIO tables as much as possible.

In addition, we allow all the following innovatively constructed industry-level equations to be flexibly added or removed from the model, when model users sometimes don’t care about the details of industries under the whole economy, or want to reduce the scale of the model as much as possible. Then the structure and parameters of some of the whole-economy-level equations or international-market-level equations introduced in Appendix II need to be modified accordingly. In Appendix II, we only introduce the forms of all the whole-economy-level equations and international-market-level equations in the complete AGMFM, which contains all the following innovatively constructed industry-level equations, and leave their modified forms in the simplified model without all those industry-level equations to be introduced in the following Part C of Section IV.

In all the following equations:

\[ \text{Variable}_i(t) \] is used to represent the linear deviation of \( \text{Variable}_i(t) \) from its steady state equilibrium value \( \text{Variable}_i(t) \):

\[ \text{Variable}_i(t) = \text{Variable}_i(t) - \text{Variable}_i(t) \]

\( \ln \text{Variable}_i(t) \) is used to represent the logarithmic deviation of \( \text{Variable}_i(t) \) from its steady state equilibrium value \( \text{Variable}_i(t) \):

\[ \ln \text{Variable}_i(t) = \ln \text{Variable}_i(t) - \ln \text{Variable}_i(t) = \frac{\text{Variable}_i(t) - \text{Variable}_i(t)}{\text{Variable}_i(t)} \]

Following the unified settings of subscripts in Section III: variable subscript "i" or "j" or "i*" represents the i-th or j-th or i*-th economy, \( i \) or \( j \) \( \in \{1,2, \ldots, N\} \), \( N \) is the total number of economies contained in the large-scale AGMFM, and for most cases \( i^* \) is adopted to represent that the \( i^*-th \) economy issues the anchor currency for the currency of the i-th economy; variable subscript "k" or "m" represents the k-th or m-th industry, and \( k \) or \( m \) \( \in \{1,2, \ldots, 45\} \).

Unless otherwise specified, all price variables in both this section and Appendix II refer to the corresponding price level denominated in domestic currency.

**A. Details within and Mechanisms between Industries**

The innovatively constructed equation (4.A.1) shows the logarithmic deviation of the real output of the k-th industry of the i-th economy \( \ln \bar{Y}_{i,k}(t) \) not only depends on the logarithmic deviation of the contemporaneous utilized private physical capital stock of the i-th economy \( \left( \ln \bar{K}_i(t) + \ln \bar{K}_i(t) \right) \) and the logarithmic deviation of the contemporaneous effective employment of the k-th industry of the i-th economy \( \left( \ln \bar{A}_{i,k}(t) + \ln \bar{L}_i(t) \right) \), but also depends on both the ratio of domestic and foreign industrial intermediate goods or services (used as intermediate inputs for domestic production sector) to industrial real output and the input-output elasticity weighted average of the logarithmic deviation of the contemporaneous real output of any specific industry of any specific economy

\[ \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m-i}^{l-k} \times \text{elasticity}_{j,m-i,k} \ln \bar{Y}_{j,m}(t)}{Y_{i,k}} \], according to the detailed industry-level production...
function supported by each column of the OECD ICIO table. Here we introduce the logarithmic deviation of the output demand shock for the intermediate or final goods or services from the k-th industry of the i-th economy that would serve as intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector \( \ln Y_{i,k}^{output} \) and \( \ln Y_{i,k}^{input} \), and the logarithmic deviation of the intermediate input shock for the intermediate goods or services from the m-th industry of the j-th economy to the k-th industry of the i-th economy \( \ln Y_{j,m}^{input} \) into this equation, in order to better serve the specific needs of various research topics:

\[
\ln Y_{i,k}(t) + \ln Y_{i,k}^{output} = Y_{i,k} - \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m}^{input}}{Y_{i,k}} \text{elasticity}_{GVA-i,k} \left[ \left( 1 - \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m}^{input}}{Y_{i,k}} \right) \left( \ln \tilde{u}_{i,k}^{K}(t) + \ln \tilde{K}_{i}(t) + \frac{\theta_{Y}^{v}}{\theta_{Y}^{v-1}} \frac{\partial_{v} W_{i}^{I}}{\partial_{v-1} Y_{i}} \left( \ln \tilde{u}_{i,k}^{K}(t) + \ln \tilde{K}_{i}(t) \right) \right) + \right.
\]

\[
\left. \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m}^{input}}{Y_{i,k}} \text{elasticity}_{j,m-i,k} \left( \ln Y_{j,m}(t) - \ln Y_{j,m}^{input} \right) \right] (4.A.1)
\]

Here,

\[
\frac{Y_{i,k} - \sum_{j=1}^{N} \sum_{m=1}^{45} Y_{j,m}^{input}}{Y_{i,k}} \text{ represents the ratio of industrial real output minus all intermediate inputs of the k-th industry of the i-th economy to the real output of the k-th industry of the i-th economy, in other words, it is the ratio of the GVA part of the k-th industry of the i-th economy, produced by capital and labor as input factors, to the real output of the k-th industry of the i-th economy;}
\]

\[
\frac{Y_{j,m}^{input}}{Y_{i,k}} \text{ represents the ratio of goods or services from the m-th industry of the j-th economy that are adopted as intermediate inputs of the k-th industry of the i-th economy to the real output of the k-th industry of the i-th economy.}
\]

In addition, \( \text{elasticity}_{GVA-i,k} \) represents the input-output elasticity specifically for the GVA part of the k-th industry of the i-th economy (produced by capital and labor as input factors) and the real output of the k-th industry of the i-th economy; \( \text{elasticity}_{j,m-i,k} \) represents the input-output elasticity specifically for the input from the m-th industry of the j-th economy to the k-th industry of the i-th economy and the real output of the k-th industry of the i-th economy. All these input-output elasticities are usually estimated based on the annual OECD ICIO tables forming a sufficiently long time series. Under the most simplified treatment, if the importance of GVA and goods or services from different industries of different countries to serve as intermediate inputs for the production of the k-th industry of the i-th economy are regarded as the same, the values of all the above input-output elasticities can be directly taken as 1.

Compared with the above equation defining the industry-level real output, the following innovatively constructed equation (4.A.2) has a similar structure to the above equation (4.A.1), their only difference is that equation (4.A.2) uses the logarithmic deviation of the contemporaneous potential output of the m-th industry of the j-th economy \( \ln Y_{j,m}^{K}(t) \) to replace the logarithmic deviation of the contemporaneous real output of the m-th industry of the j-th economy \( \ln Y_{j,m}(t) \), uses the logarithmic deviation of the contemporaneous private physical capital stock of the i-th economy \( \ln \tilde{K}_{i}(t) \) instead of the logarithmic deviation of the contemporaneous utilized private physical capital stock of the i-th economy \( \left( \ln \tilde{u}_{i,k}^{K}(t) + \ln \tilde{K}_{i}(t) \right) \), and uses the logarithmic deviation of the contemporaneous effective labor force of
the i-th economy \( \ln \tilde{A}_i(t) + \ln \tilde{N}_i(t) \) instead of the logarithmic deviation of the contemporaneous effective employment of the i-th economy \( \ln \tilde{A}_i(t) + \ln \tilde{L}_i(t) \), to define the potential output of the k-th industry of the i-th economy (the inferred industry-level total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \ln \tilde{Y}_{i,k}(t) \):

\[
\ln \tilde{Y}_{i,k}(t) + \ln Y_{output}^{all\,economies}(t) = \frac{Y_{i,k} - \sum_{j=1}^{N} \sum_{m=1}^{45} Y_{j,m-i,k}}{Y_{i,k}} \text{elasticity}_{GVA-i,k} \left[ 1 - \frac{\theta Y}{\theta Y - 1} \frac{W_{l,i}}{p_{l,i}Y_i} \left( \ln \tilde{A}_i(t) + \ln \tilde{N}_i(t) \right) \right] + \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m-i,k}}{Y_{i,k}} \text{elasticity}_{j,m-i,k} \left( \ln \tilde{Y}_{j,m}(t) - \ln Y_{input}^{i,m-i,k}(t) \right)
\]

(4.A.2)

The innovatively constructed equation (4.A.3) shows the logarithmic deviation of the total exports from the k-th industry of the i-th economy \( \ln \tilde{X}_{i,k}(t) \) not only depends on the ratio of exported industrial intermediate goods or services (used as intermediate inputs for foreign production sector) to industrial total exports weighted average of the logarithmic deviation of the contemporaneous real output of the m-th industry of the j-th economy (since part of the exports of the k-th industry of the i-th economy serve as intermediate inputs for the m-th industry of the j-th economy, we simply assume that the upstream export quantity of the k-th industry of the i-th economy can be affected by the rise and fall of the downstream output quantity of the m-th industry of the j-th economy) \( \sum_{j=1}^{N} \sum_{m=1}^{45} Y_{j,m-i,k} / Y_{i,k} \left( \ln \tilde{Y}_{j,m}(t) \right) \), but also depends on the ratio of exported industrial final goods or services (used as final goods or services for foreign private consumption, private investment or public purchase) to industrial total exports weighted average of the logarithmic deviation of the contemporaneous private consumption, private investment or public purchase in the j-th economy (since part of the exports of the k-th industry of the i-th economy serve as final goods or services for private consumption, private investment or public purchase of the j-th economy, we simply assume that the upstream export quantity of the k-th industry of the i-th economy can be affected by the rise and fall of the downstream demand quantity of the m-th industry of the j-th economy) \( \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m-i,k}}{Y_{i,k}} \ln Y_{i,k}(t) \), but also introduces the logarithmic deviation of private consumption shock for final goods or services used for private consumption in the j-th economy from the k-th industry of the i-th economy \( \ln \tilde{C}_{i,k-j}(t) \), and the logarithmic deviation of private investment shock for final goods or services used for private investment in the j-th economy from the k-th industry of the i-th economy \( \ln \tilde{I}_{i,k-j}(t) \), and the logarithmic deviation of public purchase shock for final goods or services used for public consumption or public investment in the j-th economy from the k-th industry of the i-th economy \( \ln \tilde{G}_{i,k-j}(t) \) into it to support various research needs:
\[
\ln X_{i,k}(t) = \sum_{j=1}^{N} \sum_{m=1}^{M_{k \rightarrow j}} \frac{X_{i,k \rightarrow j,m}}{M_{i,k}} \left( \ln \hat{Y}_{j,m}(t) - \ln \hat{Y}_{k \rightarrow j,m}(t) \right) + \sum_{j=1}^{N} \left[ \frac{X_{i,k \rightarrow j,\text{private consumption}}}{M_{i,k}} \left( \ln \hat{C}_{j}(t) - \ln \hat{C}_{k \rightarrow j}(t) \right) + \frac{X_{i,k \rightarrow j,\text{private investment}}}{M_{i,k}} \left( \ln \hat{I}_{j}(t) - \ln \hat{I}_{k \rightarrow j}(t) \right) + \frac{X_{i,k \rightarrow j,\text{public purchase}}}{M_{i,k}} \left( \ln \hat{G}_{j}(t) - \ln \hat{G}_{k \rightarrow j}(t) \right) \right]
\]

(4.A.3)

Here, \( \frac{X_{i,k \rightarrow j,m}}{X_{i,k}} \) represents the ratio of the exported intermediate goods or services from the k-th industry of the i-th economy that are adopted as intermediate inputs of the m-th industry of the j-th economy to the total exports of the k-th industry of the i-th economy; \( \frac{X_{i,k \rightarrow j,\text{private consumption}}}{X_{i,k}}, \frac{X_{i,k \rightarrow j,\text{private investment}}}{X_{i,k}} \) and \( \frac{X_{i,k \rightarrow j,\text{public purchase}}}{X_{i,k}} \) correspondingly represent the ratio of the exported final goods or services from the k-th industry of the i-th economy for private consumption, private investment or public purchase of the j-th economy to the total imports of the k-th industry of the i-th economy.

The innovatively constructed equation (4.A.4) shows the logarithmic deviation of the total imports of the i-th economy from the k-th industry of all other economies \( \ln M_{i,k}(t) \) not only depends on the ratio of imported industrial intermediate goods or services (used as intermediate inputs for domestic production sector) to industrial total imports weighted average of the logarithmic deviation of the contemporaneous output of the k-th industry of the j-th economy (since part of the imports from the k-th industry of the j-th economy serve as intermediate inputs for all industries of the i-th economy, we simply assume that the downstream import quantity of all industries of the i-th economy can be affected by the rise and fall of the upstream output quantity of the k-th industry of the j-th economy)

\[
\sum_{j=1}^{N} \frac{M_{i,k \rightarrow j,\text{intermediate inputs}}}{M_{i,k}} \left( \ln \hat{Y}_{j,k}(t) \right), \text{ but also depends on the ratio of imported industrial final goods or services (used as final goods or services for domestic private consumption, private investment or public purchase) to industrial total imports weighted average of the logarithmic deviation of the contemporaneous private consumption, private investment or public purchase in the i-th economy (since part of the imports from the k-th industry of the j-th economy serve as final goods or services for private consumption, private investment or public purchase of the i-th economy, we simply assume that the import quantity of the absorption sector of the i-th economy for final goods or services from the k-th industry of the j-th economy can be affected by the rise and fall of the downstream private and public demand of the i-th economy)} + \sum_{j=1}^{N} \left[ \frac{M_{i,k \rightarrow j,\text{private consumption}}}{M_{i,k}} \left( \ln \hat{C}_{j}(t) \right) + \frac{M_{i,k \rightarrow j,\text{private investment}}}{M_{i,k}} \left( \ln \hat{I}_{j}(t) \right) + \frac{M_{i,k \rightarrow j,\text{public purchase}}}{M_{i,k}} \left( \ln \hat{G}_{j}(t) \right) \right], \text{ according to the detailed industry-level import flow supported by corresponding column of the OECD ICIO table. For this equation, we not only introduce the logarithmic deviation of the output demand shock for the intermediate goods or services from the k-th industry of the j-th economy demanded by all industries of the i-th economy \( \ln \hat{Y}_{k \rightarrow j,\text{intermediate inputs}}(t) \), but also introduce the logarithmic deviation of private consumption shock for final goods or services used for private consumption in the i-th economy from the k-th industry of the j-th economy \( \ln \hat{C}_{k \rightarrow j}(t) \), the logarithmic deviation of private investment shock for final goods or services used for private investment in the i-th economy from the k-th industry of the j-th economy \( \ln \hat{I}_{k \rightarrow j}(t) \), and the logarithmic deviation of public purchase shock for final goods or services used for public
consumption or public investment in the i-th economy from the k-th industry of the j-th economy \( \ln \tilde{M}_{k ightarrow i}(t) \) into it to support relevant research needs:

\[
\ln \tilde{M}_{k ightarrow i}(t) = \sum_{j=1}^{N} \frac{\sum_{m=1}^{45} Y_{k ightarrow j}^{G}(t) \cdot (\ln \tilde{G}_{k ightarrow j}(t) - \ln \tilde{Y}_{k ightarrow j}^{G}(t))}{M_{k ightarrow i}} \\
+ \sum_{j=1}^{N} \frac{M_{j ightarrow k ightarrow private consumption}}{M_{k ightarrow i}} \left( \ln \tilde{C}_{j}(t) - \ln \tilde{Y}_{k ightarrow j}(t) \right) + \frac{M_{j ightarrow k ightarrow private investment}}{M_{k ightarrow i}} \left( \ln \tilde{I}_{j}(t) - \ln \tilde{Y}_{k ightarrow j}(t) \right) + \\
\sum_{j=1}^{N} \frac{M_{j ightarrow k ightarrow public purchase}}{M_{k ightarrow i}} \left( \ln \tilde{G}_{j}(t) - \ln \tilde{Y}_{k ightarrow j}(t) \right) \\
\text{(4.A.4)}
\]

Here, \( \frac{M_{j ightarrow k ightarrow intermediate inputs}}{M_{k ightarrow i}} \) represents the ratio of the imported intermediate goods or services from the k-th industry of the j-th economy that are adopted as intermediate inputs of all industries of the i-th economy to the total imports of the i-th economy from foreign k-th industry; \( \frac{M_{j ightarrow k ightarrow private consumption}}{M_{k ightarrow i}}, \frac{M_{j ightarrow k ightarrow private investment}}{M_{k ightarrow i}} \) and \( \frac{M_{j ightarrow k ightarrow public purchase}}{M_{k ightarrow i}} \) correspondingly represent the ratio of the imported final goods or services from the k-th industry of the j-th economy for private consumption, private investment or public purchase of the i-th economy to the total imports of the i-th economy from foreign k-th industry.

The innovatively constructed equation (4.A.5) shows the logarithmic deviation of the US dollar denominated output price level for the output of the k-th industry of the i-th economy \( \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) \) not only depends on the ratio of industrial intermediate goods or services (used as intermediate inputs for domestic and foreign production sector) to industrial output weighted average of the logarithmic deviation of the contemporaneous US dollar denominated output price level for the output of the m-th industry of the j-th economy (since part of the output of the k-th industry of the i-th economy serves as intermediate input for the m-th industry of the j-th economy, we simply assume that the upstream output price of the k-th industry of the i-th economy can be affected by the rise and fall of the downstream output price of the m-th industry of the j-th economy) \( \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j ightarrow m}}{Y_{k ightarrow i}} \left( \ln E_{1,j}(t) + \ln P_{1,m}(t) \right) \), but also depends on the ratio of final goods or services (used for private consumption, private investment or public purchase in domestic and foreign economies) to industrial output weighted average of the logarithmic deviation of the contemporaneous US dollar denominated domestic core price or export price for final goods or services provided by the k-th industry of the i-th economy for private consumption, private investment or public purchase in the j-th economy (since part of the output of the k-th industry of the i-th economy serves as final goods or services for private consumption, private investment or public purchase of the j-th economy, we simply assume that the upstream output price of the k-th industry of the i-th economy can be affected by the rise and fall of both the downstream core price for final goods or services in the i-th economy and the downstream export price of final goods or services from the i-th economy to the j-th economy)

\[
\left( \frac{Y_{j ightarrow k ightarrow private consumption}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) + \frac{Y_{j ightarrow k ightarrow private investment}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) + \\
\frac{Y_{j ightarrow k ightarrow public purchase}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) \right) \\
+ \sum_{i=1}^{N} \left( \frac{Y_{j ightarrow k ightarrow private consumption}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) + \frac{Y_{j ightarrow k ightarrow private investment}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) + \\
\frac{Y_{j ightarrow k ightarrow public purchase}}{Y_{k ightarrow i}} \left( \ln E_{1,i}(t) + \ln P_{1,i}(t) \right) \right),
\]

according to the detailed industry-level output distribution supported by each row of the OECD ICIO table:
\[
\ln \hat{E}_{1i}(t) + \ln \hat{P}^Y(t) = \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{1k-m}}{Y_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}^Y(t) \right) + \\
\frac{Y_{1k-i, private \ consumption}}{Y_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \frac{Y_{1k-i, private \ investment}}{Y_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \\
\frac{Y_{1k-i, public \ purchase}}{Y_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \frac{Y_{1k-i, public \ purchase}}{Y_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \\
\frac{Y_{1k-j, private \ consumption}}{Y_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \frac{Y_{1k-j, private \ investment}}{Y_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \\
\frac{Y_{1k-j, public \ purchase}}{Y_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right)
\]

(4.A.5)

Here, \( \frac{Y_{1k-i, m}}{Y_{1k}} \) represents the ratio of the intermediate goods or services from the k-th industry of the i-th economy that are adopted as intermediate inputs of the m-th industry of the j-th economy to the real output of the k-th industry of the i-th economy; \( \frac{Y_{1k-i, private \ consumption}}{Y_{1k}} \), \( \frac{Y_{1k-i, private \ investment}}{Y_{1k}} \) and \( \frac{Y_{1k-i, public \ purchase}}{Y_{1k}} \) correspondingly represent the ratio of the final goods or services from the k-th industry of the i-th economy for private consumption, private investment or public purchase of the j-th economy to the real output of the k-th industry of the i-th economy.

The innovatively constructed equation (4.A.6) shows the logarithmic deviation of the US dollar denominated consumption price level in the i-th economy for final goods or services (used for private consumption, private investment or public purchase) from the k-th industry of all economies \( \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) \) depends on the ratio of industrial final goods or services from a specific economy (used for private consumption, private investment or public purchase of the i-th economy) to industrial final goods or services from all economies weighted average of the logarithmic deviation of the contemporaneous US dollar denominated domestic core price or import price for final goods or services from foreign k-th industry (since part of the output of the k-th industry of all economies serves as final goods or services for private consumption, private investment or public purchase of the i-th economy, we simply assume that the consumption price in the i-th economy for final goods or services from domestic and foreign k-th industry can be affected by the rise and fall of both the core price for final goods or services in the i-th economy and the import price of final goods or services from the j-th economy to the i-th economy).

\[
\frac{C_{1k-i, private \ consumption}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \frac{C_{1k-i, private \ investment}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \\
\frac{C_{1k-i, public \ purchase}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \frac{C_{1k-j, private \ consumption}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \\
\frac{C_{1k-j, private \ investment}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \frac{C_{1k-j, public \ purchase}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right)
\]

(4.A.6)

according to the detailed industry-level consumption distribution supported by corresponding column of the OECD ICIO table:

\[
\ln \hat{E}_{1i}(t) + \ln \hat{P}(t) = \frac{C_{1k-i, private \ consumption}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \\
\frac{C_{1k-i, private \ investment}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \frac{C_{1k-i, public \ purchase}}{C_{1k}} \left( \ln \hat{E}_{1i}(t) + \ln \hat{P}(t) \right) + \\
\sum_{j=1}^{N} \frac{C_{1k-j, private \ consumption}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \frac{C_{1k-j, private \ investment}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right) + \\
\sum_{j=1}^{N} \frac{C_{1k-j, public \ purchase}}{C_{1k}} \left( \ln \hat{E}_{1j}(t) + \ln \hat{P}(t) \right)
\]

(4.A.6)
Here, \( \frac{C_{jk,i}}{C_{ik}} \), \( \frac{C_{jk,i}}{C_{ik}} \), and \( \frac{C_{jk,i}}{C_{ik}} \) correspondingly represent the ratio of the final goods or services from the k-th industry of the j-th economy for private consumption, private investment or public purchase of the i-th economy to the final goods or services from the k-th industry of all economies for private consumption, private investment or public purchase of the i-th economy.

The new equation (4.A.7) added accordingly defines the linear deviation of the export price inflation for the exported output of the k-th industry of the i-th economy \( \pi_{i,k}^X(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the price of exports for the exported output of the k-th industry of the i-th economy \( \ln\Pi_{i,k}^X(t) \):

\[
\pi_{i,k}^X(t) = \ln\Pi_{i,k}^X(t) - \ln\Pi_{i,k}^X(t-1)
\]

(4.A.7)

The following innovatively constructed equations about the logarithmic deviation of the price of exports for the exported output of different industries in any economy are different, and they are the decomposed equations of the equation (Appendix.G.6) in the original model defining the whole-economy-level export price that does not distinguish between industries (please refer to the following Part C of this section for details of this original equation).

For those industries of the i-th economy that produce internationally homogeneous energy commodities, equation (4.A.8a) shows the linear deviation of the export price inflation for the exported output of the k-th industry of the i-th economy \( \pi_{i,k}^X(t) \) not only depends on a linear combination of its past and expected future values, driven by the difference between the logarithmic deviation of the contemporaneous i-th economy’s currency denominated price of internationally homogeneous energy commodities \( \ln\hat{E}^1_{i,1}(t) + \ln\hat{P}_e^Y(t) \) and the logarithmic deviation of the contemporaneous price of exports for the exported output of the k-th industry of the i-th economy \( \ln\hat{P}_{i,k}^X(t) \), but also depends on the contemporaneous, past and expected future values of the logarithmic deviation of the i-th economy’s currency denominated price of internationally homogeneous energy commodities \( \ln\hat{E}^1_{i,1}(t) + \ln\hat{P}_e^Y(t) \), and is also influenced by the logarithmic deviation of the contemporaneous industry-level export price markup shock \( \ln\hat{\delta}_i^{X,k}(t) \), according to industry-level export price Phillips curve:

\[
\pi_{i,k}^X(t) = \frac{\gamma X(1-\mu)}{1+\beta Y(1-\mu)} \hat{n}_{i,k}^X(t-1) + \frac{\beta}{1+\beta Y(1-\mu)} E_i \hat{n}_{i,k}^X(t+1) + \frac{(1-\omega X)(1-\omega Y)}{\omega X(1+\beta Y(1-\mu))} \left[ (\ln\hat{E}^1_{i,1}(t) + \ln\hat{P}_e^Y(t)) - \frac{1}{1+\beta} (\ln\hat{E}^1_{i,1}(t-1) + \ln\hat{P}_e^Y(t-1)) \right] - \frac{\beta}{1+\beta} E_i \left( \ln\hat{E}^1_{i,1}(t+1) + \ln\hat{P}_e^Y(t+1) \right) \]

(4.A.8a)

For those industries that produce internationally homogeneous nonenergy commodities, equation (4.A.8b) shows the linear deviation of the export price inflation for the exported output of the k-th industry of the i-th economy \( \pi_{i,k}^X(t) \) not only depends on a linear combination of its past and expected future values, driven by the difference between the logarithmic deviation of the contemporaneous i-th economy’s currency denominated price of
internationally homogeneous nonenergy commodities \((\ln E_{i,1}(t) + \ln P_{ne}^Y(t))\) and the logarithmic deviation of the contemporaneous price of exports for the exported output of the k-th industry of the i-th economy \(\ln P_{i,k}^X(t)\), but also depends on the contemporaneous, past and expected future values of the logarithmic deviation of the i-th economy’s currency denominated price of internationally homogeneous nonenergy commodities \((\ln E_{i,1}(t) + \ln P_{ne}^Y(t))\), and is also influenced by the logarithmic deviation of the contemporaneous industry-level export price markup shock \(\ln \delta_{i,k}^X(t)\), according to industry-level export price Phillips curve:

\[
\pi_{i,k}^X(t) = \gamma_{i,k}^X(1-\mu_{i,k})_{1+\beta_{i,k}Y(1-\mu_{i,k})} E_i \pi_{i,k}^X(t+1) + \frac{(1-\omega_{i,k}Y)(1-\omega_{i,k}^Y\beta)}{\omega_{i,k}^Y(1+\beta_{i,k}Y(1-\mu_{i,k}))} \left[(\ln E_{i,1}(t) + \ln P_{ne}^Y(t) - \ln P_{i,k}^X(t) + \ln \delta_{i,k}^X(t)\right] + \frac{\mu_{i,k}Y}{1+\beta_{i,k}Y(1-\mu_{i,k})} \left[(\ln E_{i,1}(t) + \ln P_{ne}^Y(t) - E_t \ln P_{i,k}^X(t+1) + (1-\omega_{i,k}Y)(1-\omega_{i,k}^Y\beta)\omega_{i,k}^Y(1+\beta_{i,k}Y(1-\mu_{i,k}))\right]
\]

(4.A.8b)

For those industries that produce internationally heterogeneous goods or services, equation (4.A.8c) shows the linear deviation of the export price inflation for the exported output of the k-th industry of the i-th economy \(\pi_{i,k}^X(t)\) not only depends on a linear combination of its past and expected future values, driven by the difference between the logarithmic deviation of the contemporaneous core price level of the i-th economy \(\ln P_{i}(t)\) and the logarithmic deviation of the contemporaneous price of exports for the exported output of the k-th industry of the i-th economy \(\ln P_{i,k}^X(t)\), but also depends on the contemporaneous, past and expected future values of the linear deviation of the core inflation of the i-th economy \(\pi_{i}(t)\), according to industry-level export price Phillips curve:

\[
\pi_{i,k}^X(t) = \gamma_{i,k}^X(1-\mu_{i,k})_{1+\beta_{i,k}Y(1-\mu_{i,k})} \pi_{i,k}(t-1) + \frac{\beta}{1+\beta_{i,k}Y(1-\mu_{i,k})} E_i \pi_{i,k}^X(t+1) + \frac{(1-\omega_{i,k}Y)(1-\omega_{i,k}^Y\beta)}{\omega_{i,k}^Y(1+\beta_{i,k}Y(1-\mu_{i,k}))} \left[(\ln E_{i,1}(t) + \ln P_{ne}^Y(t) - \ln P_{i,k}^X(t) + \ln \delta_{i,k}^X(t) - E_t \ln P_{i,k}^X(t+1) + (1-\omega_{i,k}Y)(1-\omega_{i,k}^Y\beta)\omega_{i,k}^Y(1+\beta_{i,k}Y(1-\mu_{i,k}))\right]
\]

(4.A.8c)

The new equation (4.A.9) added accordingly defines the linear deviation of the post-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies \(\pi_{i,k}^M(t)\), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the post-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies \(\ln P_{i,k}^M(t)\):

\[
\pi_{i,k}^M(t) = \ln P_{i,k}^M(t) - \ln P_{i,k}^M(t-1)
\]

(4.A.9)

The new equation (4.A.10) added accordingly defines the linear deviation of the pre-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies \(\pi_{i,k}^{MT}(t)\), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the pre-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies \(\ln P_{i,k}^{MT}(t)\):
The innovatively constructed equation (4.A.11) defines the logarithmic deviation of the pre-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies \( \ln P_{i,k}^{M,T}(t) \), which depends on both the logarithmic deviation of the post-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies \( \ln P_{i,k}^{M,T}(t) \) and the linear deviation of the contemporaneous import tariff rate for the total imports of the i-th economy from the k-th industry of all other economies \( \tau_{i,k}^M(t) \), with \( \tau_{i,k}^M(t) \) the contemporaneous industry-level import tariff rate variable and \( \tau_{i,k}^M \) the industry-level tariff rate constant for the i-th economy's imports from the k-th industry of all other economies in period t:

\[
\ln P_{i,k}^{M,T}(t) = \ln P_{i,k}^{M,T}(t) - \frac{1}{1 + \tau_{i,k}^{M,T}} (\tau_{i,k}^M(t) - \tau_{i,k}^M)
\]

(4.A.11)

Here \( \tau_{i,k}^{Tariff} \) is the broad level of import tariff rates for the total imports of the i-th economy from the k-th industry of all other economies.

The innovatively constructed equation (4.A.12) shows the linear deviation of the import tariff rate for the total imports of the i-th economy from the k-th industry of all other economies \( (\tau_{i,k}^M(t) - \tau_{i,k}^M) \) depends on its past value \( (\tau_{i,k}^M(t-1) - \tau_{i,k}^M) \), and is also influenced by the linear deviation of the contemporaneous industry-level import tariff rate shock \( \vartheta_{i,k}^M(t) \), according to industry-level fiscal revenue rule:

\[
(\tau_{i,k}^M(t) - \tau_{i,k}^M) = \rho_{i,k}^M (\tau_{i,k}^M(t-1) - \tau_{i,k}^M) + \vartheta_{i,k}^M(t)
\]

(4.A.12)

Here \( \rho_{i,k}^M \) represents the country differentiated and industry differentiated coefficient for the intertemporal change of country differentiated and industry differentiated import tariff rate, which is estimated based on the industry specific tariff reduction commitments of each country in the tariff terms of for example RCEP.

The innovatively constructed equations about the logarithmic deviation of the pre-tariff price of imports for the total imports of any economy from different industries of all other economies are different, and they are the decomposed equations of the equation (Appendix.G.8) in the original model defining the whole-economy-level import price that does not distinguish between industries (please refer to the following Part C of this section for details of this original equation).

For those industries that produce internationally homogeneous energy commodities, equation (4.A.13a) shows the linear deviation of the pre-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies \( \pi_{i,k}^{M,T}(t) \) not only depends on a linear combination of its past and expected future values, driven by the difference between the industry-level import weighted average of the logarithmic deviation of the contemporaneous i-th economy's currency denominated price of exports for the exported output of the k-th industry of any specific economy \( \sum_{j=1}^{N} \omega_{i,j,k} \ln P_{i,j,k}^E(t) \) and the
For those industries that produce internationally homogeneous nonenergy commodities, equation (4.A.13b) shows the linear deviation of the contemporaneous industry-level import price markup shock from the industry's currency denominated price of exports for the i-th economy from the k-th industry of all other economies:

\[ \pi_i^M(t) = \frac{1}{\omega_M} \left( \sum_{j=1}^{N_k} \omega_j (\ln P_{ikj}(t-1) + \ln P_{ikj}(t) + \ln P_{ikj}(t+1)) \right) - \frac{1}{1+\beta M} \left( \ln P_{ik}(t-1) + \ln P_{ik}(t) + \ln P_{ik}(t+1) \right) + \beta \gamma M \left[ \ln P_{ik}(t) - \ln P_{ik}(t-1) \right] \]

This equation shows the linear deviation of the pre-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies, and is also influenced by the logarithmic deviation of the contemporaneous import weighted average of the logarithmic deviation of the pre-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies.
of the contemporaneous pre-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies \( \ln P_{i,k}^{M,T} \), and is also influenced by the logarithmic deviation of the contemporaneous industry-level import price markup shock \( \ln \delta_{i,k}^{M} \), according to industry-level import price Phillips curve:

\[
\dot{\pi}_{i,k}^{M,T}(t) = \frac{\gamma M(1-\mu_{i}^{M})}{1+\beta M(1-\mu_{i}^{M})} \pi_{i,k}^{M,T}(t-1) + \frac{\beta}{1+\beta M(1-\mu_{i}^{M})} E_{t} \pi_{i,k}^{M,T}(t+1) + \frac{(1-\omega_{i})}{\omega_{i}(1+\beta M(1-\mu_{i}^{M}))} \left( \sum_{j=1}^{N} \omega_{i,k}^{M} \ln \tilde{E}_{ij}(t) + \ln \delta_{i,k}^{M} \right)
\]

(4.A.13c)

The innovatively constructed equation (4.A.14) shows the logarithmic deviation of the labor productivity for the k-th industry of the i-th economy \( \ln \tilde{\Lambda}_{i,k}(t) \) not only depends on the logarithmic deviation of the contemporaneous cross-border cross-industry productivity shifter

\[
\lambda^{A} \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m,i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}} \ln \tilde{\nu}_{j,m,i,k}^{A} + \left( 1 - \lambda^{A} \sum_{j'=1}^{N} \sum_{m'=1}^{45} \frac{Y_{j',m'-i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}} \right) \ln \tilde{\nu}_{i,k}^{A}(t)
\]

that satisfies dynamic factor process to reflect technology diffusion among all industries of all economies, but also depends on the difference between the logarithmic deviation of the contemporaneous industry-level public physical capital stock \( \ln \tilde{K}_{i,k}^{G}(t) \) and the logarithmic deviation of the contemporaneous total labor force \( \ln \tilde{N}_{i}(t) \) (here we simply assume the logarithmic deviation of industry-level labor force equal to the logarithmic deviation of total labor force), which reflects the promotion effect of per capita public physical capital stock on labor productivity improvement, through ways such as infrastructure construction and public expenditure for R & D:

\[
\ln \tilde{\Lambda}_{i,k}(t) = \phi^{A} \left[ \lambda^{A} \sum_{j=1}^{N} \sum_{m=1}^{45} \frac{Y_{j,m,i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}} \ln \tilde{\nu}_{j,m,i,k}^{A} + \left( 1 - \lambda^{A} \sum_{j'=1}^{N} \sum_{m'=1}^{45} \frac{Y_{j',m'-i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}} \right) \ln \tilde{\nu}_{i,k}^{A}(t) \right] + (1 - \phi^{A}) \left( \ln \tilde{K}_{i,k}^{G}(t) - \ln \tilde{N}_{i}(t) \right)
\]

(4.A.14)

Here,

\[
\frac{Y_{j,m,i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}}
\]

represents the ratio of goods or services from the m-th industry of the j-th economy that are adopted as intermediate inputs of the k-th industry of the i-th economy to goods or services from all industries of all economies that are adopted as intermediate inputs of the k-th industry of the i-th economy;

\[
\frac{Y_{i,k-i,k}}{\sum_{j'=1}^{N} \sum_{m'=1}^{45} Y_{j',m',i,k}}
\]

represents the ratio of intermediate goods or services from the k-th industry of the i-th economy that continue to be used as intermediate inputs for the k-th industry of the i-th economy to goods or services from all industries of all economies that are adopted as intermediate inputs of the k-th industry of the i-th economy.

All these parameters are used to reflect that the degree of cross-border cross-industry technology diffusion is directly proportional to the scale of input-output relation between different industries in different countries.

The structure of the innovatively constructed equation (4.A.15) defining the industry-level public investment and the equation (Appendix.J.8) in the original model that defines the whole-economy-level public investment (please refer to the following Part C of this section
for details of this original equation) is similar. The equation (4.A.15) shows the logarithmic deviation of the public investment for the $k$-th industry of the $i$-th economy $\ln G_{i,k}^I(t)$ depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporaneous industry-level public investment shock $\nu_{i,k}^G(t)$, while the logarithmic deviation of the desired public investment for the $k$-th industry of the $i$-th economy equals to the logarithmic deviation of the contemporaneous potential output of the $i$-th economy $\ln \bar{Y}_i(t)$, according to the industry-level fiscal expenditure rule adjusted correspondingly:

$$\ln G_{i,k}^I(t) = \rho_G \ln G_{i,k}^I(t-1) + (1 - \rho_G) \ln \bar{Y}_i(t) + \nu_{i,k}^G(t)$$

(4.A.15)

The structure of the innovatively constructed equation (4.A.16) defining the industry-level public physical capital stock and the equation (Appendix.J.9) in Appendix II that defines the whole-economy-level public physical capital stock is similar. The equation (4.A.16) shows the logarithmic deviation of the past public investment for the $k$-th industry of the $i$-th economy $\ln G_{i,k}^I(t-1)$ is accumulated to form the logarithmic deviation of the contemporaneous public physical capital stock for the $k$-th industry of the $i$-th economy $\ln K_{i,k}^G(t)$, according to the perpetual inventory method:

$$\ln K_{i,k}^G(t) = \delta_G \ln K_{i,k}^G(t-1) + \delta_G \ln G_{i,k}^I(t-1)$$

(4.A.16)

The innovatively constructed equation (4.A.17) defining the industry-level corporate profits has a similar structure to the equation (Appendix.D.10) in Appendix II that defines the whole-economy-level corporate profits. It shows the logarithmic deviation of the corporate profits of the $k$-th industry of the $i$-th economy $\ln \Pi_{i,k}^S(t)$ not only depends on the logarithmic deviation of the contemporaneous output price level for the output of the $k$-th industry of the $i$-th economy $\ln P_{i,k}^Y(t)$ and the logarithmic deviation of the contemporaneous real output of the $k$-th industry of the $i$-th economy $\ln Y_{i,k}(t)$, but also depends on the linear deviation of the contemporaneous corporate income tax rate $\tau_{i,k}(t)$, according to the industry-level corporate profit function modified correspondingly:

$$\ln \Pi_{i,k}^S(t) = \ln P_{i,k}^Y(t) + \ln Y_{i,k}(t) - \ln \left(1 - \frac{1}{1 - \tau_{i,k}} \left(1 - \frac{W_{i,k}}{P_{i,k}^Y} \right) \bar{Y}_i(t) \right)$$

(4.A.17)

The structure of the innovatively constructed equation (4.A.18) defining the industry-level corporate equity price and the equation (Appendix.D.9) in Appendix II that defines the whole-economy-level corporate equity price is similar. The equation (4.A.18) shows the logarithmic deviation of the price of corporate equity of the $k$-th industry of the $i$-th economy $\ln V_{i,k}^S(t)$ not only depends on its expected future value, driven by the logarithmic deviation of the expected future corporate profits of the $k$-th industry of the $i$-th economy $\ln \Pi_{i,k}^S(t+1)$, but also depends on the linear deviation of the contemporaneous nominal short term bond yield of the $i$-th economy $\tilde{i}_i(t)$, and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign equity risk premium for the $k$-th industry of the $i$-th economy $\ln \nu_{i,k}^S(t)$ defined in the following equation, according to the industry-level equity market relationship adjusted correspondingly:
\[
\ln \tilde{\nu}^S_{i,k}(t) = \beta E_t \ln \tilde{\nu}^S_{i,k}(t + 1) + (1 - \beta) E_t \ln \hat{\nu}^S_{i,k}(t + 1) - \left( \hat{\nu}^S(t) + \ln \nu_{i,k}^S(t) \right)
\]
(4.18)

Since we assume that the equity market contagion mechanism for equity price of listed companies in the same industry of various countries is similar to the equity market contagion mechanism among various countries for equity price of all listed companies that do not distinguish between industries, the structure of the innovatively constructed equation (4.19) defining the industry-level weighted average of domestic and foreign equity risk premium is similar to that of the equation (Appendix.K.10) in the original model that defines the whole-economy-level weighted average of domestic and foreign equity risk premium (please refer to the following Part C of this section for details of this original equation). Specifically, the equation (4.19) defines the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the k-th industry of the i-th economy \( \ln \nu_{i,k}^S(t) \) as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous equity risk premium shock for the k-th industry of any specific economy except the i-th economy \( \sum_{j=1}^{N} \omega_j^A \ln \nu_{j,k}^S(t) - \omega_i^A \ln \nu_{i,k}^S(t) \), adds the logarithmic deviation of the contemporaneous equity risk premium shock for the k-th industry of the i-th economy \( \ln \nu_{i,k}^S(t) \):

\[
\ln \nu_{i,k}^S(t) = \lambda_i^S \sum_{j=1}^{N} \omega_j^A \ln \nu_{j,k}^S(t) + (1 - \lambda_i^S \omega_i^A) \ln \nu_{i,k}^S(t)
\]
(4.19)

Here the parameter \( \lambda_i^S \) represents the i-th economy's capital market contagion level, the same as that in the equation (Appendix.K.10) in the original model (please refer to the following Part C of this section for details of this original equation).

The structure of the innovatively constructed equation (4.20) defining the industry-level corporate loan default rate and the equation (Appendix.E.14) in Appendix II that defines the whole-economy-level corporate loan default rate is similar. The equation (4.20) shows the linear deviation of the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy \( \delta_{i,k}^C(t) \) depends on its past and attractor values, and is also influenced by the linear deviation of the contemporaneous industry-level corporate loan default shock \( \nu_{i,k}^S(t) \), by the intertemporal change in the logarithmic deviation of the price of corporate equity of the k-th industry of the i-th economy \( \ln \nu_{i,k}^S(t) - \ln \nu_{i,k}^S(t - 1) \), according to the industry-level corporate loan default rate relationship modified correspondingly:

\[
\delta_{i,k}^C(t) = \rho_{\delta} \delta_{i,k}^C(t - 1) - (1 - \rho_{\delta}) \left[ \zeta^{\delta,C,Y} \left( \ln Y_{i,k}(t) - \ln \bar{Y}_{i,k}(t) \right) + \zeta^{\delta,C,V} \left( \ln \nu_{i,k}^S(t) - \ln \nu_{i,k}^S(t - 1) \right) \right] + \nu_{i,k}^C(t)
\]
(4.20)
B. Industry-Level Exogenous Shocks

Since we usually consider shocks of short duration on the intermediate inputs from upstream industries to downstream industries in the global supply chain when applying this model, the innovatively constructed equation (4.B.1) shows the logarithmic deviation of the intermediate input shock for the intermediate goods or services from the m-th industry of the j-th economy to the k-th industry of the i-th economy $\ln \nu_{j,m\rightarrow i,k}^{Y,\text{input}}(t)$, which directly influences the quantity of intermediate goods or services from the m-th industry of the j-th economy to the k-th industry of the i-th economy, follows normally distributed white noise process, and $\epsilon_{j,m\rightarrow i,k}^{Y,\text{input}}(t) \sim \text{iid } N\left(0, \left(\sigma_{j,m\rightarrow i,k}^{Y,\text{input}}\right)^2\right)$. Here $\sigma_{j,m\rightarrow i,k}^{Y,\text{input}}$ represents the standard deviation of the normally distributed innovations $\epsilon_{j,m\rightarrow i,k}^{Y,\text{input}}(t)$ to reflect the upstream industry differentiated and downstream industry differentiated scale of supply shock for intermediate inputs:

$$\ln \nu_{j,m\rightarrow i,k}^{Y,\text{input}}(t) = \epsilon_{j,m\rightarrow i,k}^{Y,\text{input}}(t)$$

(4.B.1)

However, the setting of this industry-level intermediate input supply shock can be changed to follow stationary first order autoregressive process or even other processes, according to the specific research needs such as persistent shrinkage of intermediate inputs.

Given that we usually consider demand shocks of short duration on the upstream industries’ output of intermediate or final goods or services needed by domestic and foreign downstream industries or absorption sectors when applying this model to supply chain related research topics, the innovatively constructed equation (4.B.2) shows the logarithmic deviation of the output demand shock for the intermediate or final goods or services from the k-th industry of the i-th economy that would serve as intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector $\ln \nu_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}(t)$, which directly influences the real output of the k-th industry of the i-th economy, follows normally distributed white noise process, and $\epsilon_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}\right)^2\right)$. Here $\sigma_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}$ represents the standard deviation of the normally distributed innovations $\epsilon_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}(t)$ to reflect the upstream industry differentiated and downstream industry or absorption sector differentiated scale of demand shock for intermediate or final goods or services:

$$\ln \nu_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}(t) = \epsilon_{i,k\rightarrow \text{all economies}}^{Y,\text{output}}(t)$$

(4.B.2)

However, the setting of this industry-level intermediate or final output demand shock can be changed to follow stationary first order autoregressive process or even other processes, according to the specific research needs such as persistent shrinkage of demand for intermediate or final goods or services.

The above industry-level output demand shock can be further decomposed into more granular shocks, such as the following five demand-side shocks described by equation (4.B.3) to equation (4.B.7), that are introduced into the equation (4.A.3) and (4.A.4) accordingly.
The innovatively constructed equation (4.B.3) shows the logarithmic deviation of the output demand shock for the intermediate goods or services from the k-th industry of the i-th economy demanded by the m-th industry of the j-th economy \( \ln \nu_{i,k\rightarrow j,m}^{Y_{output}}(t) \), which directly influences the intermediate goods or services output from the k-th industry of the i-th economy to the m-th industry of the j-th economy, follows normally distributed white noise process, and \( \epsilon_{i,k\rightarrow j,m}^{Y_{output}}(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k\rightarrow j,m}^{Y_{output}}\right)^2\right) \). Here \( \sigma_{i,k\rightarrow j,m}^{Y_{output}} \) represents the standard deviation of the normally distributed innovations \( \epsilon_{i,k\rightarrow j,m}^{Y_{output}}(t) \) to reflect the upstream industry differentiated and downstream industry differentiated scale of demand shock for intermediate goods or services:

\[
\ln \nu_{i,k\rightarrow j,m}^{Y_{output}}(t) = \epsilon_{i,k\rightarrow j,m}^{Y_{output}}(t)
\]

(4.B.3)

The innovatively constructed equation (4.B.4) shows the logarithmic deviation of the output demand shock for the intermediate goods or services from the k-th industry of the j-th economy demanded by all industries of the i-th economy \( \ln \nu_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}(t) \), which directly influences the intermediate goods or services output from the k-th industry of the j-th economy to all industries of the i-th economy, follows normally distributed white noise process, and \( \epsilon_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}(t) \sim \text{iid } N\left(0, \left(\sigma_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}\right)^2\right) \). Here \( \sigma_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}} \) represents the standard deviation of the normally distributed innovations \( \epsilon_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}(t) \) to reflect the upstream industry differentiated and downstream economy differentiated scale of demand shock for intermediate goods or services:

\[
\ln \nu_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}(t) = \epsilon_{j,k\rightarrow i,\text{intermediate inputs}}^{Y_{output}}(t)
\]

(4.B.4)

The innovatively constructed equation (4.B.5) shows the logarithmic deviation of private consumption shock for final goods or services used for private consumption in the j-th economy from the k-th industry of the i-th economy \( \ln \nu_{i,k\rightarrow j}^{C}(t) \), which directly influences the quantity of final goods or services used for private consumption in the j-th economy from the k-th industry of the i-th economy, follows normally distributed white noise process, and \( \epsilon_{i,k\rightarrow j}^{C}(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k\rightarrow j}^{C}\right)^2\right) \). Here \( \sigma_{i,k\rightarrow j}^{C} \) represents the standard deviation of the normally distributed innovations \( \epsilon_{i,k\rightarrow j}^{C}(t) \) to reflect the producing and consuming countries differentiated and industry differentiated private consumption fluctuation:

\[
\ln \nu_{i,k\rightarrow j}^{C}(t) = \epsilon_{i,k\rightarrow j}^{C}(t)
\]

(4.B.5)

The innovatively constructed equation (4.B.6) shows the logarithmic deviation of private investment shock for final goods or services used for private investment in the j-th economy from the k-th industry of the i-th economy \( \ln \nu_{i,k\rightarrow j}^{I}(t) \), which directly influences the quantity of final goods or services used for private investment in the j-th economy from the k-th industry of the i-th economy, follows normally distributed white noise process, and
\[ \varepsilon_{i,k\rightarrow j}^{\nu I}(t) \sim \text{iid } N \left( 0, \left( \sigma_{i,k\rightarrow j}^{\nu I} \right)^2 \right). \] Here \( \sigma_{i,k\rightarrow j}^{\nu I} \) represents the standard deviation of the normally distributed innovations \( \varepsilon_{i,k\rightarrow j}^{\nu I} \) to reflect the producing and consuming countries differentiated and industry differentiated private investment fluctuation:

\[ \ln \nu_{i,k\rightarrow j}^{\nu I}(t) = \varepsilon_{i,k\rightarrow j}^{\nu I}(t) \] (4.B.6)

The innovatively constructed equation (4.B.7) shows the logarithmic deviation of public purchase shock for final goods or services used for public consumption or public investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy \( \ln \nu_{i,k\rightarrow j}^{\nu G}(t) \), which directly influences the quantity of final goods or services used for public consumption or public investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy, follows normally distributed white noise process, and \( \varepsilon_{i,k\rightarrow j}^{\nu G}(t) \sim \text{iid } N \left( 0, \left( \sigma_{i,k\rightarrow j}^{\nu G} \right)^2 \right) \). Here \( \sigma_{i,k\rightarrow j}^{\nu G} \) represents the standard deviation of the normally distributed innovations \( \varepsilon_{i,k\rightarrow j}^{\nu G} \) to reflect the producing and consuming countries differentiated and industry differentiated public consumption and public investment fluctuation:

\[ \ln \nu_{i,k\rightarrow j}^{\nu G}(t) = \varepsilon_{i,k\rightarrow j}^{\nu G}(t) \] (4.B.7)

However, the settings of all the above five types of demand-side shocks can be changed to follow stationary first order autoregressive process or even other processes, depending on the specific scenario under which the demand-side shocks with what time duration should be set.

Since we often want to set price markup shocks of short duration on the output or export of or import from any industry of any economy in many studies that need to apply this model, we introduce the following three price markup shocks into the industry-level part of the AGMFM accordingly.

The innovatively constructed equation (4.B.8) shows the logarithmic deviation of the output price markup shock for the output of the \( k \)-th industry of the \( i \)-th economy \( \ln \vartheta_{i,k}^{\nu Y}(t) \), which directly influences the output price of the \( k \)-th industry of the \( i \)-th economy, follows normally distributed white noise process, and \( \varepsilon_{i,k}(t) \sim \text{iid } N \left( 0, \left( \sigma_{i,k}^{\nu Y} \right)^2 \right) \). Here \( \sigma_{i,k}^{\nu Y} \) represents the standard deviation of the normally distributed innovations \( \varepsilon_{i,k}^{\nu Y}(t) \) to reflect the country differentiated and industry differentiated output price fluctuation:

\[ \ln \vartheta_{i,k}^{\nu Y}(t) = \varepsilon_{i,k}^{\nu Y}(t) \] (4.B.8)

The innovatively constructed equation (4.B.9) shows the logarithmic deviation of the export price markup shock for the exports of the \( k \)-th industry of the \( i \)-th economy \( \ln \vartheta_{i,k}^{\nu X}(t) \), which directly influences the export price for exports from the \( k \)-th industry of the \( i \)-th economy, follows normally distributed white noise process, and \( \varepsilon_{i,k}(t) \sim \text{iid } N \left( 0, \left( \sigma_{i,k}^{\nu X} \right)^2 \right) \). Here \( \sigma_{i,k}^{\nu X} \) represents
represents the standard deviation of the normally distributed innovations $\epsilon_{i,k}^x(t)$ to reflect the country differentiated and industry differentiated export price fluctuation:

$$\ln \delta_{i,k}^X(t) = \epsilon_{i,k}^x(t)$$

(4.B.9)

The innovatively constructed equation (4.B.10) shows the logarithmic deviation of the import price markup shock for the imports of the i-th economy from foreign k-th industry $\ln \delta_{i,k}^M(t)$, which directly influences the import price for imports of the i-th economy from the k-th industry of all other economies, follows normally distributed white noise process, and $\epsilon_{i,k}^M(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k}^M\right)^2\right)$. Here $\sigma_{i,k}^M$ represents the standard deviation of the normally distributed innovations $\epsilon_{i,k}^M(t)$ to reflect the country differentiated and industry differentiated import price fluctuation:

$$\ln \delta_{i,k}^M(t) = \epsilon_{i,k}^M(t)$$

(4.B.10)

It is worth noting that the setting of the above three industry-level price markup shocks can be changed to follow stationary first order autoregressive process or even other processes, due to the specific research settings or assumptions.

The innovatively constructed equation (4.B.11) shows the linear deviation of the import tariff rate shock for the total imports of the i-th economy from the k-th industry of all other economies $\nu_{i,k}^T(t)$, which directly influences the import tariff rate for the total imports of the i-th economy from the k-th industry of all other economies, follows stationary first order autoregressive process, with normally distributed innovations $\epsilon_{i,k}^T(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k}^T\right)^2\right)$:

$$\nu_{i,k}^T(t) = \rho_{i,k}^T \nu_{i,k}^T(t-1) + \epsilon_{i,k}^T(t)$$

(4.B.11)

Here $\rho_{i,k}^T$ represents the country differentiated and industry differentiated coefficient for the intertemporal change of country differentiated and industry differentiated import tariff rate shock, $\sigma_{i,k}^T$ represents the standard deviation of the normally distributed innovations $\epsilon_{i,k}^T(t)$ to reflect the country differentiated and industry differentiated tax reduction intensity and volatility in the tariff terms of for example RCEP, which are all estimated based on the specific tariff reduction commitments of each country.

Similar to the setting of the whole-economy-level labor productivity shock in the previous equation (Appendix.L.1) in the original model (please refer to the following Part C of this section for details of this original equation), the innovatively adjusted equation (4.B.12) shows the logarithmic deviation of the labor productivity shock for the production of the k-th industry of the i-th economy $\ln \nu_{i,k}^A(t)$, which directly influences the labor productivity for the k-th industry of the i-th economy, follows stationary first order autoregressive process, with normally distributed innovations $\epsilon_{i,k}^A(t) \sim \text{iid } N\left(0, \left(\sigma_{i,k}^A\right)^2\right)$. Here $\sigma_{i,k}^A$ represents the standard
deviation of the normally distributed innovations $\varepsilon_{ik}(t)$ to reflect the country differentiated and industry differentiated evolution path of labor productivity:

$$\ln \nu_{ik}(t) = \rho_{ik} \ln \nu_{ik}(t-1) + \varepsilon_{ik}(t)$$

(4.B.12)

The newly added equation (4.B.13) shows the linear deviation of the public investment shock for the k-th industry of the i-th economy $\nu_{i,k}^{GI}(t)$, which directly influences the public investment for the k-th industry of the i-th economy, follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon_{i,k}^{GI}(t) \sim iid N\left(0, \left(\sigma_{i,k}^{GI}\right)^2\right)$. Here $\sigma_{i,k}^{GI}$ represents the standard deviation of the normally distributed innovations $\varepsilon_{i,k}^{GI}(t)$ to reflect the government's differentiated public investment support for sunrise and sunset industries:

$$\nu_{i,k}^{GI}(t) = \rho_{i,k}^{GI} \nu_{i,k}^{GI}(t-1) + \varepsilon_{i,k}^{GI}(t)$$

(4.B.13)

Similar to the setting of the whole-economy-level equity risk premium shock in the previous equation (Appendix.L.16) in the original model (please refer to the following Part C of this section for details of this original equation), the innovatively modified equation (4.B.14) shows the logarithmic deviation of the equity risk premium shock for the k-th industry of the i-th economy $\ln \nu_{i,k}^{S}(t)$, which directly influences the equity price of listed companies in the k-th industry of the i-th economy, follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon_{i,k}^{S}(t) \sim iid N\left(0, \left(\sigma_{i,k}^{S}\right)^2\right)$. Here $\sigma_{i,k}^{S}$ represents the standard deviation of the normally distributed innovations $\varepsilon_{i,k}^{S}(t)$ to reflect the country differentiated and industry differentiated corporate equity risk premium:

$$\ln \nu_{i,k}^{S}(t) = \rho_{i,k}^{S} \ln \nu_{i,k}^{S}(t-1) + \varepsilon_{i,k}^{S}(t)$$

(4.B.14)

Similar to the setting of the whole-economy-level corporate loan default shock in the previous equation (Appendix.L.21) in the original model (please refer to the following Part C of this section for details of this original equation), the innovatively adjusted equation (4.B.15) shows the linear deviation of the corporate loan default shock for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy $\nu_{i,k}^{DC}(t)$, which directly influences the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy, follows normally distributed white noise process, and $\varepsilon_{i,k}^{DC}(t) \sim iid N\left(0, \left(\sigma_{i,k}^{DC}\right)^2\right)$. Here $\sigma_{i,k}^{DC}$ represents the standard deviation of the normally distributed innovations $\varepsilon_{i,k}^{DC}(t)$ to reflect the country differentiated and industry differentiated corporate loan default rate fluctuation:
The modified and simplified equation (Appendix.A.2) has its aggregated consumer prices at the industry level replaced with the weighted average of core price level and import price level at the whole economy level. Specifically, this simplified equation shows the logarithmic deviation of the domestic comprehensive consumption price level $\ln \bar{P}_i^C(t)$ depends on the total domestic demand and total imports weighted average of the logarithmic deviation of the contemporaneous domestic core price level $\ln \bar{P}_i(t)$ and the logarithmic deviation of the contemporaneous domestic currency denominated post-tariff price of imports $\ln \bar{P}_i^M(t)$ (the structure of this equation is similar to that of the corresponding equation in the original model, but the parameters before price variables are adjusted to make its setting more suitable for the situation that the total import value of some small and highly open economies we add to the model can almost equal or even exceed their GDP), according to the simplified consumption price relationship:

$$\ln \bar{P}_i^C(t) = \frac{D_i}{D_i + M_i} \ln \bar{P}_i(t) + \frac{M_i}{D_i + M_i} \ln \bar{P}_i^M(t)$$

(Appendix.A.2)

Here, $\frac{D_i}{D_i + M_i}$ represents the proportion of the domestic currency denominated value of the i-th economy’s total domestic demand to the domestic currency denominated value of the sum of the i-th economy’s total domestic demand and total imports; $\frac{M_i}{D_i + M_i}$ represents the proportion of the domestic currency denominated value of the i-th economy’s total imports to the domestic currency denominated value of the sum of the i-th economy’s total domestic demand and total imports.

The modified and simplified equation (Appendix.D.2) has its aggregated output prices at the industry level replaced with the weighted average of core price level and export price level at the whole economy level. Specifically, this simplified equation shows the logarithmic deviation of the domestic comprehensive output price level $\ln \bar{P}_i^Y(t)$ depends on the total domestic demand and total exports weighted average of the logarithmic deviation of the contemporaneous domestic core price level $\ln \bar{P}_i(t)$ and the logarithmic deviation of the
contemporaneous domestic currency denominated price of exports $\ln P^X_i(t)$ (the structure of this equation is similar to that of the corresponding equation in the original model, but the parameters before price variables are adjusted to make its setting more suitable for the situation that the total export value of some small and highly open economies we add to the model can almost equal or even exceed their GDP), according to the simplified output price relationship:

$$\ln P^Y_i(t) = \frac{D_i}{D_i+X_i} \ln P^Y_i(t) + \frac{X_i}{D_i+X_i} \ln P^X_i(t)$$

(Appendix.D.2)

Here, $\frac{D_i}{D_i+X_i}$ represents the proportion of the domestic currency denominated value of the i-th economy’s total domestic demand to the domestic currency denominated value of the sum of the i-th economy’s total domestic demand and total exports; $\frac{X_i}{D_i+X_i}$ represents the proportion of the domestic currency denominated value of the i-th economy’s total exports to the domestic currency denominated value of the sum of the i-th economy’s total domestic demand and total exports.

The simplified equation (Appendix.D.3) only uses capital and labor as input factors for the overall production function of national total output, and has the industry-level output weighted average of the logarithmic deviation of the contemporaneous total output of any specific industry of the i-th economy $\sum_{k=1}^{M^I} Contribution_{Y_{i,k}} \ln Y_{i,k}(t)$ removed. Specifically, this simplified equation shows the logarithmic deviation of the total output of the i-th economy $\ln Y_i(t)$ depends on the weighted average of the logarithmic deviation of the contemporaneous utilized private physical capital stock of the i-th economy $\ln u_i K(t) + \ln K_i(t)$ and the logarithmic deviation of the contemporaneous effective employment of the i-th economy $\ln \Lambda_i(t) + \ln L_i(t)$, according to the simplified total production function:

$$\ln Y_i(t) = \left[ \left( 1 - \frac{\theta Y}{\theta Y - 1} \frac{W_i L_i}{P_i Y_i} \right) (\ln \tilde{u}_i K(t) + \ln \tilde{K}_i(t)) + \frac{\theta Y}{\theta Y - 1} \frac{W_i L_i}{P_i Y_i} (\ln \tilde{A}_i(t) + \ln \tilde{L}_i(t)) \right]$$

(Appendix.D.3)

In addition, since the complete replication of the original model shows that it could be too far to pass the verification of rank condition, while adjusting the value of elasticity $\theta^Y$ in the above modified production function not only could significantly move the model closer to pass the rank condition verification, but also is consistent with the reality that the production function of most countries usually have a slight scale effect. Since capital as input factor is usually easier to show scale effect than labor, then the overall parameter $\left( 1 - \frac{\theta^Y}{\theta^Y - 1} \frac{W_i L_i}{P_i Y_i} \right)$ in front of the utilized private physical capital stock $\left( \ln \tilde{u}_i K(t) + \ln \tilde{K}_i(t) \right)$ has been slightly increased to reflect this slight scale effect (the same adjustment are also made to the previous equation (4.A.1) and (4.A.2), and the following equation (Appendix.D.4)), which make the scale effect of the adjusted total production function range from 1.1 to 1.6, depending on the ratio of return on labor to return on capital in different countries. However, this parameter adjustment is worthy of further discussion in the improvement of the model in the future, since this elasticity $\theta^Y$ is precisely the parameter that the model's simulation results are usually extremely sensitive to the change of its value. It is also necessary to
conduct a large number of sensitivity analysis around these non-economy-specific important parameters in order to identify all result sensitive parameters in the AGMFM in the future.

The simplified equation (Appendix.D.4) has the industry-level output weighted average of the logarithmic deviation of the contemporaneous potential output of any specific industry of the i-th economy \( \sum_{k=1}^{15} \text{Contribution}_{Y_{ik}} \ln \tilde{Y}_{i(k)}(t) \) replaced with the capital and labor as input factors for the overall production function of national potential output. Specifically, this simplified equation shows the logarithmic deviation of the potential output of the i-th economy (the inferred total output given full utilization of private physical capital and effective labor) \( \ln \tilde{Y}_{i}(t) \) depends on the weighted average of the logarithmic deviation of the contemporaneous private physical capital stock of the i-th economy \( \ln K_{i}(t) \) and the logarithmic deviation of the contemporaneous effective labor force of the i-th economy \( \ln \tilde{N}_{i}(t) \): (Appendix.D.4)

\[
\ln \tilde{Y}_{i}(t) = \left( 1 - \frac{\theta Y}{\theta Y - 1} \right) \ln \tilde{R}_{i}(t) + \frac{\theta Y}{\theta Y - 1} \left( \ln \tilde{A}_{i}(t) + \ln \tilde{N}_{i}(t) \right)
\]

The modified and simplified equation (Appendix.D.12) has its aggregated labor productivity at the industry level replaced with the weighted average of labor productivity at the whole economy level. Specifically, this simplified equation shows the logarithmic deviation of the labor productivity \( \ln \tilde{A}_{i}(t) \) not only depends on the logarithmic deviation of the contemporaneous productivity shifter \( \ln \tilde{\nu}_{i}^{A}(t) \) that satisfies dynamic factor process to reflect technology diffusion among economies, but also depends on the difference between the logarithmic deviation of the contemporaneous public physical capital stock \( \ln K_{iG}(t) \) and the logarithmic deviation of the contemporaneous total labor force \( \ln \tilde{N}_{i}(t) \), which reflects that the increase of per capita public physical capital stock can improve labor productivity, through ways such as infrastructure construction and public expenditure for R & D:

\[
\ln \tilde{A}_{i}(t) = \phi_{A} \ln \tilde{\nu}_{i}^{A}(t) + \left( 1 - \lambda A \omega_{i}^{Y} \right) \ln \tilde{\nu}_{i}^{A}(t) + \left( 1 - \phi_{A} \right) \left( \ln \tilde{R}_{iG}(t) - \ln \tilde{N}_{i}(t) \right)
\]

(Appendix.D.12)

The equation (Appendix.G.1) from the original model shows the logarithmic deviation of the total exports of the i-th economy \( \ln \tilde{X}_{i}(t) \) depends on the export weighted average of the logarithmic deviation of the contemporaneous total imports of any specific economy \( \sum_{j=1}^{N} \omega_{i}^{X} \ln \tilde{M}_{j}(t) \), and is also influenced by the logarithmic deviation of the contemporaneous export demand shock of the i-th economy \( \ln \tilde{\nu}_{i}^{X}(t) \), according to export demand relationship:

\[
\ln \tilde{X}_{i}(t) = \sum_{j=1}^{N} \omega_{i}^{X} \left( \ln \tilde{M}_{j}(t) - \ln \tilde{\nu}_{i}^{X}(t) \right)
\]

(Appendix.G.1)

The equation (Appendix.G.2) from the original model shows the logarithmic deviation of the total imports of the i-th economy \( \ln \tilde{M}_{i}(t) \) depends on the logarithmic deviation of the contemporaneous total domestic demand of the i-th economy \( \ln \tilde{D}_{i}(t) \), as well as the logarithmic deviation of the contemporaneous external terms of trade of the i-th economy \( \ln \tilde{\nu}_{i}^{M}(t) \), and is also influenced by the logarithmic deviation of the contemporaneous import demand shock of the i-th economy \( \ln \tilde{\nu}_{i}^{M}(t) \), according to import demand relationship:
\[
\ln \tilde{M}_i(t) = \left( \ln \tilde{D}_i(t) - \ln \tilde{M}^M(t) \right) - \psi^M \ln \tilde{M}^M(t)
\]

(Appendix.G.2)

The equation (Appendix.G.6) from the original model shows the logarithmic deviation of the export price inflation of the i-th economy \( \tilde{n}^X_i(t) \) not only depends on a linear combination of its past and expected future values, driven by the difference between the logarithmic deviation of the contemporaneous core price level (for all those internationally heterogeneous goods or services) of the i-th economy \( \ln \tilde{P}_i(t) \) and the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln \tilde{P}^X_i(t) \), the difference between the logarithmic deviation of the contemporaneous domestic currency denominated price of internationally homogeneous energy commodities of the i-th economy \( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}^Y_{ne}(t) \) and the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln \tilde{P}^X_i(t) \), and the difference between the logarithmic deviation of the contemporaneous domestic currency denominated price of internationally homogeneous nonenergy commodities of the i-th economy \( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}^Y_{ne}(t) \) and the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln \tilde{P}^X_i(t) \), but also depends on the contemporaneous, past and expected future values of the linear deviation of the core inflation (for all those internationally heterogeneous goods or services) of the i-th economy \( \tilde{n}_i(t) \), the contemporaneous, past and expected future values of the logarithmic deviation of the domestic currency denominated price of internationally homogeneous energy commodities of the i-th economy \( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}^Y_{ne}(t) \), and is also influenced by the logarithmic deviation of the contemporaneous export price markup shock of the i-th economy \( \ln \delta^X_i(t) \), according to export price Phillips curve:

\[
\tilde{n}^X_i(t) = \frac{\gamma^X(1-\mu^X)}{1+\beta^X(1-\mu^X)} \tilde{n}^X_i(t-1) + \frac{\beta}{1+\beta^X(1-\mu^X)} E_t \tilde{n}^X_i(t+1) + \frac{(1-\omega^X)(1-\omega^X)}{\omega^X(1+\beta^X)(1-\mu^X)} \left\{ \left( 1 - \frac{X_{i,e}^X}{X_i} \right) \ln \tilde{P}_i(t) - \ln \tilde{P}^X_i(t) \right\} + \frac{X_{i,e}^X}{X_i} \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}^Y_{ne}(t) - \ln \delta^X_i(t) + \frac{\beta}{1+\beta^X(1-\mu^X)} E_t \tilde{n}_i(t+1) + \frac{\mu^X(1+\beta)}{1+\beta^X(1-\mu^X)} \left\{ \left( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}^Y_{ne}(t) \right) - \frac{1}{1+\beta^X} \left( \ln \tilde{E}_{i,1}(t-1) + \ln \tilde{P}^Y_{ne}(t-1) \right) \right\}
\]

(\text{Appendix.G.6})

Compared with the corresponding equation characterizing the formation mechanism of the import price inflation without considering tariff in the original model, the correspondingly modified equation (Appendix.G.8) shows the logarithmic deviation of the pre-tariff import price inflation of the i-th economy \( \tilde{n}^M_i(t) \) not only depends on a linear combination of its past and expected future values, driven by the difference between the import weighted average of the logarithmic deviation of the contemporaneous domestic currency denominated price of exports of any specific economy \( \sum_{i=1}^{N} \omega^M_{i,j} \) \( \tilde{n}^M_{i,j}(t) + \ln \tilde{P}^X_{i,j}(t) \) and the logarithmic deviation of the contemporaneous pre-tariff price of imports of the i-th economy...
\[ \ln P_i(t), \text{ but also depends on the contemporaneous, past and expected future values of the logarithmic deviation of the domestic currency denominated price of internationally homogeneous energy commodities of the } i-\text{th economy } \left( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}_e^Y(t) \right), \text{ and the contemporaneous, past and expected future values of the logarithmic deviation of the domestic currency denominated price of internationally homogeneous nonenergy commodities of the } i-\text{th economy } \left( \ln \tilde{E}_{i,1}(t) + \ln \tilde{P}_{ne}^Y(t) \right), \text{ and is also influenced by the logarithmic deviation of the contemporaneous import price markup shock of the } i-\text{th economy } \ln \vartheta_{iM}(t), \text{ according to import price Phillips curve:} \]

\[
\pi_{iM}(t) = \gamma M(1-\mu_iM)\left( \frac{1}{1+\beta M(1-\mu_iM)^2} \sum_{j=1}^{N} \omega_j^M \ln \nu_{jS}(t) + \ln \nu_{iS}(t) \right) + \frac{1}{1+\beta M(1-\mu_iM)^2} \ln \nu_{iS}(t) \]

(Appendix.G.8)

The equation (Appendix.J.8) from the original model shows the logarithmic deviation of the public investment \( \ln G_{iI}(t) \) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporaneous public investment shock \( \nu_{iGI}(t) \), while the logarithmic deviation of the desired public investment equals to the logarithmic deviation of the contemporaneous potential output \( \ln \tilde{Y}_i(t) \), according to fiscal expenditure rule:

\[
\ln G_{iI}(t) = \rho_G \ln G_{iI}(t-1) + (1-\rho_G) \ln \tilde{Y}_i(t) + \nu_{iGI}(t)
\]

(Appendix.J.8)

The equation (Appendix.K.10) from the original model defines the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the i-th economy (the price of corporate equity \( V_i^S(t) \) is adjusted by this equity risk premium) \( \ln \tilde{\nu}_i^S(t) \) as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous equity risk premium shock of any specific economy except the i-th economy \( \left( \sum_{j=1}^{N} \omega_j^A \ln \nu_{jS}(t) - \omega_i^A \ln \nu_{iS}(t) \right) \), adds the logarithmic deviation of the contemporaneous equity risk premium shock of the i-th economy \( \ln \nu_{iS}(t) \):

\[
\ln \tilde{\nu}_i^S(t) = \lambda_i^S \sum_{j=1}^{N} \omega_j^A \ln \nu_{jS}(t) + (1-\lambda_i^S \omega_i^A) \ln \nu_{iS}(t)
\]

(Appendix.K.10)

Here \( \lambda_i^S \) represents the i-th economy's capital market contagion level.

The equation (Appendix.L.8) from the original model shows the logarithmic deviation of the output price markup shock \( \ln \tilde{\vartheta}_{iY}(t) \) (which directly influences the core price level) follows normally distributed white noise process, and \( \vartheta_{iY}^e(t)\sim \text{iid } N(0, \sigma_{\vartheta_Y}^2) \):

\[
\ln \tilde{\vartheta}_{iY}(t) = \rho_{\vartheta} \ln \tilde{\vartheta}_{iY}(t-1) + (1-\rho_{\vartheta}) \ln \tilde{\vartheta}_{iY}(t) + \vartheta_{iY}^e(t)
\]
\[ \ln \hat{\theta}_Y(t) = \epsilon_i^Y(t) \]  
(Appendix.L.8)

The equation (Appendix.L.16) from the original model shows the logarithmic deviation of the equity risk premium shock \( \ln \hat{\nu}^S(t) \) (which directly influences the price of corporate equity) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_i^{\nu^S}(t) \sim \text{iid } N(0, \sigma^2_{\nu^S}) \):

\[ \ln \hat{\nu}^S(t) = \rho_{\nu^S} \ln \hat{\nu}^S(t - 1) + \epsilon_i^{\nu^S}(t) \]  
(Appendix.L.16)

The equation (Appendix.L.21) from the original model shows the linear deviation of the corporate loan default shock \( \hat{\nu}^{\delta^C}(t) \) (which directly influences both the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy, and the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks) follows normally distributed white noise process, and \( \epsilon_i^{\nu^{\delta^C}}(t) \sim \text{iid } N(0, \sigma^2_{\nu^{\delta^C}}) \):

\[ \hat{\nu}^{\delta^C}(t) = \epsilon_i^{\nu^{\delta^C}}(t) \]  
(Appendix.L.21)

The following six equations defining some whole-economy-level exogenous shocks come from the original model, and they are used to fill in the blanks of those corresponding whole-economy-level exogenous shocks after those previously introduced equations defining industry-level exogenous shocks are removed for the purpose of model simplification.

The equation (Appendix.L.1) from the original model shows the logarithmic deviation of the labor productivity shock \( \ln \hat{\nu}^A(t) \) (which directly influences the labor productivity and the trend labor productivity) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_i^{\nu^A}(t) \sim \text{iid } N(0, \sigma^2_{\nu^A}) \):

\[ \ln \hat{\nu}^A(t) = \rho_{\nu^A} \ln \hat{\nu}^A(t - 1) + \epsilon_i^{\nu^A}(t) \]  
(Appendix.L.1)

The equation (Appendix.L.6) from the original model shows the logarithmic deviation of the export demand shock \( \ln \hat{\nu}^X(t) \) (which directly influences the total exports) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_i^{\nu^X}(t) \sim \text{iid } N(0, \sigma^2_{\nu^X}) \):

\[ \ln \hat{\nu}^X(t) = \rho_{\nu^X} \ln \hat{\nu}^X(t - 1) + \epsilon_i^{\nu^X}(t) \]  
(Appendix.L.6)

The equation (Appendix.L.7) from the original model shows the logarithmic deviation of the import demand shock \( \ln \hat{\nu}^M(t) \) (which directly influences the total imports) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_i^{\nu^M}(t) \sim \text{iid } N(0, \sigma^2_{\nu^M}) \):
\[ \ln \nu_i^M(t) = \rho_{\nu M} \ln \nu_i^M(t - 1) + \epsilon_i^{\nu M}(t) \]

(Appendix.L.7)

The equation (Appendix.L.10) from the original model shows the logarithmic deviation of the export price markup shock \( \ln \delta_i^X(t) \) (which directly influences the price of exports) follows normally distributed white noise process, and \( \epsilon_i^{\delta X}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\delta X}^2) \):

\[ \ln \delta_i^X(t) = \epsilon_i^{\delta X}(t) \]

(Appendix.L.10)

The equation (Appendix.L.11) from the original model shows the logarithmic deviation of the import price markup shock \( \ln \delta_i^M(t) \) (which directly influences the pre-tariff price of imports) follows normally distributed white noise process, and \( \epsilon_i^{\delta M}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\delta M}^2) \):

\[ \ln \delta_i^M(t) = \epsilon_i^{\delta M}(t) \]

(Appendix.L.11)

The equation (Appendix.L.24) from the original model shows the linear deviation of the public investment shock \( \nu_i^{GI}(t) \) (which directly influences the public investment) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_i^{\nu GI}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\nu GI}^2) \):

\[ \nu_i^{GI}(t) = \rho_{\nu GI} \nu_i^{GI}(t - 1) + \epsilon_i^{\nu GI}(t) \]

(Appendix.L.24)
Appendix I. Model Coverage

A. List of Industries

(1) { Agriculture, hunting, forestry };
(2) { Fishing and aquaculture };
(3) { Mining and quarrying, energy producing products };
(4) { Mining and quarrying, non-energy producing products };
(5) { Mining support service activities };
(6) { Food products, beverages and tobacco };
(7) { Textiles, textile products, leather and footwear };
(8) { Wood and products of wood and cork };
(9) { Paper products and printing };
(10) { Coke and refined petroleum products };
(11) { Chemical and chemical products };
(12) { Pharmaceuticals, medicinal chemical and botanical products };
(13) { Rubber and plastics products };
(14) { Other non-metallic mineral products };
(15) { Basic metals };
(16) { Fabricated metal products };
(17) { Computer, electronic and optical equipment };
(18) { Electrical equipment };
(19) { Machinery and equipment, nec };
(20) { Motor vehicles, trailers and semi-trailers };
(21) { Other transport equipment };
(22) { Manufacturing nec; repair and installation of machinery and equipment };
(23) { Electricity, gas, steam and air conditioning supply };
(24) { Water supply; sewerage, waste management and remediation activities };
(25) { Construction };
(26) { Wholesale and retail trade; repair of motor vehicles };
(27) { Land transport and transport via pipelines };
(28) { Water transport };
(29) { Air transport };
(30) { Warehousing and support activities for transportation };
(31) { Postal and courier activities };
(32) { Accommodation and food service activities };
(33) { Publishing, audiovisual and broadcasting activities };
(34) { Telecommunications };
(35) { IT and other information services };
(36) { Financial and insurance activities };
(37) { Real estate activities };
(38) { Professional, scientific and technical activities };
(39) { Administrative and support services };
(40) { Public administration and defence; compulsory social security };
(41) { Education };
(42) { Human health and social work activities };
(43) { Arts, entertainment and recreation };
(44) { Other service activities };
(45) { Activities of households as employers; undifferentiated goods- and services-
producing activities of households for own use }.

B. List of Economies

- ASEAN+3 economies (Brunei, Cambodia, China, Hong Kong SAR that belongs to
China, Indonesia, Japan, Korea, Laos, Malaysia, Myanmar, Philippines, Singapore,
Thailand, Vietnam);

- United States (since the quotation currency for transactions in the foreign exchange
market is issued by the United States, this is the only reason why the United States is
always positioned as the 1st economy in this model);

- Euro Area as a whole economy or major Euro Area countries (Austria, Belgium,
Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain);
• Other important developed countries (Australia, Canada, Czech Republic, Denmark, Israel, New Zealand, Norway, Sweden, Switzerland, United Kingdom);

• Other four BRICS countries (Brazil, India, Russian Federation, South Africa);

• Other important emerging market countries (Argentina, Chile, Colombia, Mexico, Poland, Saudi Arabia, Turkey).
Appendix II: Whole Economy or International Market Equations

In this part of the appendix, we introduce all whole-economy-level or international-market-level equations, corresponding to the characterization of the whole economy that do not distinguish between industries or around the characterization of the international markets, all of which constitute the complete AGMFM, together with all those innovatively constructed industry-level equations introduced in the previous Section IV.

For the meaning of the linear deviation or the logarithmic deviation of variables in the following equations, please refer to the specific explanation at the beginning of Section IV.

A. Household Sector

Equation (Appendix.A.1) defines the linear deviation of the consumption price inflation \( \pi^C_i(t) \), which is equal to the difference between the contemporaneous and past values of the logarithmic deviation of the consumption price level \( \ln P^C_i(t) \):

\[
\pi^C_i(t) = \ln P^C_i(t) - \ln P^C_i(t - 1)
\]

(Appendix.A.1)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.A.2) defines the domestic comprehensive consumption price level as the aggregated consumer prices at the industry level. Specifically, this equation shows the logarithmic deviation of the domestic comprehensive consumption price level \( \ln P^C_i(t) \) depends on the industry-level domestic private and public demand (refer to the i-th economy’s private consumption, private investment and public purchase of final goods or services from the k-th industry of all economies including the i-th economy) weighted average of the logarithmic deviation of the contemporaneous consumption price level in the i-th economy for the final goods or services from the domestic and foreign k-th industry \( \sum_{k=1}^{45} \text{Contribution}_{C_{i,k}} \ln P^C_i(t) \), according to the modified consumption price relationship:

\[
\ln P^C_i(t) = \sum_{k=1}^{45} \text{Contribution}_{C_{i,k}} \ln P^C_{i,k}(t)
\]

(Appendix.A.2)

Here \( \text{Contribution}_{C_{i,k}} \) represents the proportion of the i-th economy’s private consumption, private investment and public purchase of final goods or services from the k-th industry of all economies including the i-th economy to the i-th economy’s private consumption, private investment and public purchase of final goods or services from all industries of all economies including the i-th economy.

Equation (Appendix.A.3) shows the logarithmic deviation of the total consumption by all three kinds of households (bank intermediated households, capital market intermediated households, and credit constrained households) \( \ln C_i(t) \) not only depends on a weighted average of its past and expected future values, driven by the household type proportion weighted average of the linear deviation of the expected future real property return (property balances of bank intermediated households are distributed across the values of bank deposits and domestic real estate portfolio) \( E_t \left( \tilde{r}^{A,B,H}_{i}(t + 1) - \tilde{\pi}^C_i(t + 1) \right) \) and the linear deviation of the expected future real portfolio return (portfolio balances of capital market intermediated households are allocated across the values of internationally diversified short term bonds, internationally diversified and vintage diversified long term bonds, and
internationally diversified and industry diversified and firm diversified stocks)

\[ E_t \left( \pi^{A,H}_i (t + 1) - \pi^C_i (t + 1) \right) \], but also depends on the contemporaneous, past and expected future values of the logarithmic deviation of the credit constrained consumption \( \ln C^C_i (t) \) to reflect the existence of the specific consumption by credit constrained households, and is also influenced by the logarithmic deviation of the contemporaneous and expected future consumption demand shocks \( \ln \nu^C_i (t) \) and \( E_t \ln \nu^C_i (t + 1) \), according to consumption demand relationship:

\[
\ln \hat{C}_i(t) = \frac{\alpha C}{1+\alpha C} \ln C_i(t-1) + \frac{1}{1+\alpha C} E_t \ln \hat{C}_i(t+1) - \left( 1 - \phi C \right) \sigma^{1-\alpha C}_{1+\alpha C} E_t \left[ \frac{\phi B}{1-\phi C} (t^{A,H}_i (t + 1) - \pi^C_i (t + 1)) + \phi C [\ln C^C_i (t) - \frac{a C}{1+\alpha C} \ln C^C_i (t-1) - \frac{1}{1+\alpha C} E_t \ln C^C_i (t + 1)] \right]
\]

(Appendix.A.3)

Compared with the corresponding equation in the original model, the parameters before the logarithmic deviation of the terms of trade (which measures the relative price of exports to imports) \( \ln \tilde{T}_i(t) \) in the innovatively adjusted equation (Appendix.A.4) characterizing the consumption by credit constrained households have been changed, in order to make the consumption level characterization more accurate for those small and highly open economies we add to the model, whose total export value or total import value can be as large as or even larger than their GDP, which lead to the impact of their export or import price level on the consumption level much higher than that of other economies. Specifically, the innovatively adjusted equation (Appendix.A.4) shows the logarithmic deviation of the consumption by credit constrained households \( \ln \hat{C}_i^C(t) \) depends on the logarithmic deviation of the contemporaneous total output \( \ln \hat{Y}_i(t) \), depends on the total exports and total imports weighted average of the logarithmic deviation of the contemporaneous global terms of trade shifter (which is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows) \( \ln \tilde{\nu}^T_i(t) \), the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln \tilde{P}^X_i(t) \), and the logarithmic deviation of the contemporaneous post-tariff price of imports of the i-th economy \( \ln \tilde{P}^M_i(t) \), while for small and highly open economies, this part is replaced by the logarithmic deviation of the contemporaneous terms of trade of the i-th economy \( \ln \tilde{T}_i(t) \) multiplied by a threshold value (to reflect such small and highly open economies are usually good at balancing their imports and exports and maintaining a stable structure between imports, exports, domestic demand and domestic production), and also depends on the tax base weighted average of the linear deviation of the contemporaneous corporate income tax rate \( \tau^K_i(t) \) and the linear deviation of the contemporaneous labor income tax rate \( \tau^L_i(t) \), as well as the linear deviation of the ratio of contemporaneous nondiscretionary and discretionary lump sum transfer payments by the fiscal authority to contemporaneous nominal output \( \frac{\tilde{\tau}^C_i(t) \tilde{P}^Y_i(t)}{\tilde{P}^Y_i(t) \tilde{Y}_i(t)} \), according to the adjusted credit constrained consumption demand relationship:

\[
\ln \hat{C}_i^C(t) = \ln \hat{Y}_i(t) + \left( \frac{0.5 + X_i + 0.5}{Y_i} \ln \tilde{\nu}^T(t) + \frac{X_i}{Y_i} \ln \tilde{P}^X_i(t) - \frac{M_i}{Y_i} \ln \tilde{P}^M_i(t) \right) - \frac{1}{1-t_i} \left[ 1 - \left( \frac{W_i \tilde{P}^Y_i(t)}{\tilde{P}^Y_i(t) \tilde{Y}_i(t)} + \frac{W_i \tilde{P}^Y_i(t)}{\tilde{P}^Y_i(t) \tilde{Y}_i(t)} \right) \right] + \frac{\tilde{T}^C_i(t)}{\phi C \tilde{P}^Y_i(t) \tilde{Y}_i(t)}
\]

(Appendix.A.4)
\[
\ln C_i(t) = \ln Y_i(t) + (\text{threshold value}) \ln T_i(t) - \frac{1}{1-\tau_i} \left[ 1 - \frac{W_i L_i}{p_i Y_i} \right] \tilde{v}_i^K(t) + \frac{W_i L_i + \tilde{v}_i^H(t)}{p_i Y_i} + \frac{\phi C_i T_i(t)}{p_i^C Y_i(t)}
\]

(Appendix.A.4)

Here, the adjusted parameter \(\frac{0.5 \cdot X_i + 0.5 \cdot M_i}{Y_i}\) represents the proportion of the domestic currency denominated average value of the i-th economy's total exports and imports to the domestic currency denominated nominal GDP of the i-th economy;

the adjusted parameter \(\frac{X_i}{Y_i}\) represents the proportion of the domestic currency denominated value of the i-th economy's total exports to the domestic currency denominated nominal GDP of the i-th economy;

the adjusted parameter \(\frac{M_i}{Y_i}\) represents the proportion of the domestic currency denominated value of the i-th economy's total imports to the domestic currency denominated nominal GDP of the i-th economy.

Equation (Appendix.A.5) shows the linear deviation of the nominal property return (property balances of bank intermediated households are distributed across the values of bank deposits and domestic real estate portfolio) \(\tilde{i}_i^{B,H}(t)\) depends on the linear deviation of the past nominal interbank loans rate \(\tilde{i}_i^B(t-1)\), and is also influenced by the logarithmic deviation of the past housing risk premium shock \(\ln \tilde{\nu}_i^H(t-1)\), according to property return function:

\[
\tilde{i}_i^{A,H}(t) = \tilde{i}_i^B(t-1) + \phi_i^H \ln \tilde{\nu}_i^H(t-1)
\]

(Appendix.A.5)

Compared with the corresponding equation in the original model, the timeline of this equation has been pushed back for one period (period \(t+1\) is adjusted to period \(t\), period \(t\) is adjusted to period \(t-1\), etc), while the structure of this equation and the values of its parameters have not been changed.

Equation (Appendix.A.6) shows the linear deviation of the nominal portfolio return (portfolio balances of capital market intermediated households are allocated across the values of internationally diversified short term bonds, internationally diversified and vintage diversified long term bonds, and internationally diversified and industry diversified and firm diversified stocks) \(\tilde{i}_i^{A,H}(t)\) depends on the linear deviation of the past nominal short term bond yield \(\tilde{i}_i^S(t-1)\), and is also influenced by both the long term bond investment weighted average of the logarithmic deviation of the past weighted average of domestic and foreign duration risk premium for any specific economy \(\sum_{j=1}^N \omega_i^j \ln \tilde{\nu}_j^B(t-1)\) and the equity investment weighted average of the logarithmic deviation of the past weighted average of domestic and foreign equity risk premium for any specific economy \(\sum_{j=1}^N \omega_i^j \ln \tilde{\nu}_j^S(t-1)\) to reflect the existence of a portfolio balance mechanism, according to portfolio return function:

\[
\tilde{i}_i^{A,H}(t) = \tilde{i}_i^S(t-1) + \phi_i^B \sum_{j=1}^N \omega_i^j \ln \tilde{\nu}_j^B(t-1) + \phi_i^S \sum_{j=1}^N \omega_i^j \ln \tilde{\nu}_j^S(t-1)
\]

(Appendix.A.6)
The timeline of this equation has been pushed back for one period compared with the corresponding equation in the original model, while the structure of this equation and the values of its parameters have no adjustment.

B. Labor Supply Sector

Equation (Appendix.B.1) defines the linear deviation of the nominal wage inflation $\hat{\pi}^W_i(t)$, which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the nominal wage $\ln \hat{W}_i(t)$:

$$\hat{\pi}^W_i(t) = \ln \hat{W}_i(t) - \ln \hat{W}_i(t - 1)$$

(Appendix.B.1)

Equation (Appendix.B.2) shows the logarithmic deviation of the real effective wage

$$\left( \ln \hat{W}_i(t) - \ln \hat{P}^C_i(t) - \ln \hat{A}_i(t) \right)$$

not only depends on a weighted average of its past and expected future values, driven by the linear deviation of the contemporaneous and past unemployment rates $\hat{u}^L_i(t)$ and $\hat{u}^L_i(t - 1)$, but also depends on both the contemporaneous, past and expected future values of the linear deviation of the consumption price inflation $\hat{\pi}^C_i(t)$ and the contemporaneous, past and expected future values of the intertemporal change in the logarithmic deviation of the trend labor productivity $\Delta \ln \hat{A}_i(t)$, and is also influenced by the logarithmic deviation of the contemporaneous wage markup shock $\ln \hat{\theta}^L_i(t)$, according to wage Phillips curve:

$$\ln \hat{W}_i(t) - \ln \hat{P}^C_i(t) - \ln \hat{A}_i(t) = \frac{1}{1+\beta} \left( \ln \hat{W}_i(t - 1) - \ln \hat{P}^C_i(t - 1) - \ln \hat{A}_i(t - 1) \right) + \frac{\beta}{1+\beta} E_t \left( \ln \hat{W}_i(t + 1) - \ln \hat{P}^C_i(t + 1) - \ln \hat{A}_i(t + 1) - \frac{(1-\omega^L)(1-\omega^C)}{\omega^L(1+\beta)} \left[ \frac{1}{\eta(1-\omega^C)} \left( \hat{u}^L_i(t) - \hat{u}^L_i(t - 1) \right) \right] - \frac{1+\gamma^L}{1+\beta} \left( \hat{\pi}^C_i(t) + \Delta \ln \hat{A}_i(t) \right) - \frac{1}{1+\gamma^L} \left( \hat{\pi}^C_i(t - 1) + \Delta \ln \hat{A}_i(t - 1) \right) \right)$$

(Appendix.B.2)

Equation (Appendix.B.3) defines the linear deviation of the unemployment rate $\hat{u}^L_i(t)$, which equals to the difference between the logarithmic deviation of the contemporaneous total labor force $\ln \hat{N}_i(t)$ and the logarithmic deviation of the contemporaneous employed labor force $\ln \hat{L}_i(t)$:

$$\hat{u}^L_i(t) = \ln \hat{N}_i(t) - \ln \hat{L}_i(t)$$

(Appendix.B.3)

Equation (Appendix.B.4) shows the linear deviation of the unemployment rate $\hat{u}^L_i(t)$ depends on its past value, driven by both the logarithmic deviation of the contemporaneous employed labor force $\ln \hat{L}_i(t)$ and the logarithmic deviation of the contemporaneous real effective wage

$$\left( \ln \hat{W}_i(t) - \ln \hat{P}^C_i(t) - \ln \hat{A}_i(t) \right)$$

and is also influenced by the logarithmic deviation of the contemporaneous labor supply shock $\ln \hat{\nu}^N_i(t)$, according to labor supply relationship:

$$\hat{u}^L_i(t) = \alpha^L \hat{u}^L_i(t - 1) - (1 - \alpha^L) \left[ \left( \ln \hat{L}_i(t) - \ln \hat{\nu}^N_i(t) \right) - \eta \left( \ln \hat{W}_i(t) - \ln \hat{P}^C_i(t) - \ln \hat{A}_i(t) \right) \right]$$

(Appendix.B.4)
C. Construction Sector

Equation (Appendix.C.1) shows the logarithmic deviation of the residential investment $\ln I^H_i(t)$ depends on a weighted average of its past and expected future values, driven by the logarithmic deviation of the contemporaneous relative shadow price of housing $\left(\ln Q^H_i(t) - \ln P^C_i(t)\right)$, and is also influenced by the logarithmic deviation of the contemporaneous residential investment demand shock $\ln \nu^I_i(t)$, according to residential investment demand relationship:

$$\ln I^H_i(t) = \frac{1}{1+\beta} \ln I^H_i(t-1) + \frac{\beta}{1+\beta} E_t \ln I^H_i(t+1) + \frac{1}{\chi^H(1+\beta)} (\ln \nu^I_i(t) + \ln Q^H_i(t) - \ln P^C_i(t))$$

(Appendix.C.1)

Equation (Appendix.C.2) shows the logarithmic deviation of the relative shadow price of housing $\left(\ln Q^H_i(t) - \ln P^C_i(t)\right)$ not only depends on its expected future value, as well as the linear deviation of the expected future real property return $E_t \left(\hat{I}^R_{iH}(t + 1) - \bar{P}_{iC}(t + 1)\right)$ and the linear deviation of the expected future domestic real mortgage loan rate $E_t \left(\hat{I}^M_i(t) - \bar{P}_{iC}(t + 1)\right)$, but also depends on the logarithmic deviation of the expected future real rental price of housing $E_t \left(\ln \nu^I_i(t + 1) - \ln P^C_i(t + 1)\right)$ and the linear deviation of the contemporaneous regulatory mortgage loan to value ratio limit $\phi^D_i(t)$ to reflect the existence of a financial accelerator mechanism, according to residential investment Euler equation:

$$\ln Q^H_i(t) - \ln P^C_i(t) = E_t \left(\beta(1 - \delta^H) \left(\ln Q^H_i(t + 1) - \ln P^C_i(t + 1)\right) - \left(1 - \phi^D_i\right) \left(\hat{I}^R_{iH}(t + 1) - \bar{P}_{iC}(t + 1)\right) + \phi^D_i \beta \left(1+\frac{1}{\delta^H} - \frac{1}{\beta}\right) \left(\ln \nu^I_i(t + 1) - \ln P^C_i(t + 1)\right) + \phi^D_i(t)\right)$$

(Appendix.C.2)

Equation (Appendix.C.3) shows the logarithmic deviation of the real rental price of housing $\left(\ln Q^H_i(t) - \ln P^C_i(t)\right)$ depends on the difference between the logarithmic deviation of the contemporaneous total consumption by all three kinds of households $\ln C_i(t)$ and the logarithmic deviation of the contemporaneous housing stock $\ln H_i(t)$, according to rental price of housing relationship:

$$\ln Q^H_i(t) - \ln P^C_i(t) = \xi \left(\ln C_i(t) - \ln H_i(t)\right)$$

(Appendix.C.3)

Equation (Appendix.C.4) shows the logarithmic deviation of the past residential investment $\ln I^H_i(t-1)$ is accumulated to form the logarithmic deviation of the contemporaneous housing stock $\ln H_i(t)$, according to the perpetual inventory method. The logarithmic deviation of the contemporaneous housing stock $\ln H_i(t)$ is also influenced by the logarithmic deviation of the past residential investment demand shock $\ln \nu^I_i(t - 1)$:
\[ \ln \hat{F}_i(t) = (1 - \delta_H) \ln \hat{R}_i(t - 1) + \delta_H \left( \ln \hat{V}_i^H(t - 1) + \ln \hat{I}_i^H(t - 1) \right) \]  
(Appendix.C.4)

Compared with the corresponding equation in the original model, only the timeline of this equation has been pushed back for one period.

Equation (Appendix.C.5) shows the logarithmic deviation of the price of housing \( \ln \hat{V}_i^H(t) \) not only depends on its expected future value, driven by the logarithmic deviation of the expected future real estate developer profits \( E_t \ln \hat{P}_i^H(t + 1) \), but also depends on the linear deviation of the contemporaneous nominal interbank loans rate \( \hat{i}_i^B(t) \), and is also influenced by the logarithmic deviation of the contemporaneous housing risk premium shock \( \ln \nu_i^H(t) \), according to housing market relationship:

\[ \ln \hat{V}_i^H(t) = \beta E_t \ln \hat{V}_i^H(t + 1) + (1 - \beta) E_t \ln \Pi_i^H(t + 1) - \Delta \hat{i}_i^B(t) + \ln \nu_i^H(t) \]  
(Appendix.C.5)

Equation (Appendix.C.6) shows the logarithmic deviation of the real estate developer profits \( \ln \hat{P}_i^H(t) \) depends on the logarithmic deviation of the contemporaneous real estate rental price \( \ln \hat{P}_i^Y(t) \) and the logarithmic deviation of the contemporaneous housing stock \( \ln \hat{H}_i(t) \), according to developer profit function:

\[ \ln \hat{P}_i^H(t) = \ln \hat{P}_i^Y(t) + \ln \hat{H}_i(t) \]  
(Appendix.C.6)

**D. Production Sector**

Equation (Appendix.D.1) defines the linear deviation of the output price inflation \( \hat{p}_i^Y(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the output price level \( \ln \hat{P}_i^Y(t) \):

\[ \hat{p}_i^Y(t) = \ln \hat{P}_i^Y(t) - \ln \hat{P}_i^Y(t - 1) \]  
(Appendix.D.1)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.D.2) defines the domestic comprehensive output price level as the aggregated output prices at the industry level. Specifically, this equation shows the logarithmic deviation of the domestic comprehensive output price level \( \ln \hat{P}_i^Y(t) \) depends on the domestic industrial output weighted average of the logarithmic deviation of the contemporaneous domestic currency denominated output price level for the output of the k-th industry of the i-th economy \( \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln \hat{P}_{i,k}^Y(t) \), according to the adjusted output price relationship:

\[ \ln \hat{P}_i^Y(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln \hat{P}_{i,k}^Y(t) \]  
(Appendix.D.2)

Here \( \text{Contribution}_{Y_i,k} \) represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.
Compared with the corresponding equation showing the production function of national total output in the original model, the following modified equation (Appendix.D.3) replaces the input factors for the national production function from capital and labor with goods or services produced by all 45 domestic industries, in order to be consistent with the innovatively added equations characterizing industries under the whole economy and their inter-country cross-industry input-output relations in the model.

Specifically, the correspondingly modified equation (Appendix.D.3) shows the logarithmic deviation of the total output of the i-th economy \( \ln Y_i(t) \) depends on the industry-level output weighted average of the logarithmic deviation of the contemporaneous total output of the k-th industry of the i-th economy \( \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln Y_{i,k}(t) \), according to the adjusted total production function:

\[
\ln Y_i(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln Y_{i,k}(t)
\]

(Appendix.D.3)

Here \( \text{Contribution}_{Y_i,k} \) represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Compared with the above equation defining the national real output, the following modified equation (Appendix.D.4) has a similar structure to the above modified equation (Appendix.D.3), their only difference is that equation (Appendix.D.4) uses the logarithmic deviation of the potential output of the k-th industry of the i-th economy (the inferred industry-level total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \ln \tilde{Y}_{i,k}(t) \) instead of the logarithmic deviation of the real output of the k-th industry of the i-th economy \( \ln Y_{i,k}(t) \), to define the logarithmic deviation of the potential output of the i-th economy (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \ln \tilde{Y}_i(t) \):

\[
\ln \tilde{Y}_i(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln \tilde{Y}_{i,k}(t)
\]

(Appendix.D.4)

Since we want to further incorporate the structure of FDI and ODI into this model, compared with the corresponding equation characterizing the logarithmic deviation of the business investment \( \ln I^K_i(t) \) in the original model, the following innovatively adjusted equation (Appendix.D.5) replaces all the logarithmic deviation of the business investment \( \ln I^K_i(t) \) in the original equation with the logarithmic deviation of the domestic and foreign business investment \( \ln I^K_{i,\text{dom}}(t) \) for the production sector of the i-th economy. The innovatively modified equation shows the logarithmic deviation of the domestic and foreign business investment \( \ln I^K_{i,\text{dom}}(t) \) depends on a weighted average of its past and expected future values, driven by the logarithmic deviation of the contemporaneous relative shadow price of private physical capital \( \ln Q^K_i(t) - \ln P^C_i(t) \), and is also influenced by the logarithmic deviation of the contemporaneous business investment demand shock \( \ln \nu^K_i(t) \), according to the adjusted business investment demand relationship:
\[
\ln I_{i}^{\text{K,in}}(t) = \frac{1}{1+\beta} \ln I_{i}^{\text{K,in}}(t-1) + \frac{\beta}{1+\beta} E_{t} \ln I_{i}^{\text{K,in}}(t+1) + \frac{1}{\chi_{i}(1+\beta)} \left( \ln Q_{i}^{\text{K}}(t) + \ln Q_{i}^{\text{C}}(t) - \ln P_{i}^{\text{C}}(t) \right)
\]

(Appendix.D.5)

Compared with the corresponding equation characterizing the accumulation of business investment \(\ln I_{i}^{\text{K}}(t)\) in the original model, the following innovatively modified equation (Appendix.D.6) replaces the logarithmic deviation of the business investment \(\ln I_{i}^{\text{K,in}}(t)\) in the original equation with the logarithmic deviation of the domestic and foreign business investment \(\ln I_{i}^{\text{K,in}}(t-1)\) for the production sector of the i-th economy. This innovatively adjusted equation shows the logarithmic deviation of the past domestic and foreign business investment \(\ln I_{i}^{\text{K,in}}(t-1)\) for the production sector of the i-th economy is accumulated to form the logarithmic deviation of the contemporaneous private physical capital stock \(\ln K_{i}(t)\), according to the perpetual inventory method, while the logarithmic deviation of the contemporaneous private physical capital stock \(\ln K_{i}(t)\) is also influenced by the logarithmic deviation of the expected future capital utilization rate \(\ln u_{i}^{\text{K}}(t+1)\), the logarithmic deviation of the expected future corporate income tax rate \(E_{t} \ln \tau_{i}^{\text{C}}(t+1)\) and the linear deviation of the contemporaneous regulatory corporate loan to value ratio limit \(\phi_{i}^{\text{F}}(t)\) to reflect the existence of a financial accelerator mechanism, according to business investment Euler equation:

\[
\ln Q_{i}^{\text{K}}(t) - \ln P_{i}^{\text{C}}(t) = E_{t} \left( \beta (1 - \delta_{i}^{\text{C}}) \ln Q_{i}^{\text{K}}(t+1) + \ln P_{i}^{\text{C}}(t+1) \right) - \left( (1 - \phi_{i}^{\text{F}}) \left( \ln I_{i}^{\text{A,H}}(t+1) - \tilde{\pi}_{i}^{\text{C}}(t+1) \right) + \phi_{i}^{\text{F}} \left( \frac{\delta_{i}^{\text{C}}}{\beta_{c-1}} \frac{1+\kappa_{i}^{\text{C}}(1-\chi_{i}^{\text{C}}\delta_{i}^{\text{C}})}{\beta} \right) \left( \ln I_{i}^{\text{C,E}}(t+1) - \tilde{\pi}_{i}^{\text{C}}(t+1) \right) \right) + \left( (1 - \beta(1 - \delta_{i}^{\text{K}})) \right) + \left( \delta_{i}^{\text{K}} \ln u_{i}^{\text{K}}(t+1) - \frac{1}{\chi_{i}(1+\beta)} \phi_{i}^{\text{F}} \left( \ln Q_{i}^{\text{K}}(t+1) + \ln Q_{i}^{\text{C}}(t+1) - \ln P_{i}^{\text{C}}(t+1) \right) + \tilde{\phi}_{i}^{\text{F}}(t) \right)
\]

(Appendix.D.7)

Equation (Appendix.D.8) shows the logarithmic deviation of the capital utilization rate \(\ln u_{i}^{\text{K}}(t)\) depends on the logarithmic deviation of the contemporaneous real wage \(\ln W_{i}(t) - \ln P_{i}^{\text{C}}(t)\), as well as the difference between the logarithmic deviation of the contemporaneous private
physical capital stock $\ln K_i(t)$ and the logarithmic deviation of the contemporaneous employed labor force $\ln L_i(t)$, according to capital utilization relationship:

$$\ln u^K_i(t) = \frac{1}{1 + \eta K} \left[ (\ln W_i(t) - \ln P_i(t)) - (\ln K_i(t) - \ln L_i(t)) \right]$$  
(Appendix.D.8)

Equation (Appendix.D.9) shows the logarithmic deviation of the price of corporate equity $\ln V_i^S(t)$ not only depends on its expected future value, driven by the logarithmic deviation of the expected future corporate profits $E_t \ln \Pi_i^S(t + 1)$, but also depends on the linear deviation of the contemporaneous nominal short term bond yield $i^K_i(t)$, and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign equity risk premium for the $i$-th economy $\ln \upsilon_i^S(t)$, according to equity market relationship:

$$\ln V_i^S(t) = \beta E_t \ln V_i^S(t + 1) + (1 - \beta) E_t \ln \Pi_i^S(t + 1) - i^K_i(t) + \ln \upsilon_i^S(t)$$  
(Appendix.D.9)

Equation (Appendix.D.10) shows the logarithmic deviation of the corporate profits $\ln \Pi_i^S(t)$ not only depends on the logarithmic deviation of the contemporaneous output price level $\ln P_i^Y(t)$ and the logarithmic deviation of the contemporaneous total output $\ln Y_i(t)$, but also depends on the linear deviation of the contemporaneous corporate income tax rate $\tau^K_i(t)$, according to corporate profit function:

$$\ln \Pi_i^S(t) = \ln \tilde{\Pi}_i^Y(t) + \ln Y_i(t) - \tau^K_i(t)$$  
(Appendix.D.10)

Equation (Appendix.D.11) defines the logarithmic deviation of the output gap $\ln \bar{Y}_i(t)$, which equals to the difference between the logarithmic deviation of the contemporaneous total output $\ln \tilde{Y}_i(t)$ and the logarithmic deviation of the contemporaneous potential output (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) $\ln \bar{Y}_i(t)$:

$$\ln \bar{Y}_i(t) = \ln \tilde{Y}_i(t) - \ln \bar{Y}_i(t)$$  
(Appendix.D.11)

Compared with the corresponding equation in the original model, the innovatively adjusted equation (Appendix.D.12) defines the domestic comprehensive labor productivity as the aggregated labor productivity at the industry level. Specifically, this equation shows the logarithmic deviation of the domestic comprehensive labor productivity $\ln \tilde{A}_i(t)$ depends on the domestic industrial output weighted average of the logarithmic deviation of the contemporaneous labor productivity for the $k$-th industry of the $i$-th economy $\sum_{k=1}^{45} \text{Contribution}_{Y_{ik}} \ln \tilde{A}_{ik}(t)$:

$$\ln \tilde{A}_i(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_{ik}} \ln \tilde{A}_{ik}(t)$$  
(Appendix.D.12)

Here $\text{Contribution}_{Y_{ik}}$ represents the proportion of the real output of the $k$-th industry of the $i$-th economy to the real output of all industries of the $i$-th economy.
Equation (Appendix.D.13) shows the logarithmic deviation of the trend labor productivity (which exhibits partial adjustment dynamics of labor productivity) $\ln \tilde{A}_i(t)$ depends on its past value, driven by the logarithmic deviation of the contemporaneous labor productivity $\ln A_i(t)$:

$$\ln \tilde{A}_i(t) = \rho A \ln \tilde{A}_i(t-1) + (1 - \rho A) \ln A_i(t)$$  
(Anyx.D.13)

Equation (Appendix.D.14) defines the intertemporal change in the logarithmic deviation of the trend labor productivity $\Delta \ln \tilde{A}_i(t)$, which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the trend labor productivity $\ln \tilde{A}_i(t)$:

$$\Delta \ln \tilde{A}_i(t) = \ln \tilde{A}_i(t) - \ln \tilde{A}_i(t-1)$$  
(Anyx.D.14)

### E. Banking Sector

Equation (Appendix.E.1) shows the logarithmic deviation of the bank credit stock (also called the aggregate bank assets) $\ln B_i^{CB}(t)$ depends on a weighted average of the logarithmic deviation of the contemporaneous bank money stock (also called the aggregate bank funding) $\ln M_i^S(t)$ and the logarithmic deviation of the contemporaneous aggregate bank capital $\ln K_i^B(t)$, according to bank balance sheet identity:

$$\ln B_i^{CB}(t) = (1 - \kappa R) \ln M_i^S(t) + \kappa R \ln K_i^B(t)$$  
(Appendix.E.1)

Compared with the corresponding equation in the original model, only the timeline of this equation has been pushed back for one period.

Equation (Appendix.E.2) shows the logarithmic deviation of the bank credit stock (also called the aggregate bank assets) $\ln B_i^{CD}(t)$ not only depends on the logarithmic deviation of the contemporaneous total mortgage loans issued to domestic real estate developers $\ln B_i^{CD}(t)$, but also depends on the nonfinancial corporate lending weighted average of the logarithmic deviation of the past domestic currency denominated contemporaneous total final corporate loans issued by global final banks of any specific economy to firms of that specific economy $\sum_{j=1}^{N} w_{ij}^C \left( \ln B_j^{CF}(t) - \ln F_{ij}(t-1) \right)$, according to bank credit demand function:

$$\ln B_i^{CD}(t) = \omega_i^C \ln B_i^{CD}(t) + (1 - \omega_i^C) \sum_{j=1}^{N} w_{ij}^C \left( \ln B_j^{CF}(t) - \ln F_{ij}(t-1) \right)$$  
(Appendix.E.2)

Only the timeline of this equation has been pushed back for one period, compared with the corresponding equation in the original model.

Equation (Appendix.E.3) shows the logarithmic deviation of the total mortgage loans issued to domestic real estate developers $\ln B_i^{CD}(t)$ not only depends on the logarithmic deviation of the past consumption price level $\ln P_i^C(t-1)$ and the logarithmic deviation of the contemporaneous housing stock $\ln H_i(t)$, but also depends on the linear deviation of the past regulatory mortgage loan to value ratio limit $\phi_i^D(t-1)$:
\[
\ln \hat{B}_{i}^{CD}(t) = \ln \hat{B}_{i}^{C}(t-1) + \ln \hat{H}_{i}(t) + \frac{1}{\phi} \hat{\phi}_{i}^{D}(t-1)
\]
(Appendix.E.3)

The timeline rather than the structure or the parameter values of this equation has been changed, and its timeline has been pushed back for one period, compared with the corresponding equation in the original model.

Equation (Appendix.E.4) shows the logarithmic deviation of the domestic currency denominated total final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( \ln B_{i}^{C}(t) \) not only depends on the logarithmic deviation of the past consumption price level \( \ln P_{i}^{C}(t-1) \) and the logarithmic deviation of the contemporaneous private physical capital stock \( \ln K_{i}(t) \), but also depends on the linear deviation of the past regulatory corporate loan to value ratio limit \( \hat{\phi}_{i}^{D}(t-1) \):
\[
\ln \hat{B}_{i}^{C}(t) = \ln \hat{B}_{i}^{C}(t-1) + \ln \hat{K}_{i}(t) + \frac{1}{\phi} \hat{\phi}_{i}^{D}(t-1)
\]
(Appendix.E.4)

Only the timeline of this equation has been changed, and it has been pushed back for one period compared with the corresponding equation in the original model.

Equation (Appendix.E.5) shows the linear deviation of the domestic nominal mortgage loan rate \( \hat{i}_{i}^{M}(t) \) not only depends on a weighted average of its past and expected future values, driven by the difference between the linear deviation of the past nominal interbank loans rate \( \hat{i}_{i}^{B}(t-1) \) and the linear deviation of the contemporaneous domestic nominal mortgage loan rate net of the linear deviation of the contemporaneous domestic mortgage loan default rate \( (\hat{i}_{i}^{M}(t) - \delta_{i}^{M}(t)) \), but also depends on the difference between the linear deviation of the contemporaneous bank capital ratio \( \hat{k}_{i}(t) \) and the linear deviation of the contemporaneous regulatory bank capital ratio requirement \( \hat{k}_{i}^{R}(t) \), as well as the difference between the linear deviation of the contemporaneous regulatory bank capital ratio requirement \( \hat{k}_{i}^{R}(t) \) and the linear deviation of the past nominal interbank loans rate \( \hat{i}_{i}^{B}(t-1) \) that reflects the funding cost for satisfying regulatory bank capital ratio requirement, and is also influenced by the logarithmic deviation of the contemporaneous mortgage loan rate markup shock \( \ln \hat{\theta}_{i}^{CD}(t) \), according to mortgage lending rate Phillips curve:
\[
\hat{i}_{i}^{M}(t) = \frac{1}{1+\beta} \hat{i}_{i}^{M}(t-1) + \beta \frac{1}{1+\beta} \hat{E}_{i} \hat{i}_{i}^{M}(t+1) + \left( \frac{1-\omega_{i}^{C}(1-\omega_{i}^{C})}{\omega_{i}^{C}(1+\beta)} \right) \left( \hat{i}_{i}^{B}(t-1) - \left( \hat{i}_{i}^{M}(t) - \delta_{i}^{M}(t) \right) \right) - \frac{1-\beta(1-\chi^{C})}{1+\kappa^{R}(1-\beta(1-\chi^{C}))} \left[ \eta^{C} \left( \hat{k}_{i}(t) - \hat{k}_{i}^{R}(t) \right) - \left( \hat{k}_{i}^{R}(t) - \kappa^{R} \hat{i}_{i}^{B}(t-1) \right) \right] + \ln \hat{\theta}_{i}^{CD}(t)
\]
(Appendix.E.5)

Equation (Appendix.E.6) shows the linear deviation of the weighted average of domestic and foreign nominal corporate loan rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( \hat{i}_{i}^{CE}(t) \) not only depends on the nonfinancial corporate borrowing weighted average of the linear deviation of the past nominal corporate loan rate for economy specific local currency denominated final corporate loans provided by domestic final banks of any specific economy to both domestic and foreign global final banks \( \sum_{i=1}^{N} \omega_{i,j}^{F} \hat{i}_{j}^{C}(t-1) \), but also depends on the
nonfinancial corporate borrowing weighted average of the intertemporal change in the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to the currency issued by any specific economy $\sum_{j=1}^{N} \omega_{ij} \left( \ln \bar{E}_{ij}(t) - \ln \bar{E}_{ij}(t-1) \right)$, according to effective corporate loan rate function:

$$i_{i}^{CE}(t) = \sum_{j=1}^{N} \omega_{ij} \left[ i_{i}^{C}(t-1) + \left( \ln \bar{E}_{ij}(t) - \ln \bar{E}_{ij}(t-1) \right) \right]$$

(Appendix.E.6)

Equation (Appendix.E.7) shows the linear deviation of the domestic nominal corporate loan rate for economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks $i_{i}^{C}(t)$ not only depends on a weighted average of its past and expected future values, driven by the difference between the linear deviation of the past nominal interbank loans rate $i_{i}^{B}(t-1)$ and the linear deviation of the contemporaneous domestic nominal corporate loan rate net of the linear deviation of the contemporaneous weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks $\left( i_{i}^{C}(t) - \delta_{i}^{CE}(t) \right)$, but also depends on the difference between the linear deviation of the contemporaneous bank capital ratio $\kappa_{i}(t)$ and the linear deviation of the contemporaneous regulatory bank capital ratio requirement $\kappa_{i}^{R}(t)$, as well as the difference between the linear deviation of the contemporaneous regulatory bank capital ratio requirement $\kappa_{i}^{R}(t)$ and the linear deviation of the past nominal interbank loans rate $i_{i}^{B}(t-1)$ that reflects the funding cost for satisfying regulatory bank capital ratio requirement, and is also influenced by the logarithmic deviation of the contemporaneous corporate loan rate markup shock $\ln \varphi_{i}^{CF}(t)$, according to corporate lending rate Phillips curve:

$$i_{i}^{C}(t) = \frac{1}{1+\beta} i_{i}^{C}(t-1) + \frac{\beta}{1+\beta} E_{t} i_{i}^{C}(t+1) + \frac{(1-\omega_{i}^{C})(1-\omega_{i}^{C})}{\omega_{i}^{C}(1+\beta)} \left\{ \left[ i_{i}^{B}(t-1) - (i_{i}^{C}(t) - \delta_{i}^{CE}(t)) \right] - \frac{1-\beta(1-\chi^{C})}{1+\kappa^{R}(1-\beta(1-\chi^{C}))} \left[ \eta^{C} \left( \kappa_{i}(t) - \kappa_{i}^{R}(t) \right) - \left( \kappa_{i}^{R}(t) - \kappa_{i}^{R}(t-1) \right) \right] + \ln \varphi_{i}^{CF}(t) \right\}$$

(Appendix.E.7)

Equation (Appendix.E.8) shows the logarithmic deviation of the bank retained earnings $\ln \bar{I}_{i}^{B}(t)$ depends on a weighted average of its past and expected future values, driven by the logarithmic deviation of the contemporaneous shadow price of bank capital $\ln \bar{Q}_{i}^{B}(t)$, according to retained earnings relationship:

$$\ln \bar{I}_{i}^{B}(t) = \frac{1}{1+\beta} \ln \bar{I}_{i}^{B}(t-1) + \frac{\beta}{1+\beta} E_{t} \ln \bar{I}_{i}^{B}(t+1) + \frac{1}{\chi^{B}(1+\beta)} \ln \bar{\delta}_{i}^{B}(t)$$

(Appendix.E.8)

Equation (Appendix.E.9) shows the logarithmic deviation of the shadow price of bank capital $\ln \bar{Q}_{i}^{B}(t)$ not only depends on its expected future value net of the linear deviation of the expected future bank capital destruction rate $E_{t} \left( \ln \bar{Q}_{i}^{B}(t+1) - \delta_{i}^{B}(t+1) \right)$, as well as the linear deviation of the contemporaneous nominal interbank loans rate $i_{i}^{B}(t)$, but also depends on the difference between the linear deviation of the expected future bank capital
ratio $E_t\kappa_i(t+1)$ and the linear deviation of the expected future regulatory bank capital ratio requirement $E_t\kappa^R_i(t+1)$, according to retained earnings Euler equation:

$$\ln\bar{Q}^B_i(t) = E_t\left\{ \beta(1 - \chi^C\delta)\left(\ln\bar{Q}^B_i(t+1) - \delta^B_i(t+1)\right) - \left[\bar{R}^B_i(t) + \left(1 - \beta(1 - \chi^C\delta)\right)\frac{\eta^C}{\chi^R}\left(\kappa_i(t+1) - \kappa^R_i(t+1)\right)\right]\right\}$$

(Appendix.E.9)

Equation (Appendix.E.10) shows the logarithmic deviation of the past bank retained earnings $\ln\bar{I}^B_i(t-1)$ is accumulated based on the logarithmic deviation of the past aggregate bank capital net of the linear deviation of the past bank capital destruction rate $\left(\ln\bar{R}^B_i(t-1) - \delta^B_i(t-1)\right)$, to form the logarithmic deviation of the contemporaneous aggregate bank capital $\ln\bar{K}^B_i(t)$, according to the perpetual inventory method:

$$\ln\bar{K}^B_i(t) = (1 - \chi^C\delta)\left(\ln\bar{K}^B_i(t-1) - \delta^B_i(t-1)\right) + \chi^C\delta\ln\bar{I}^B_i(t-1)$$

(Appendix.E.10)

Compared with the corresponding equation in the original model, the timeline of this equation has been pushed back for one period, while the structure of this equation and the values of its parameters have no adjustment.

Equation (Appendix.E.11) shows the linear deviation of the bank capital destruction rate $\delta^B_i(t)$ depends on a weighted average of the linear deviation of the contemporaneous domestic mortgage loan default rate $\delta^M_i(t)$ and the linear deviation of the contemporaneous weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks $\delta^C,E_i(t)$:

$$\delta^B_i(t) = \chi^C\left(\omega^C_i\delta^M_i(t) + (1 - \omega^C_i)\delta^C,E_i(t)\right)$$

(Appendix.E.11)

Equation (Appendix.E.12) shows the linear deviation of the domestic mortgage loan default rate $\delta^M_i(t)$ depends on a weighted average of its past and attractor values, and is also influenced by the linear deviation of the contemporaneous mortgage loan default shock $\nu^M_i(t)$, while the linear deviation of the attractor mortgage loan default rate $\delta^M_{\text{attractor}}(t)$, the intertemporal change in the logarithmic deviation of the price of housing $\left(\ln\bar{V}_i^H(t) - \ln\bar{V}_i^H(t-1)\right)$, and the intertemporal change in the logarithmic deviation of the price of housing $\left(\ln\bar{V}_i^H(t) - \ln\bar{V}_i^H(t-1)\right)$, according to mortgage loan default rate relationship:

$$\delta^M_i(t) = \rho^M\delta^M_i(t-1) - (1 - \rho^M)\left[\chi^M\nu^M_i(t) + \chi^{M,V}\left(\ln\bar{V}_i^H(t) - \ln\bar{V}_i^H(t-1)\right)\right] + \delta^M_{\text{attractor}}(t)$$

(Appendix.E.12)

Equation (Appendix.E.13) shows the linear deviation of the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic
and foreign global final banks $\delta^{C,E}_i(t)$ depends on the nonfinancial corporate lending weighted average of the linear deviation of the contemporaneous domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of any specific economy to firms of that specific economy $\sum_{j=1}^{N} \omega_{ij} \delta^{C}_j(t)$, according to corporate credit loss rate function:

$$\delta^{C,E}_i(t) = \sum_{j=1}^{N} \omega_{ij} \delta^{C}_j(t)$$  (Appendix.E.13)

Equation (Appendix.E.14) shows the linear deviation of the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy $\delta^{C}_i(t)$ depends on a weighted average of its past and attractor values, and is also influenced by the linear deviation of the contemporaneous corporate loan default shock $\nu^{C}_i(t)$, while the linear deviation of the attractor corporate loan default rate $-\xi \delta^{C}_i(t)$, $\gamma \ln Y_i(t) + \xi \delta^{C}_i(t) + \nu^{E}_i(t)$ depends on the logarithmic deviation of the contemporaneous output gap $\ln Y_i(t)$, as well as the intertemporal change in the logarithmic deviation of the price of corporate equity $(\ln V_i(t) - \ln V_i(t-1))$ according to corporate loan default rate relationship:

$$\delta^{C}_i(t) = \rho \delta^{C}_i(t-1) - (1 - \rho) \left[ \xi^{C,Y} \ln Y_i(t) + \xi^{C,Y} \left( \ln V_i(t) - \ln V_i(t-1) \right) \right] + \nu^{C}_i(t)$$  (Appendix.E.14)

**F. Foreign Exchange Sector**

The equations for the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to US dollar (which is the quotation currency for transactions in the foreign exchange market issued by the 1-st economy ("United States")) under different exchange rate and inflation targeting arrangements are different.

Under a free floating exchange rate and flexible inflation targeting arrangement or under a managed exchange rate arrangement, equation (Appendix.F.1a) shows the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to US dollar $\ln \hat{E}_{i,1}(t)$ depends on its expected future value, driven by the linear deviation of the contemporaneous nominal short term bond yield differential between the i-th economy and the 1-st economy $(\hat{r}^{S}_i(t) - \hat{r}^{S}_1(t))$, and is also influenced by the logarithmic deviation of the contemporaneous currency risk premium shock differential between the i-th economy and the 1-st economy $(\ln \hat{V}_i(t) - \ln \hat{V}_1(t))$, according to foreign exchange market relationship:

$$\ln \hat{E}_{i,1}(t) = E_t \ln \hat{E}_{i,1}(t+1) - \left[ \left( \hat{r}^{S}_i(t) - \ln \hat{V}_i(t) \right) - \left( \hat{r}^{S}_1(t) - \ln \hat{V}_1(t) \right) \right]$$  (Appendix.F.1a)

Under a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union), compared with the corresponding equation characterizing the nominal bilateral exchange rate in the original model, the following simplified equation (Appendix.F.1b) and (Appendix.F.1c) are a more simplified characterization of reality, which avoid the problem that the complete replication of the original model is easy to give extreme results when running the simulation.
If the $i^*$-th economy issues the anchor currency for the currency issued by the $i$-th economy, equation (Appendix.F.1b) shows the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $\ln \varepsilon_{i,1}(t)$ equals the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the $i^*$-th economy to US dollar $\ln \varepsilon_{i^*,1}(t)$, while equation (Appendix.F.1c) shows the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the 1-st economy United States to US dollar $\ln \varepsilon_{1,1}(t)$ should always be equal to 0:

$$\ln \varepsilon_{i,1}(t) = \ln \varepsilon_{i^*,1}(t)$$
(Appendix.F.1b)

$$\ln \varepsilon_{1,1}(t) = 0$$
(Appendix.F.1c)

Equation (Appendix.F.2) shows the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the $i$-th economy to the currency issued by the $j$-th economy (here $j\neq 1$) $\ln \varepsilon_{i,j}(t)$ equals the difference between the logarithmic deviation of the contemporaneous nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $\ln \varepsilon_{i,1}(t)$ and the logarithmic deviation of the contemporaneous nominal bilateral exchange rate of the currency issued by the $j$-th economy to US dollar $\ln \varepsilon_{j,1}(t)$:

$$\ln \varepsilon_{i,j}(t) = \ln \varepsilon_{i,1}(t) - \ln \varepsilon_{j,1}(t)$$
(Appendix.F.2)

Equation (Appendix.F.3) defines the logarithmic deviation of the nominal effective exchange rate of the currency issued by the $i$-th economy (which is the trade weighted average price of foreign currency in terms of the $i$-th economy's currency) $\ln \hat{\varepsilon}_i(t)$ as the difference between the logarithmic deviation of the contemporaneous nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $\ln \hat{\varepsilon}_{i,1}(t)$ and the trade weighted average of the logarithmic deviation of the contemporaneous nominal bilateral exchange rate of the currency issued by any specific economy to US dollar $\sum_{j=1}^{N} \omega_{ij} \ln \varepsilon_{j,1}(t)$:

$$\ln \hat{\varepsilon}_i(t) = \ln \hat{\varepsilon}_{i,1}(t) - \sum_{j=1}^{N} \omega_{ij} \ln \varepsilon_{j,1}(t)$$
(Appendix.F.3)

Equation (Appendix.F.4) shows the logarithmic deviation of the real bilateral exchange rate of the currency issued by the $i$-th economy relative to US dollar (which is the relative price of the 1-st economy's output in terms of the $i$-th economy's output) $\ln \Omega_{i,1}(t)$ not only depends on the logarithmic deviation of the contemporaneous nominal bilateral exchange rate of the currency issued by the $i$-th economy to US dollar $\ln \hat{\varepsilon}_{i,1}(t)$, but also depends on the logarithmic deviation of the contemporaneous output price level differential between the 1-st economy and the $i$-th economy $(\ln \tilde{P}_1^Y(t) - \ln \tilde{P}_i^Y(t))$:

$$\ln \Omega_{i,1}(t) = \ln \hat{\varepsilon}_{i,1}(t) + \ln \tilde{P}_1^Y(t) - \ln \tilde{P}_i^Y(t)$$
(Appendix.F.4)

Equation (Appendix.F.5) defines the logarithmic deviation of the real effective exchange rate of the currency issued by the $i$-th economy (which is the trade weighted average price of
foreign output in terms of the i-th economy’s output) \( \ln \hat{\Omega}_i(t) \) as the difference between the logarithmic deviation of the contemporaneous real bilateral exchange rate of the currency issued by the i-th economy relative to US dollar \( \ln \hat{\Omega}_{i,1}(t) \) and the trade weighted average of the logarithmic deviation of the contemporaneous real bilateral exchange rate of the currency issued by any specific economy relative to US dollar \( \sum_{j=1}^{N} \omega_{ij} \ln \hat{\Omega}_{j,1}(t) \):

\[
\ln \hat{\Omega}_i(t) = \ln \hat{\Omega}_{i,1}(t) - \sum_{j=1}^{N} \omega_{ij} \ln \hat{\Omega}_{j,1}(t)
\]

(Appendix.F.5)

**G. Export and Import Sectors**

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.G.1) defines the domestic total exports as the aggregated domestic industry-level exports. Specifically, this correspondingly modified equation shows the logarithmic deviation of the total exports of the i-th economy \( \ln X_i(t) \) depends on the industry-level exports weighted average of the logarithmic deviation of the contemporaneous total exports from the k-th industry of the i-th economy \( \sum_{k=1}^{45} \text{Contribution}_{X_{i,k}} \ln X_{i,k}(t) \), according to the adjusted total export relationship:

\[
\ln X_i(t) = \sum_{k=1}^{45} \text{Contribution}_{X_{i,k}} \ln X_{i,k}(t)
\]

(Appendix.G.1)

Here \( \text{Contribution}_{X_{i,k}} \) represents the proportion of the total exports from the k-th industry of the i-th economy (to serve as intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector) to the total exports from all industries of the i-th economy.

Relative to the corresponding equation in the original model, the innovatively adjusted equation (Appendix.G.2) defines the domestic total imports as the aggregated industry-level imports from any specific industry of all other economies. Specifically, this adjusted equation shows the logarithmic deviation of the total imports of the i-th economy \( \ln M_i(t) \) depends on the industry-level imports weighted average of the logarithmic deviation of the contemporaneous total imports of the i-th economy from foreign k-th industry \( \sum_{k=1}^{45} \text{Contribution}_{M_{i,k}} \ln M_{i,k}(t) \), according to the adjusted total import relationship:

\[
\ln M_i(t) = \sum_{k=1}^{45} \text{Contribution}_{M_{i,k}} \ln M_{i,k}(t)
\]

(Appendix.G.2)

Here \( \text{Contribution}_{M_{i,k}} \) represents the proportion of the total imports of the i-th economy from foreign k-th industry (to serve as intermediate inputs for the i-th economy’s production sector or final output goods or services for the i-th economy’s absorption sector) to the total imports of the i-th economy from all foreign industries.

Equation (Appendix.G.3) defines the logarithmic deviation of the terms of trade of the i-th economy \( \ln T_i(t) \) as the logarithmic deviation of the contemporaneous global terms of trade shifter (which is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows) \( \ln \hat{\nu}^T(t) \), adds the difference between the logarithmic deviation of the contemporaneous internal terms of trade of the i-th economy \( \ln T_i^X(t) \) and the logarithmic deviation of the contemporaneous external terms of trade of the i-th economy \( \ln T_i^M(t) \):
\[ \ln \tilde{T}_i(t) = \ln \tilde{v}_i(t) + \ln \tilde{T}_i^X(t) - \ln \tilde{T}_i^M(t) \]  
(Appendix.G.3)

Equation (Appendix.G.4) defines the logarithmic deviation of the internal terms of trade of the i-th economy \( \ln \tilde{T}_i^X(t) \) as the difference between the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln \tilde{P}_i^X(t) \) and the logarithmic deviation of the contemporaneous core price level of the i-th economy \( \ln \tilde{P}_i(t) \):

\[ \ln \tilde{T}_i^X(t) = \ln \tilde{P}_i^X(t) - \ln \tilde{P}_i(t) \]  
(Appendix.G.4)

Equation (Appendix.G.5) defines the linear deviation of the export price inflation of the i-th economy \( \tilde{\pi}_i^X(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the price of exports of the i-th economy \( \ln \tilde{P}_i^X(t) \):

\[ \tilde{\pi}_i^X(t) = \ln \tilde{P}_i^X(t) - \ln \tilde{P}_i^X(t - 1) \]  
(Appendix.G.5)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.G.6) defines the domestic comprehensive export price level as the aggregated export prices at the industry level. Specifically, this equation shows the logarithmic deviation of the domestic comprehensive export price level of the i-th economy \( \ln \tilde{P}_i^X(t) \) depends on the industry-level exports weighted average of the logarithmic deviation of the contemporaneous export price for exports from the k-th industry of the i-th economy \( \sum_{k=1}^{45} \text{Contribution}_{X_i,k} \ln \tilde{P}_i^X,k(t) \), according to the modified export price relationship:

\[ \ln \tilde{P}_i^X(t) = \sum_{k=1}^{45} \text{Contribution}_{X_i,k} \ln \tilde{P}_i^X,k(t) \]  
(Appendix.G.6)

Here \( \text{Contribution}_{X_i,k} \) represents the proportion of the total exports from the k-th industry of the i-th economy (to serve as intermediate inputs for foreign production sector or final output goods or services for foreign absorption sector) to the total exports from all industries of the i-th economy.

Since we want to further incorporate the structure of import tariff into this model, the following five equations are either innovatively added or correspondingly modified.

Compared with the corresponding equation defining the import price inflation without considering tariff in the original model, the correspondingly modified equation (Appendix.G.7) defines the linear deviation of the pre-tariff import price inflation of the i-th economy \( \tilde{\pi}_i^{M,T}(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the pre-tariff price of imports of the i-th economy \( \ln \tilde{P}_i^{M,T}(t) \):

\[ \tilde{\pi}_i^{M,T}(t) = \ln \tilde{P}_i^{M,T}(t) - \ln \tilde{P}_i^{M,T}(t - 1) \]  
(Appendix.G.7)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.G.8) defines the domestic comprehensive pre-tariff import price level as the aggregated pre-tariff import prices for all domestic industry-level imports. Specifically, this equation shows the logarithmic deviation of the domestic comprehensive pre-tariff import
price level of the i-th economy \( \ln P_{i}^{M,T}(t) \) depends on the industry-level imports weighted average of the logarithmic deviation of the contemporaneous pre-tariff import price for imports of the i-th economy from foreign k-th industry \( \sum_{k=1}^{45} \text{Contribution}_{M,i,k} \ln P_{i,k}^{M,T}(t) \), according to the modified import price relationship:

\[
\ln P_{i}^{M,T}(t) = \sum_{k=1}^{45} \text{Contribution}_{M,i,k} \ln P_{i,k}^{M,T}(t)
\]  
(Appendix.G.8)

Here \( \text{Contribution}_{M,i,k} \) represents the proportion of the total imports of the i-th economy from foreign k-th industry (to serve as intermediate inputs for the i-th economy’s production sector or final output goods or services for the i-th economy’s absorption sector) to the total imports of the i-th economy from all foreign industries.

Compared with the original model, the innovatively added equation (Appendix.G.9) defines the linear deviation of the post-tariff import price inflation of the i-th economy \( \pi_{i}^{M}(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the post-tariff price of imports of the i-th economy \( \ln P_{i}^{M}(t) \):

\[
\pi_{i}^{M}(t) = \ln P_{i}^{M}(t) - \ln P_{i}^{M}(t - 1)
\]  
(Appendix.G.9)

The innovatively added equation (Appendix.G.10) defines the logarithmic deviation of the pre-tariff price of imports of the i-th economy \( \ln P_{i}^{M,T}(t) \), which depends on both the logarithmic deviation of the contemporaneous post-tariff price of imports of the i-th economy \( \ln P_{i}^{M}(t) \) and the linear deviation of the contemporaneous import tariff rate \( (\tau_{i}^{M}(t) - \tau_{i}^{M}) \), with \( \tau_{i}^{M}(t) \) the contemporaneous import tariff rate variable and \( \tau_{i}^{M} \) the tariff rate constant for the i-th economy's imports in period t:

\[
\ln P_{i}^{M,T}(t) = \ln P_{i}^{M}(t) - \frac{1}{1 + \tau_{i}^{\text{Tariff}}(t)} (\tau_{i}^{M}(t) - \tau_{i}^{M})
\]  
(Appendix.G.10)

Here \( \tau_{i}^{\text{Tariff}} \) is the i-th economy's broad level of import tariff rates.

Compared with the corresponding equation defining the external terms of trade of the i-th economy without considering tariff in the original model, the correspondingly modified equation (Appendix.G.11) defines the logarithmic deviation of the external terms of trade of the i-th economy \( \ln \tau_{i}^{M}(t) \) as the difference between the logarithmic deviation of the contemporaneous post-tariff price of imports of the i-th economy \( \ln P_{i}^{M}(t) \) and the logarithmic deviation of the contemporaneous core price level of the i-th economy \( \ln \bar{P}_{i}(t) \):

\[
\ln \tau_{i}^{M}(t) = \ln P_{i}^{M}(t) - \ln \bar{P}_{i}(t)
\]  
(Appendix.G.11)

H. Balance of Payments Sector

Equation (Appendix.H.1) shows the linear deviation of the ratio of the current account balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{CA_{i}(t)}{E_{i,i}(t)P_{i}^{i}(t)Y_{i}(t)} \) not only depends on the linear deviation of the ratio of the
contemporaneous net foreign asset position (which equals the sum of the financial wealth of the household sector, the construction sector, the production sector, the banking sector, the export sector, the import sector, and the government sector of the i-th economy) to past nominal output of the i-th economy \( \frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} \), but also depends on the linear deviation of the ratio of the contemporaneous trade balance (denominated in US dollars) to contemporaneous nominal output of the i-th economy \( \frac{\tilde{TB}_i(t)}{E_{i,t}(t)P_i^Y(t)Y_i(t)} \), according to national dynamic budget constraint:

\[
\frac{CA_i(t)}{E_{i,t}(t)P_i^Y(t)Y_i(t)} = \frac{1-\beta}{\beta} \frac{1}{1+g} \frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} + \frac{\tilde{TB}_i(t)}{E_{i,t}(t)P_i^Y(t)Y_i(t)}
\]  

(Appendix.H.1)

Equation (Appendix.H.2) shows the linear deviation of the ratio of the net foreign asset position to past nominal output of the i-th economy \( \frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} \) not only depends on its past value, but also depends on the linear deviation of the ratio of the past current account balance (denominated in US dollars) to past nominal output of the i-th economy \( \frac{CA_i(t-1)}{E_{i,t-1}(t-1)P_i^Y(t-1)Y_i(t-1)} \) and the logarithmic deviation of the contemporaneous terms of trade \( \ln \Pi_i(t) \) of the i-th economy, which measure the relative price of exports to imports, while the structure of this equation and the values of its parameters have not been changed.

\[
\frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} = \frac{1}{1+g} \frac{\tilde{A}_i(t-1)}{P_i^Y(t-2)Y_i(t-2)} + \frac{CA_i(t-1)}{E_{i,t-1}(t-1)P_i^Y(t-1)Y_i(t-1)}
\]  

(Appendix.H.2)

Compared with the corresponding equation in the original model, the timeline of this equation has been pushed back for one period (including \( \frac{\tilde{A}_i(t+1)}{P_i^Y(t)Y_i(t)} \) is adjusted to \( \frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} \), \( \frac{\tilde{A}_i(t)}{P_i^Y(t-1)Y_i(t-1)} \) is adjusted to \( \frac{\tilde{A}_i(t-1)}{P_i^Y(t-2)Y_i(t-2)} \), and \( \frac{CA_i(t)}{E_{i,t}(t)P_i^Y(t)Y_i(t)} \) is adjusted to \( \frac{CA_i(t-1)}{E_{i,t-1}(t-1)P_i^Y(t-1)Y_i(t-1)} \), while the structure of this equation and the values of its parameters have not been changed.

Compared with the corresponding equation in the original model, the parameters before the logarithmic deviation of the terms of trade (which measures the relative price of exports to imports) \( \ln \Pi_i(t) \) in the innovatively adjusted equation (Appendix.H.3) characterizing the ratio of the trade balance to nominal output have been changed, in order to make the trade balance characterization more accurate for those small and highly open economies, whose total export value or total import value can be as large as or even larger than their GDP, which lead to the impact of their export or import value on the trade balance much higher than that of other economies. Specifically, the innovatively adjusted equation (Appendix.H.3) shows the linear deviation of the ratio of the trade balance (denominated in US dollars) to nominal output of the i-th economy \( \frac{\tilde{TB}_i(t)}{E_{i,t}(t)P_i^Y(t)Y_i(t)} \) depends on the total exports and total imports weighted average of the logarithmic deviation of the contemporaneous global terms of trade shifter (which is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows) \( \ln \tilde{U}_i^0(t) \), the logarithmic deviation of the contemporaneous global terms of trade shifter \( \ln \tilde{U}_i^0(t) \), and the logarithmic deviation of the contemporaneous post-tariff import value of the i-th economy \( \ln \tilde{M}_i(t) + \ln \tilde{M}_i(t) \), while for small and highly open economies, this part is replaced by the difference between the logarithmic deviation of the contemporaneous total exports of the i-th economy \( \ln \tilde{X}_i(t) \) and the logarithmic deviation of the contemporaneous total imports of the i-
th economy \(\ln M_i(t)\), adding the logarithmic deviation of the contemporaneous terms of trade of the i-th economy (which measures the relative price of exports to imports of the i-th economy) \(\ln T_i(t)\), multiplied by a threshold value (to reflect such small and highly open economies are usually good at balancing their imports and exports and maintaining a stable structure between imports, exports, domestic demand and domestic production):

\[
\frac{\tau_B_i(t)}{E_{i,t}(P_i^Y(t)Y_i(t))} = 0.5\cdot \frac{X_i+0.5\cdot M_i}{Y_i} \ln 0^T(t) + \frac{X_i}{Y_i} \left[ \ln \tilde{P}_i^X(t) + \ln \tilde{X}_i(t) \right] - \frac{M_i}{Y_i} \left[ \ln \tilde{P}_i^M(t) + \ln \tilde{M}_i(t) \right]
\]

(Appendix.H.3)

or

\[
\frac{\tau_B_i(t)}{E_{i,t}(P_i^Y(t)Y_i(t))} = \text{(threshold value)} \left[ \left( \ln \tilde{X}_i(t) - \ln \tilde{M}_i(t) \right) + \ln \tilde{T}_i(t) \right]
\]

(Appendix.H.3)

Here,

the adjusted parameter \(0.5\cdot \frac{X_i+0.5\cdot M_i}{Y_i}\) represents the proportion of the domestic currency denominated average value of the i-th economy’s total exports and imports to the domestic currency denominated nominal GDP of the i-th economy;

the adjusted parameter \(\frac{X_i}{Y_i}\) represents the proportion of the domestic currency denominated value of the i-th economy’s total exports to the domestic currency denominated nominal GDP of the i-th economy;

the adjusted parameter \(\frac{M_i}{Y_i}\) represents the proportion of the domestic currency denominated value of the i-th economy’s total imports to the domestic currency denominated nominal GDP of the i-th economy.

Since we want to further incorporate the structure of international direct investment, including the structure of FDI and ODI into this model, the following three equations are innovatively added.

The innovatively added equation (Appendix.H.4) shows the logarithmic deviation of the domestic and foreign business investment \(\ln I^K_{i,\text{in}}(t)\) for the production sector of the i-th economy depends on the FDI (from the j-th economy to the i-th economy) weighted average of the logarithmic deviation of the business investment from any specific economy \(\sum_{j=1}^{N} \omega_{ij} D_{i,j} \ln I^K_j(t)\):

\[
\ln I^K_{i,\text{in}}(t) = \sum_{j=1}^{N} \omega_{ij} D_{i,j} \ln I^K_j(t)
\]

(Appendix.H.4)

The innovatively added equation (Appendix.H.5) shows the linear deviation of the ratio of the direct investment balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \(\frac{CFAD_{i,t}}{E_{i,t}(P_i^Y(t)Y_i(t))}\) depends on the FDI (from the j-th economy to the i-th economy) weighted average of the logarithmic deviation of the contemporaneous US dollar denominated value of total ODI of any specific economy \(\sum_{j=1}^{N} \omega_{ij} D_{i,j} \left( \ln E_{i,j}(t) + \ln \tilde{P}_j^C(t) + \ln \tilde{I}_j^K(t) \right)\), minus the logarithmic deviation of the contemporaneous US dollar denominated value of total ODI of the i-th economy \(\left( \ln E_{i,\text{in}}(t) + \ln \tilde{P}_i^C(t) + \ln \tilde{I}_i^K(t) \right)\):
\[ \frac{\text{CA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} = \frac{1}{Y_i} \sum_{j=1}^{N} \omega_i \left( \ln \text{E}_{1,i}(t) + \ln \text{P}_{j}^C(t) + \ln \text{Y}_{j}^K(t) - \left( \ln \text{E}_{1,i}(t) + \ln \text{P}_{j}^C(t) + \ln \text{Y}_{j}^K(t) \right) \right) \]  
(Appendix.H.5)

The innovatively added equation (Appendix.H.6) shows the weighted average of the linear deviation of the ratio of the current account balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{\text{CA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} \), the linear deviation of the ratio of the direct investment balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{\text{CA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} \), and the linear deviation of the ratio of the rest of capital and financial account balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{\text{CA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} \) should equal to zero, according to balance of payments constraint:

\[ \text{CA}_i \cdot \frac{\text{CA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} + \text{CFA}_i \cdot \frac{\text{CFA}_i(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} + \text{CFA}_{\text{Rest}} \cdot \frac{\text{CFA}_{\text{Rest}}(t)}{\text{E}_i(t)\text{P}_i(t)\text{Y}_i(t)} = 0 \]  
(Appendix.H.6)

### I. Virtual “Absorption Sector”

Equation (Appendix.I.1) defines the linear deviation of the core inflation \( \pi_{\text{c}}(t) \), which equals to the difference between the contemporaneous and past values of the logarithmic deviation of the core price level \( \ln \text{P}_{\text{c}}(t) \):

\[ \pi_{\text{c}}(t) = \ln \text{P}_{\text{c}}(t) - \ln \text{P}_{\text{c}}(t-1) \]  
(Appendix.I.1)

Equation (Appendix.I.2) shows the linear deviation of the core inflation \( \pi_{\text{c}}(t) \) depends on a linear combination of its past and expected future values, driven by the logarithmic deviation of the contemporaneous real labor cost per unit of output \( \ln \text{W}_{\text{c}}(t) + \ln \text{L}_{\text{c}}(t) - \ln \text{P}_{\text{c}}(t), \) and is also influenced by the logarithmic deviation of the contemporaneous output price markup shock \( \ln \text{Y}^Y(t) \), according to Phillips curve:

\[ \tilde{\pi}_{\text{c}}(t) = \frac{\gamma}{1+\gamma^Y} \pi_{\text{c}}(t-1) + \frac{\beta}{1+\gamma^Y} \tilde{\pi}_{\text{c}}(t+1) + \frac{(1-\omega)(1-\omega^Y)}{\omega^Y(1+\gamma^Y)} \left[ \left( \ln \text{W}_{\text{c}}(t) + \ln \text{L}_{\text{c}}(t) - \ln \text{P}_{\text{c}}(t) \right) - \ln \text{Y}_{\text{c}}^Y(t) \right] \]  
(Appendix.I.2)

The structure of the following innovatively adjusted equation is the same as that of the corresponding equation in the original model, however, the parameters before total domestic demand, export and import variables are adjusted to make its setting more suitable for those small and highly open economies we add to the model, whose total export value or total import value can almost equal or even exceed their GDP, which may lead to excessive values of their macroeconomic variables in the simulation. Referring to the reality, such small and highly open economies are usually good at balancing their imports and exports and maintaining a stable structure between imports, exports, domestic demand and domestic production. Therefore, we set a threshold value for both the parameter \( \frac{X_i}{Y_i} \) (the proportion of the domestic currency denominated value of the i-th economy’s total exports to the domestic
currency denominated value of the i-th economy’s total output) before the total export variable \( \ln X_i(t) \), and the parameter \( \frac{M_i}{Y_i} \) (the proportion of the domestic currency denominated value of the i-th economy’s total imports to the domestic currency denominated value of the i-th economy’s total output) before the total import variable \( \ln M_i(t) \). For small and highly open economies whose parameter value is greater than this threshold value, we directly take the value of its parameter \( \frac{X_i}{Y_i} \) or \( \frac{M_i}{Y_i} \) as the threshold value. Finally, the innovatively adjusted equation (Appendix.I.3) shows the logarithmic deviation of the total output \( \ln Y_i(t) \) not only depends on the logarithmic deviation of the contemporaneous total domestic demand \( \ln D_i(t) \), multiplied by the proportion of the domestic currency denominated value of the i-th economy’s total domestic demand to the domestic currency denominated value of the i-th economy’s total output \( \frac{D_i}{Y_i} \) (for small and highly open economies, this parameter takes 1 to realize the constraint that the sum of the three parameters should be 1), but also depends on the gap between the logarithmic deviation of the contemporaneous total exports \( \ln X_i(t) \) multiplied by the proportion of the domestic currency denominated value of the i-th economy’s total exports to the domestic currency denominated value of the i-th economy’s total output \( \frac{X_i}{Y_i} \) or the threshold value, and the logarithmic deviation of the contemporaneous total imports \( \ln M_i(t) \) multiplied by the proportion of the domestic currency denominated value of the i-th economy’s total imports to the domestic currency denominated value of the i-th economy’s total output \( \frac{M_i}{Y_i} \) or the threshold value, according to the modified output demand relationship:

\[
\ln Y_i(t) = \frac{D_i}{Y_i} \ln D_i(t) + \left( \frac{X_i}{Y_i} \right) \ln X_i(t) - \left( \frac{M_i}{Y_i} \right) \ln M_i(t)
\]

(Appendix.I.3)

or

\[
\ln Y_i(t) = \ln D_i(t) + \text{(threshold value)} \ast \left( \ln X_i(t) - \ln M_i(t) \right)
\]

(Appendix.I.3)

Compared with the corresponding equation in the original model, the parameters before the logarithmic deviation of total consumption \( \ln C_i(t) \), the logarithmic deviation of total investment \( \ln I_i(t) \) and the logarithmic deviation of domestic public demand \( \ln G_i(t) \) in the innovatively adjusted equation (Appendix.I.4) characterizing the total domestic demand have been changed, in order to make the total domestic demand characterization more accurate for those economies whose gap between total exports and total imports, or gap between total domestic demand and total output, can be quite large. Specifically, the innovatively adjusted equation (Appendix.I.4) shows the logarithmic deviation of the total domestic demand \( \ln D_i(t) \) depends on a weighted average of the logarithmic deviation of contemporaneous total consumption \( \ln C_i(t) \), the logarithmic deviation of contemporaneous total investment \( \ln I_i(t) \) and the logarithmic deviation of contemporaneous domestic public demand \( \ln G_i(t) \), according to the modified domestic demand relationship:

\[
\ln D_i(t) = \frac{C_i}{D_i} \ln C_i(t) + \frac{I_i}{D_i} \ln I_i(t) + \frac{G_i}{D_i} \ln G_i(t)
\]

(Appendix.I.4)

Here,
\( \frac{C_i}{D_i} \) represents the proportion of the domestic currency denominated value of the i-th economy's total private consumption to the domestic currency denominated value of the i-th economy's total domestic demand;

\( \frac{I_i}{D_i} \) represents the proportion of the domestic currency denominated value of the i-th economy's total private investment to the domestic currency denominated value of the i-th economy's total domestic demand;

\( \frac{G_i}{D_i} \) represents the proportion of the domestic currency denominated value of the i-th economy's total public demand to the domestic currency denominated value of the i-th economy's total domestic demand.

Equation (Appendix.I.5) shows the logarithmic deviation of the total investment (both residential and business investment) \( \ln I_i^\bar{I} (t) \) depends on a weighted average of the logarithmic deviation of contemporaneous residential investment \( \ln I_i^H(t) \) and the logarithmic deviation of contemporaneous business investment \( \ln I_i^K(t) \), according to investment demand relationship:

\[
\frac{I_i}{Y_i} \ln I_i^\bar{I} (t) = \frac{I_i^H}{Y_i} \ln I_i^H (t) + \frac{I_i^K}{Y_i} \ln I_i^K (t)
\]

(Appendix.I.5)

### J. Government Sector

The equations for the linear deviation of the nominal policy interest rate under different exchange rate and inflation targeting arrangements are different.

Under a free floating exchange rate and flexible inflation targeting arrangement, equation (Appendix.J.1a) shows the linear deviation of the nominal policy interest rate \( \hat{i}_i^P(t) \) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporanous monetary policy shock \( \hat{\nu}_i^P(t) \), while the linear deviation of the desired nominal policy interest rate \( \hat{\xi}_i \hat{\pi}_i^C(t + 1) + \xi Y \ln Y_i (t) \) responds to the linear deviation of the expected future consumption price inflation \( \hat{\pi}_i^C(t + 1) \) and the logarithmic deviation of the contemporaneous output gap \( \ln Y_i (t) \), according to monetary policy rule:

\[
\hat{i}_i^P(t) = \rho \hat{i}_i^P(t - 1) + (1 - \rho) \left( \hat{\xi}_i \hat{\pi}_i^C(t + 1) + \xi Y \ln Y_i (t) \right) + \hat{\nu}_i^P(t)
\]

(Appendix.J.1a)

Under a managed exchange rate arrangement, equation (Appendix.J.1b) shows the linear deviation of the nominal policy interest rate \( \hat{i}_i^P(t) \) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporanous monetary policy shock \( \hat{\nu}_i^P(t) \), while the linear deviation of the desired nominal policy interest rate \( \left[ \hat{\xi}_i \hat{\pi}_i^C(t + 1) + \xi Y \ln Y_i (t) + \xi \left( \hat{\ln E}_i (t) - \ln \hat{E}_i (t - 1) \right) \right] \) responds to the linear deviation of the expected future consumption price inflation \( \hat{\pi}_i^C(t + 1) \), the logarithmic deviation of the contemporaneous output gap \( \ln Y_i (t) \), and the intertemporal change in the logarithmic deviation of the nominal effective exchange rate of the currency issued by the i-th economy \( \left( \ln \hat{E}_i (t) - \ln \hat{E}_i (t - 1) \right) \), according to monetary policy rule:
\begin{align*}
\hat{i}_i^p (t) &= \rho \hat{i}_i^p (t - 1) + (1 - \rho) \left[ \xi \hat{E}_i \hat{\pi}_i^c (t + 1) + \xi \ln \hat{Y}_i (t) + \xi \left( \ln \hat{E}_i (t) - \ln \hat{E}_i (t - 1) \right) \right] + \hat{\nu}^p_i (t) \\
&= \frac{1}{1 - \rho} \left[ \xi \left( \ln \hat{E}_i (t) - \ln \hat{E}_i (t - 1) \right) + \xi \ln \hat{Y}_i (t) \right] + \hat{\nu}^p_i (t) \\
(\text{Appendix.J.1b})
\end{align*}

Under a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union), equation (Appendix.J.1c) shows the linear deviation of the nominal policy interest rate \(i^p_i \) tracks the linear deviation of the contemporaneous nominal policy interest rate of the \(i^*\)-th economy that issues the anchor currency for the currency issued by the \(i\)-th economy \(i^*\), and also responds to the intertemporal change in the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the \(i\)-th economy to the anchor currency issued by the \(i^*\)-th economy \(\ln \frac{\hat{Y}_i}{\hat{Y}_{i^*}}\), according to monetary policy rule:

\begin{align*}
\hat{i}_i^p (t) &= \hat{i}_{i^*}^p (t) + \xi \left( \ln \hat{E}_i (t) - \ln \hat{E}_{i^*} (t - 1) \right) \\
&= \hat{i}_{i^*}^p (t) + \xi \left( \ln \hat{E}_i (t) - \ln \hat{E}_{i^*} (t - 1) \right) \\
(\text{Appendix.J.1c})
\end{align*}

Equation (Appendix.J.2) shows the linear deviation of the bank capital ratio \(\hat{\kappa}_i(t)\) depends on the difference between the logarithmic deviation of the contemporaneous aggregate bank capital \(\ln K_i^B(t)\) and the logarithmic deviation of the contemporaneous aggregate bank assets (also called the bank credit stock) \(\ln B_i^C(t)\):

\begin{align*}
\hat{\kappa}_i(t) &= \kappa^R \left( \ln \hat{R}_i^B (t) - \ln \hat{B}_i^C (t) \right) \\
&= \kappa^R \left( \ln \hat{R}_i^B (t) - \ln \hat{B}_i^C (t) \right) \\
(\text{Appendix.J.2})
\end{align*}

Compared with the corresponding equation in the original model, the timeline rather than the structure or the parameter values of this equation has been changed, and it has been pushed back for one period.

Equation (Appendix.J.3) shows the linear deviation of the regulatory bank capital ratio requirement \(\hat{\kappa}_i^R(t)\) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the past bank capital requirement shock \(\xi \kappa_i^R (t - 1)\), while the linear deviation of the desired regulatory bank capital ratio requirement \(\xi \kappa_i^R (t - 1)\), and also returns to the intertemporal change in the logarithmic deviation of the aggregate bank assets (also called the bank credit stock) \(\ln B_i^{CB}(t)\), the intertemporal change in the logarithmic deviation of the price of housing \(\ln V_i^H(t - 1) - \ln V_i^H(t - 2)\), and the intertemporal change in the logarithmic deviation of the price of corporate equity \(\ln V_i^S(t - 1) - \ln V_i^S(t - 2)\), according to countercyclical capital buffer rule:

\begin{align*}
\hat{\kappa}_i^R(t) &= \rho \hat{\kappa}_i^R (t - 1) + (1 - \rho) \left[ \xi \kappa_i^R \left( \ln \hat{B}_i^{CB}(t) - \ln \hat{B}_i^{CB}(t - 1) \right) + \xi \kappa_i^V \left( \ln \hat{V}_i^H(t - 1) - \ln \hat{V}_i^H(t - 2) \right) + \xi \kappa_i^S \left( \ln \hat{V}_i^S(t - 1) - \ln \hat{V}_i^S(t - 2) \right) \right] + \hat{\nu}_i^R (t - 1) \\
&= \rho \hat{\kappa}_i^R (t - 1) + (1 - \rho) \left[ \xi \kappa_i^R \left( \ln \hat{B}_i^{CB}(t) - \ln \hat{B}_i^{CB}(t - 1) \right) + \xi \kappa_i^V \left( \ln \hat{V}_i^H(t - 1) - \ln \hat{V}_i^H(t - 2) \right) + \xi \kappa_i^S \left( \ln \hat{V}_i^S(t - 1) - \ln \hat{V}_i^S(t - 2) \right) \right] + \hat{\nu}_i^R (t - 1) \\
(\text{Appendix.J.3})
\end{align*}
The timeline rather than the structure or the parameter values of this equation has been changed, and it has been pushed back for one period, compared with the corresponding equation in the original model.

Equation (Appendix.J.4) shows the linear deviation of the regulatory mortgage loan to value ratio limit \( \hat{\phi}^D_i(t) \) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporaneous mortgage loan to value limit shock \( \nu^D_i \), while the linear deviation of the desired regulatory mortgage loan to value ratio limit \( \nu^D_i(t) \) responds to the intertemporal change in the logarithmic deviation of the total mortgages issued to domestic real estate developers \( (\ln B^C_i(t) - \ln B^C_i(t - 1)) \) and the intertemporal change in the logarithmic deviation of the price of housing \( (\ln V^H_i(t) - \ln V^H_i(t - 1)) \), according to mortgage loan to value limit rule:

\[
\hat{\phi}^D_i(t) = \rho_{\phi^D} \hat{\phi}^D_i(t - 1) - (1 - \rho_{\phi^D}) \left[ \xi_{\phi^D,B} \left( \ln B^C_i(t) - \ln B^C_i(t - 1) \right) + \xi_{\phi^D,V} \left( \ln V^H_i(t) - \ln V^H_i(t - 1) \right) \right] + \nu^D_i(t)
\]

(Appendix.J.4)

Equation (Appendix.J.5) shows the linear deviation of the regulatory corporate loan to value ratio limit \( \hat{\phi}^F_i(t) \) depends on a weighted average of its past and desired values, and is also influenced by the linear deviation of the contemporaneous corporate loan to value limit shock \( \nu^F_i \), while the linear deviation of the desired regulatory corporate loan to value ratio limit \( \nu^F_i(t) \) responds to the intertemporal change in the logarithmic deviation of the domestic currency denominated total final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( (\ln B^C_i(t) - \ln B^C_i(t - 1)) \), and the intertemporal change in the logarithmic deviation of the price of corporate equity \( (\ln V^S_i(t) - \ln V^S_i(t - 1)) \), according to corporate loan to value limit rule:

\[
\hat{\phi}^F_i(t) = \rho_{\phi^F} \hat{\phi}^F_i(t - 1) - (1 - \rho_{\phi^F}) \left[ \xi_{\phi^F,B} \left( \ln B^C_i(t) - \ln B^C_i(t - 1) \right) + \xi_{\phi^F,V} \left( \ln V^S_i(t) - \ln V^S_i(t - 1) \right) \right] + \nu^F_i(t)
\]

(Appendix.J.5)

Equation (Appendix.J.6) shows the logarithmic deviation of the domestic public demand \( \ln G_i(t) \) depends on a weighted average of the logarithmic deviation of the contemporaneous public consumption \( \ln G_i^C(t) \) and the logarithmic deviation of the contemporaneous public investment \( \ln G_i^I(t) \), according to domestic public demand relationship:

\[
\frac{c_i}{y_i} \ln G_i(t) = \frac{c_i^C}{y_i} \ln G_i^C(t) + \frac{c_i^I}{y_i} \ln G_i^I(t)
\]

(Appendix.J.6)

Equation (Appendix.J.7) shows the logarithmic deviation of the public consumption \( \ln G_i^C(t) \) depends on a weighted average of its past and desired values, and is also influenced by the
linear deviation of the contemporaneous public consumption shock $\nu_i^C(t)$, while the logarithmic deviation of the desired public consumption equals to the logarithmic deviation of the contemporaneous potential output (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) $\ln Y_i(t)$, according to fiscal expenditure rule:

$$\ln G_i^C(t) = \rho_G \ln G_i^C(t-1) + (1-\rho_G) \ln Y_i(t) + \nu_i^C(t)$$
(Appendix.J.7)

Compared with the corresponding equation in the original model, the following innovatively modified equation (Appendix.J.8) defines the domestic total public investment as the aggregated domestic industry-level public investment. Specifically, this correspondingly modified equation shows the logarithmic deviation of the total public investment $\ln G_i^I(t)$ depends on the industry-level output weighted average of the logarithmic deviation of the contemporaneous public investment for the k-th industry of the i-th economy $\sum_{k=1}^{45} \text{Contribution}_{Y_{i,k}} \ln G_{i,k}^I(t)$, according to the adjusted total public investment relationship:

$$\ln G_i^I(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_{i,k}} \ln G_{i,k}^I(t)$$
(Appendix.J.8)

Here $\text{Contribution}_{Y_{i,k}}$ represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Equation (Appendix.J.9) shows the logarithmic deviation of the past public investment $\ln G_i^I(t-1)$ is accumulated to form the logarithmic deviation of the contemporaneous public physical capital stock $\ln K_i^G(t)$, according to the perpetual inventory method:

$$\ln K_i^G(t) = (1-\delta^G) \ln K_i^G(t-1) + \delta^G \ln G_i^I(t-1)$$
(Appendix.J.9)

Compared with the corresponding equation in the original model, the timeline of this equation has been pushed back for one period, while there is no adjustment for the structure of this equation or the values of its parameters.

Equation (Appendix.J.10) shows the linear deviation of the corporate income tax rate $\tau_i^K(t)$ depends on its past value, and is also influenced by the linear deviation of the contemporaneous corporate income tax rate shock $\nu_i^K(t)$, according to fiscal revenue rule:

$$\tau_i^K(t) = \rho_{\tau^K} \tau_i^K(t-1) + \nu_i^K(t)$$
(Appendix.J.10)

Equation (Appendix.J.11) shows the linear deviation of the labor income tax rate $\tau_i^L(t)$ depends on its past value, and is also influenced by the linear deviation of the contemporaneous labor income tax rate shock $\nu_i^L(t)$, according to fiscal revenue rule:

$$\tau_i^L(t) = \rho_{\tau^L} \tau_i^L(t-1) + \nu_i^L(t)$$
(Appendix.J.11)
Since we want to further incorporate the structure of import tariff into this model, the following equation (Appendix.J.12) is innovatively added and the following equation (Appendix.J.13) correspondingly modified.

The innovatively added equation (Appendix.J.12) shows the linear deviation of the import tariff rate \( \left( \tau_i^M(t) - \tau_i^M \right) \) depends on its past value \( \left( \tau_i^M(t - 1) - \tau_i^M \right) \), and is also influenced by the linear deviation of the contemporaneous import tariff rate shock \( \varphi_i^M(t) \), according to fiscal revenue rule:

\[
\left( \tau_i^M(t) - \tau_i^M \right) = \rho_i^T \left( \tau_i^M(t - 1) - \tau_i^M \right) + \varphi_i^M(t)
\]  
(Appendix.J.12)

Here \( \rho_i^T \) represents the country differentiated coefficient for the intertemporal change of country differentiated import tariff rate, which is estimated based on the specific tariff reduction commitments of each country in the tariff terms of for example RCEP.

The correspondingly modified equation (Appendix.J.13) has added import tariff revenue to the total tax revenue, it shows the logarithmic deviation of the ratio of the tax revenue of the fiscal authority to nominal output

\[
\ln \frac{T_i(t)}{P_i(t)Y_i(t)} = \frac{1}{\tau_i} \left[ \frac{Y_i}{P_i Y_i} \tau^K(t) + \frac{W_i L_i}{P_i Y_i} \tau^L(t) + \frac{M_i}{P_i Y_i} \left( \tau_i^M(t) - \tau_i^M \right) \right]
\]

(Appendix.J.13)

Equation (Appendix.J.14) defines the linear deviation of the ratio of both nondiscretionary and discretionary lump sum transfer payments by the fiscal authority to nominal output \( \frac{\theta_i(t)}{P_i(t)Y_i(t)} \) as the sum of the linear deviation of the ratio of contemporaneous nondiscretionary lump sum transfer payments by the fiscal authority to contemporaneous nominal output \( \frac{\theta_i^{CN}(t)}{P_i(t)Y_i(t)} \) and the linear deviation of the ratio of contemporaneous discretionary lump sum transfer payments by the fiscal authority to contemporaneous nominal output \( \frac{\theta_i^{CD}(t)}{P_i(t)Y_i(t)} \):

\[
\frac{\theta_i(t)}{P_i(t)Y_i(t)} = \frac{\theta_i^{CN}(t)}{P_i(t)Y_i(t)} + \frac{\theta_i^{CD}(t)}{P_i(t)Y_i(t)}
\]

(Appendix.J.14)

Equation (Appendix.J.15) shows the linear deviation of the ratio of nondiscretionary lump sum transfer payments by the fiscal authority to nominal output (a budget neutral nondiscretionary lump sum transfer program redistributes national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households) \( \frac{\theta_i^{CN}(t)}{P_i(t)Y_i(t)} \) responds to the linear deviation of the ratio of the contemporaneous net foreign asset position to past nominal output \( \frac{A_i(t)}{P_i(t-1)Y_i(t-1)} \), according to nondiscretionary transfer payment rule:
\frac{\nu^{C,N}_i(t)}{P^i_Y(t)Y_i(t)} = \zeta^N \frac{\Delta_i(t)}{P^i_Y(t-1)Y_i(t-1)}
\tag{Appendix.J.15}

Equation (Appendix.J.16) shows the linear deviation of the ratio of discretionary lump sum transfer payments by the fiscal authority to nominal output (a discretionary lump sum transfer program only provides income support to credit constrained households) \( \frac{\nu^{C,D}_i(t)}{P^i_Y(t)Y_i(t)} \) responds to the linear deviation of the ratio of the contemporaneous net government asset of the fiscal authority (also called accumulated public financial wealth) to past nominal output \( \frac{\Delta_i(t)}{P^i_Y(t-1)Y_i(t-1)} \), and is also influenced by the linear deviation of the contemporaneous transfer payment shock \( \hat{\nu}^T_i(t) \), according to discretionary transfer payment rule:
\frac{\nu^{C,D}_i(t)}{P^i_Y(t)Y_i(t)} = \zeta^D \frac{\Delta_i(t)}{P^i_Y(t-1)Y_i(t-1)} + \hat{\nu}^T_i(t)
\tag{Appendix.J.16}

Equation (Appendix.J.17) shows the linear deviation of the ratio of the fiscal balance of the fiscal authority to nominal output \( \frac{\nu^{B}_i(t)}{P^i_Y(t)Y_i(t)} \) not only depends on a weighted average of the linear deviation of the past nominal short term bond yield \( i^S_i(t-1) \) and the linear deviation of the past nominal effective long term market interest rate \( i^{LE}_i(t-1) \), but also depends on the linear deviation of the ratio of the contemporaneous net government asset of the fiscal authority (also called accumulated public financial wealth) to past nominal output \( \frac{\Delta_i(t)}{P^i_Y(t-1)Y_i(t-1)} \), adjusted by the intertemporal change in the logarithmic deviation of the nominal output \( \left( \ln P^Y_i(t) + \ln Y_i(t) - \ln P^Y_i(t-1) - \ln Y_i(t-1) \right) \), and also depends on the linear deviation of the ratio of the contemporaneous primary fiscal balance of the fiscal authority to contemporaneous nominal output \( \frac{\nu^{B}_i(t)}{P^i_Y(t)Y_i(t)} \), according to government dynamic budget constraint:
\frac{\nu^{B}_i(t)}{P^i_Y(t)Y_i(t)} = \frac{1}{\beta(1+g)} \left( A^G_i \left[ \frac{B^S_i G^i S_i}{A^P_i} i^S_i(t-1) + \frac{B^L_i G^i L_i}{A^P_i} i^{LE}_i(t-1) \right] + (1 - \beta) \left[ \frac{\Delta_i(t)}{P^i_Y(t-1)Y_i(t-1)} - \frac{A^G_i}{P^i_Y(t-1)} \left( \ln P^Y_i(t) + \ln Y_i(t) - \ln P^Y_i(t-1) - \ln Y_i(t-1) \right) \right] + \frac{\nu^{B}_i(t)}{P^i_Y(t)Y_i(t)} \right)
\tag{Appendix.J.17}

Compared with the corresponding equation in the original model, the parameters before the logarithmic deviation of the terms of trade (which measures the relative price of exports to imports) \( \ln P^Y_i(t) \) in the innovatively adjusted equation (Appendix.J.18) characterizing the ratio of the primary fiscal balance of the fiscal authority to nominal output have been changed, in order to make the primary fiscal balance characterization more accurate for those small and highly open economies whose total export value or total import value can be as large as or even larger than their GDP, which lead to the impact of their export or import price level on the primary fiscal balance much higher than that of other economies. Specifically, the innovatively adjusted equation (Appendix.J.18) shows the linear deviation of the ratio of the primary fiscal balance of the fiscal authority to nominal output \( \frac{\nu^{B}_i(t)}{P^i_Y(t)Y_i(t)} \) not only depends on the logarithmic deviation of the contemporaneous real tax revenue of the fiscal authority...
\( \ln T_i(t) - \ln P_i^{Y}(t) \), but also depends on the logarithmic deviation of the contemporaneous domestic public demand \( \ln G_i(t) \), as well as the total exports and total imports weighted average of the logarithmic deviation of the contemporaneous global terms of trade shifter (which is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows) \( \ln T_i(t) \), the logarithmic deviation of the contemporaneous price of exports of the i-th economy \( \ln P_i^{X}(t) \), and the logarithmic deviation of the contemporaneous post-tariff price of imports of the i-th economy \( \ln P_i^{M}(t) \), while for small and highly open economies, this part is replaced by the logarithmic deviation of the contemporaneous terms of trade of the i-th economy \( \ln \eta_i(t) \) multiplied by a threshold value (to reflect such small and highly open economies are usually good at balancing their imports and exports and maintaining a stable structure between imports, exports, domestic demand and domestic production), and also depends on the linear deviation of the ratio of contemporaneous discretionary lump sum transfer payments by the fiscal authority to contemporaneous nominal output \( \frac{\Phi^{C,D}(t)}{P_i^{Y}(t)Y_i(t)} \): 

\[
\frac{\Phi^{B}(t)}{P_i^{Y}(t)Y_i(t)} = \frac{G_i}{Y_i} \left( \ln T_i(t) - \ln P_i^{Y}(t) \right) - \left( \ln G_i(t) - \left( \frac{0.5\times X_i + 0.5 \times M_i}{Y_i} \ln \eta_i(t) + \frac{X_i}{Y_i} \ln P_i^{X}(t) - \frac{M_i}{Y_i} \ln P_i^{M}(t) \right) \right) \]  

(Appendix J.18) 

or

\[
\frac{\Phi^{B}(t)}{P_i^{Y}(t)Y_i(t)} = \frac{G_i}{Y_i} \left( \ln T_i(t) - \ln P_i^{Y}(t) \right) - \left( \ln G_i(t) - \text{(threshold value)} \ln T_i(t) \right) \]  

(Appendix J.18) 

Here, 

the adjusted parameter \( \frac{0.5\times X_i + 0.5 \times M_i}{Y_i} \) represents the proportion of the domestic currency denominated average value of the i-th economy's total exports and imports to the domestic currency denominated nominal GDP of the i-th economy;  

the adjusted parameter \( \frac{X_i}{Y_i} \) represents the proportion of the domestic currency denominated value of the i-th economy's total exports to the domestic currency denominated nominal GDP of the i-th economy;  

the adjusted parameter \( \frac{M_i}{Y_i} \) represents the proportion of the domestic currency denominated value of the i-th economy's total imports to the domestic currency denominated nominal GDP of the i-th economy. 

Equation (Appendix J.19) shows the linear deviation of the ratio of the net government asset of the fiscal authority (also called accumulated public financial wealth) to past nominal output \( \frac{\Delta A_i^{F}(t)}{P_i^{Y}(t-1)Y_i(t-1)} \) not only depends on its past value, adjusted by the intertemporal change in the logarithmic deviation of the past nominal output \( \left( \ln P_i^{Y}(t-1) + \ln Y_i(t-1) - \ln P_i^{Y}(t-2) - \ln Y_i(t-2) \right) \), but also depends on the linear deviation of the ratio of the past fiscal balance of the fiscal authority to past nominal output \( \frac{\Phi^{B}(t-1)}{P_i^{Y}(t-1)Y_i(t-1)} \):
\[
\frac{\dddot{A}^G_i(t)}{P^i(t-1)Y^i(t-1)} = \frac{1}{1+g} \left[ \frac{\dddot{A}^G_i(t-1)}{P^i(t-2)Y^i(t-2)} - \frac{\dddot{A}^G_i(t)}{P^i(t-1)Y^i(t-1)} \left( \ln P^Y_i(t-1) + \ln Y^i(t-1) - \ln P^i(t-2) - \ln Y^i(t-2) \right) \right] + \frac{\bar{e}^B_i(t-1)}{P^i(t-1)Y^i(t-1)}
\]
(Appendix.J.19)

Compared with the corresponding equation in the original model, the timeline of this equation has been pushed back for one period (such as \(\frac{\dddot{A}^G_i(t+1)}{P^i(t)Y^i(t)}\) is adjusted to \(\frac{\dddot{A}^G_i(t)}{P^i(t-1)Y^i(t-1)}\), etc), while the structure of this equation and the values of its parameters have not been changed.

**K. Combination of Domestic Interbank, Money, Bond, and Equity Markets and International Financial Markets**

Equation (Appendix.K.1) shows the linear deviation of the nominal interbank loans rate of the \(i\)-th economy \(\dddot{i}^B_i(t)\) depends on the linear deviation of the contemporaneous nominal short term bond yield of the \(i\)-th economy \(\dddot{i}^S_i(t)\), and is also influenced by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign liquidity risk premium for the \(i\)-th economy \(\ln \nu^B_i(t)\), according to interbank market relationship:

\[
\dddot{i}^B_i(t) = \dddot{i}^S_i(t) + \ln \nu^B_i(t)
\]
(Appendix.K.1)

Equation (Appendix.K.2) defines the logarithmic deviation of the weighted average of domestic and foreign liquidity risk premium for the \(i\)-th economy (the above nominal interbank loans rate is adjusted by this liquidity risk premium) \(\ln \nu^B_i(t)\) as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous liquidity risk premium shock of any specific economy except the \(i\)-th economy \(\sum_{j=1}^{N} \omega_j \nu^B_j(t) - \omega_i \nu^B_i(t)\), adds the logarithmic deviation of the contemporaneous liquidity risk premium shock of the \(i\)-th economy \(\ln \nu^B_i(t)\):

\[
\ln \nu^B_i(t) = \lambda_i^M \sum_{j=1}^{N} \omega_j^A \nu^B_j(t) + (1 - \lambda_i^M \omega_i^A) \ln \nu^B_i(t)
\]
(Appendix.K.2)

Here \(\lambda_i^M\) represents the \(i\)-th economy's interbank market contagion level.

Equation (Appendix.K.3) shows the linear deviation of the nominal short term bond yield of the \(i\)-th economy \(\dddot{i}^S_i(t)\) depends on the linear deviation of the contemporaneous nominal policy interest rate of the \(i\)-th economy \(\dddot{i}^P_i(t)\), and is also influenced by the logarithmic deviation of the contemporaneous weighted average of the domestic and foreign credit risk premium for the \(i\)-th economy \(\ln \nu^S_i(t)\), according to money market relationship:

\[
\dddot{i}^S_i(t) = \dddot{i}^P_i(t) + \ln \nu^S_i(t)
\]
(Appendix.K.3)
Equation (Appendix.K.4) defines the linear deviation of the real short term bond yield of the i-th economy $r^i_S(t)$ as the linear deviation of the contemporaneous nominal short term bond yield of the i-th economy $\hat{i}^i_S(t)$, minus the linear deviation of the expected future consumption price inflation of the i-th economy $E_t \hat{n}^i_C(t + 1)$:

$$r^i_S(t) = \hat{i}^i_S(t) - E_t \hat{n}^i_C(t + 1)$$

(Appendix.K.4)

Equation (Appendix.K.5) defines the logarithmic deviation of the weighted average of domestic and foreign credit risk premium for the i-th economy (the above nominal short term bond yield is adjusted by this credit risk premium) $\ln \bar{\nu}^i_B(t)$ as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous credit risk premium shock of any specific economy except the i-th economy $\sum_{j=1}^{N} \omega_j \ln \bar{\nu}^j_B(t) - \omega_i \ln \bar{\nu}^i_B(t)$, adds the logarithmic deviation of the contemporaneous credit risk premium shock of the i-th economy $\ln \bar{\nu}^i_B(t)$:

$$\ln \bar{\nu}^i_B(t) = \lambda_i^B \sum_{j=1}^{N} \omega_j \ln \bar{\nu}^j_B(t) + (1 - \lambda_i^B \omega_i^B) \ln \bar{\nu}^i_B(t)$$

(Appendix.K.5)

Here $\lambda_i^B$ represents the i-th economy's capital market contagion level.

Equation (Appendix.K.6) shows the linear deviation of the nominal long term bond yield of the i-th economy $\hat{i}^i_L(t)$ not only depends on its expected future value, but also depends on the linear deviation of the contemporaneous nominal short term bond yield of the i-th economy $\hat{i}^i_S(t)$, adjusted by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign duration risk premium for the i-th economy $\ln \bar{\nu}^i_B(t)$, according to bond market relationship:

$$\hat{i}^i_L(t) = \omega_B^r E_t \hat{i}^i_L(t + 1) + \left[\frac{1 - \omega_B^r}{\omega_B^r}\right] \left(\hat{i}^i_S(t) + \ln \bar{\nu}^i_B(t)\right)$$

(Appendix.K.6)

Equation (Appendix.K.7) shows the linear deviation of the real long term bond yield of the i-th economy $\hat{r}^i_L(t)$ not only depends on its expected future value, but also depends on the linear deviation of the contemporaneous real short term bond yield of the i-th economy $\hat{r}^i_S(t)$, adjusted by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign duration risk premium for the i-th economy $\ln \bar{\nu}^i_B(t)$, according to bond market relationship:

$$\hat{r}^i_L(t) = \omega_B^r E_t \hat{r}^i_L(t + 1) + \left[\frac{1 - \omega_B^r}{\omega_B^r}\right] \left(\hat{r}^i_S(t) + \ln \bar{\nu}^i_B(t)\right)$$

(Appendix.K.7)

Equation (Appendix.K.8) shows the logarithmic deviation of the weighted average of domestic and foreign term premium for the i-th economy $\ln \bar{\mu}^i_B(t)$ depends on its expected future value, driven by the logarithmic deviation of the contemporaneous weighted average of domestic and foreign duration risk premium for the i-th economy $\ln \bar{\nu}^i_B(t)$:
\[ \ln \mu_i^B(t) = \omega_B \beta E_t \ln \mu_i^B(t + 1) + \left[ \left( \frac{1 - \omega_B}{\omega_B} \right) \left( \omega_B + \frac{1 - \omega_B}{\omega_B} \right) \right] \ln \nu_i^B(t) \]  
(Appendix.K.8)

Equation (Appendix.K.9) defines the logarithmic deviation of the weighted average of domestic and foreign duration risk premium for the i-th economy (the above long term bond yield and term premium are adjusted by this duration risk premium) \( \ln \nu_i^B(t) \) as the capital market capitalization weighted average of the logarithmic deviation of the contemporaneous duration risk premium shock of any specific economy except the i-th economy \( \sum_{j=1}^{N} \omega_j^A \ln \nu_j^B(t) - \omega_i^A \ln \nu_i^B(t) \), adds the logarithmic deviation of the contemporaneous duration risk premium shock of the i-th economy \( \ln \nu_i^B(t) \):

\[ \ln \nu_i^B(t) = \lambda_i^B \sum_{j=1}^{N} \omega_j^A \ln \nu_j^B(t) + (1 - \lambda_i^B \omega_i^A) \ln \nu_i^B(t) \]  
(Appendix.K.9)

Here \( \lambda_i^B \) represents the i-th economy's capital market contagion level.

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.K.10) defines the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the i-th economy (the price of corporate equity \( V_i^S(t) \) is adjusted by this equity risk premium) \( \ln \nu_i^S(t) \) as the domestic industrial output weighted average of the logarithmic deviation of the weighted average of contemporaneous domestic and foreign equity risk premium for the k-th industry of the i-th economy \( \sum_{k=1}^{45} \sum_{k=1}^{N} \text{Contribution}_{Y_i,k} \ln \nu_k^S(t) \), in order to be consistent with the innovatively added equations characterizing industries under the whole economy and their inter-country cross-industry input-output relations in the model:

\[ \ln \nu_i^S(t) = \sum_{k=1}^{45} \text{Contribution}_{Y_i,k} \ln \nu_k^S(t) \]  
(Appendix.K.10)

Here \( \text{Contribution}_{Y_i,k} \) represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Equation (Appendix.K.11) shows the linear deviation of the nominal effective long term market interest rate of the i-th economy (the ratio of the fiscal balance of the fiscal authority to nominal output \( \frac{FB_i(t)}{\rho_i^L(t+1)} \)) is adjusted by this domestic nominal effective long term market interest rate \( \frac{\hat{i}_{LE}^L(t)}{\rho_i^L(t+1)} \) depending on its past value, driven by the linear deviation of the contemporaneous nominal long term bond yield of the i-th economy \( \hat{i}_{i}^L(t) \):

\[ \frac{\hat{i}_{LE}^L(t)}{\rho_i^L(t+1)} = \omega_B \hat{i}_{LE}^L(t - 1) + \omega_B (1 - \omega_B) \left( \omega_B + \frac{1 - \omega_B}{\omega_B} \right) \hat{i}_{i}^L(t) \]  
(Appendix.K.11)

L. All Whole-Economy-Level Exogenous Shocks

Since some of the whole-economy-level exogenous shocks defined by equation (Appendix.L.1), (Appendix.L.6), (Appendix.L.7), (Appendix.L.10), (Appendix.L.11) and (Appendix.L.24) in the original model can be completely replaced by the weighted sum of the corresponding industry-level exogenous shocks defined in the previous Part B of Section IV,
they no longer appear in this AGMFM. However, if all industry-level equations in the AGMFM are removed due to some specific model applications do not care about the details of industries under the whole economy and want to reduce the scale of the model as much as possible, all those whole-economy-level exogenous shocks defined by these equations from the original model should be re-added to the model. This is also the reason why there are some corresponding vacancies between the numbers of the following equations defining whole-economy-level exogenous shocks, please refer to the previous Part C of Sector IV for details of those removed equations.

Equation (Appendix.L.2) shows the logarithmic deviation of the labor supply shock $\ln \nu^N_i(t)$ (which directly influences the unemployment rate) follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon^N_i(t) \sim \text{iid } \mathcal{N}(0, \sigma^2_{\nu^N})$:

$$\ln \nu^N_i(t) = \rho^N \ln \nu^N_i(t - 1) + \varepsilon^N_i(t)$$

(Appendix.L.2)

Equation (Appendix.L.3) shows the logarithmic deviation of the consumption demand shock $\ln \nu^C_i(t)$ (which directly influences the total consumption by all three kinds of households) follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon^C_i(t) \sim \text{iid } \mathcal{N}(0, \sigma^2_{\nu^C})$:

$$\ln \nu^C_i(t) = \rho^C \ln \nu^C_i(t - 1) + \varepsilon^C_i(t)$$

(Appendix.L.3)

Equation (Appendix.L.4) shows the logarithmic deviation of the residential investment demand shock $\ln \nu^{IH}_i(t)$ (which directly influences the residential investment) follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon^{IH}_i(t) \sim \text{iid } \mathcal{N}(0, \sigma^2_{\nu^{IH}})$:

$$\ln \nu^{IH}_i(t) = \rho^{IH} \ln \nu^{IH}_i(t - 1) + \varepsilon^{IH}_i(t)$$

(Appendix.L.4)

Equation (Appendix.L.5) shows the logarithmic deviation of the business investment demand shock $\ln \nu^{IK}_i(t)$ (which directly influences the business investment) follows stationary first order autoregressive process, with normally distributed innovations $\varepsilon^{IK}_i(t) \sim \text{iid } \mathcal{N}(0, \sigma^2_{\nu^{IK}})$:

$$\ln \nu^{IK}_i(t) = \rho^{IK} \ln \nu^{IK}_i(t - 1) + \varepsilon^{IK}_i(t)$$

(Appendix.L.5)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.L.8) defines the logarithmic deviation of the output price markup shock $\ln \vartheta^Y_i(t)$ (which directly influences the core price level), as the domestic industrial output weighted average of the logarithmic deviation of the contemporaneous output price markup shock for the output of the k-th industry of the i-th economy (which directly influences the output price of the k-th industry of the i-th economy) $\sum_{k=1}^{45} \text{Contribution}_{Y,ik} \ln \vartheta^Y_k(t)$, in order to be consistent with the innovatively added equations characterizing industries under the whole economy and their inter-country cross-industry input-output relations in the model:
Here, Contribution_{Y_{lk}} represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Equation (Appendix.L.9) shows the logarithmic deviation of the wage markup shock ln\hat{\nu}_i^L(t) (which directly influences the nominal wage) follows normally distributed white noise process, and \epsilon_i^{\hat{\nu}_i^L}(t)~iid \mathcal{N}(0, \sigma_i^{\hat{\nu}_i^L}):

\ln\hat{\nu}_i^L(t) = \epsilon_i^{\hat{\nu}_i^L}(t)

(Appendix.L.9)

Equation (Appendix.L.12) shows the logarithmic deviation of the liquidity risk premium shock ln\hat{\nu}_i^{\lambda_B}(t) (which directly influences the nominal interbank loans rate) follows stationary first order autoregressive process, with normally distributed innovations \epsilon_i^{\nu_i^{\lambda_B}}(t)~iid \mathcal{N}(0, \sigma_i^{\nu_i^{\lambda_B}}):

\ln\hat{\nu}_i^{\lambda_B}(t) = \rho_{\nu_i^{\lambda_B}}\ln\hat{\nu}_i^{\lambda_B}(t - 1) + \epsilon_i^{\nu_i^{\lambda_B}}(t)

(Appendix.L.12)

Equation (Appendix.L.13) shows the logarithmic deviation of the housing risk premium shock ln\hat{\nu}_i^{H}(t) (which directly influences both the nominal property return and the price of housing) follows stationary first order autoregressive process, with normally distributed innovations \epsilon_i^{\nu_i^{H}}(t)~iid \mathcal{N}(0, \sigma_i^{\nu_i^{H}}):

\ln\hat{\nu}_i^{H}(t) = \rho_{\nu_i^{H}}\ln\hat{\nu}_i^{H}(t - 1) + \epsilon_i^{\nu_i^{H}}(t)

(Appendix.L.13)

Equation (Appendix.L.14) shows the logarithmic deviation of the credit risk premium shock ln\hat{\nu}_i^{S}(t) (which directly influences the nominal short term bond yield) follows stationary first order autoregressive process, with normally distributed innovations \epsilon_i^{\nu_i^{S}}(t)~iid \mathcal{N}(0, \sigma_i^{\nu_i^{S}}):

\ln\hat{\nu}_i^{S}(t) = \rho_{\nu_i^{S}}\ln\hat{\nu}_i^{S}(t - 1) + \epsilon_i^{\nu_i^{S}}(t)

(Appendix.L.14)

Equation (Appendix.L.15) shows the logarithmic deviation of the duration risk premium shock ln\hat{\nu}_i^{B}(t) (which directly influences the nominal long term bond yield) follows stationary first order autoregressive process, with normally distributed innovations \epsilon_i^{\nu_i^{B}}(t)~iid \mathcal{N}(0, \sigma_i^{\nu_i^{B}}):

\ln\hat{\nu}_i^{B}(t) = \rho_{\nu_i^{B}}\ln\hat{\nu}_i^{B}(t - 1) + \epsilon_i^{\nu_i^{B}}(t)

(Appendix.L.15)

Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.L.16) defines the logarithmic deviation of the equity risk premium shock (which directly influences the price of corporate equity) ln\hat{\nu}_i^{S}(t) as the domestic industrial output weighted average of the logarithmic deviation of the contemporaneous equity risk premium shock for the k-th industry of the i-th economy (which directly influences the equity
price of listed companies in the k-th industry of the i-th economy
\[ \sum_{k=1}^{45} \text{Contribution}_Y_{i,k} \ln \nu_{i,k}^S(t) \]

in order to be consistent with the innovatively added equations characterizing industries under the whole economy and their inter-country cross-industry input-output relations in the model:

\[ \ln \nu_{i,k}^S(t) = \sum_{k=1}^{45} \text{Contribution}_Y_{i,k} \ln \nu_{i,k}^S(t) \]  
(Appendix.L.16)

Here Contribution$_Y_{i,k}$ represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Equation (Appendix.L.17) shows the logarithmic deviation of the currency risk premium shock \( \ln \nu_{i}^E(t) \) (which directly influences the nominal bilateral exchange rate, the nominal effective exchange rate, the real bilateral exchange rate, and the real effective exchange rate) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_{i}^E(t) \sim \text{iid } N(0, \sigma_{\nu E}^2) \):

\[ \ln \nu_{i}^E(t) = \rho_{\nu E} \ln \nu_{i}^E(t - 1) + \epsilon_{i}^E(t) \]  
(Appendix.L.17)

Equation (Appendix.L.18) shows the logarithmic deviation of the mortgage loan rate markup shock \( \ln \nu_{i}^{CD}(t) \) (which directly influences the domestic nominal mortgage loan rate) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_{i}^{\theta,CD}(t) \sim \text{iid } N(0, \sigma_{\theta,CD}^2) \):

\[ \ln \nu_{i}^{CD}(t) = \rho_{\nu E} \ln \nu_{i}^{CD}(t - 1) + \epsilon_{i}^{\theta,CD}(t) \]  
(Appendix.L.18)

Equation (Appendix.L.19) shows the logarithmic deviation of the corporate loan rate markup shock \( \ln \nu_{i}^{CF}(t) \) (which directly influences both the domestic nominal corporate loan rate for economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks, and the weighted average of domestic and foreign nominal corporate loan rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy) follows stationary first order autoregressive process, with normally distributed innovations \( \epsilon_{i}^{\theta,CF}(t) \sim \text{iid } N(0, \sigma_{\theta,CF}^2) \):

\[ \ln \nu_{i}^{CF}(t) = \rho_{\nu E} \ln \nu_{i}^{CF}(t - 1) + \epsilon_{i}^{\theta,CF}(t) \]  
(Appendix.L.19)

Equation (Appendix.L.20) shows the linear deviation of the mortgage loan default shock \( \nu_{i}^{D,M}(t) \) (which directly influences the domestic mortgage loan default rate) follows normally distributed white noise process, and \( \epsilon_{i}^{\nu D,M}(t) \sim \text{iid } N(0, \sigma_{\nu D}^2) \):

\[ \nu_{i}^{D,M}(t) = \epsilon_{i}^{\nu D,M}(t) \]  
(Appendix.L.20)
Compared with the corresponding equation in the original model, the innovatively modified equation (Appendix.L.21) defines the linear deviation of the corporate loan default shock (which directly influences both the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy, and the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks) $\nu^C_{i}(t)$, as the domestic industrial output weighted average of the linear deviation of the contemporaneous corporate loan default shock for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy (which directly influences the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the k-th industry of the i-th economy)

$$\sum_{k=1}^{\text{Contribution}_Y} \cdot \nu^C_{i}(t) \cdot \nu^C_{i}(t)$$

(Appendix.L.21)

Here $\text{Contribution}_Y$ represents the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

Equation (Appendix.L.22) shows the linear deviation of the monetary policy shock $\nu^{iP}_{i}(t)$ (which directly influences the nominal policy interest rate) follows normally distributed white noise process, and $\epsilon_{\nu^{iP}_{i}}(t) \sim \text{iid } \mathcal{N}(0, \sigma^{\nu^{iP}_{i}}_{i})$:

$$\nu^{iP}_{i}(t) = \epsilon_{\nu^{iP}_{i}}(t)$$

(Appendix.L.22)

Equation (Appendix.L.23) shows the linear deviation of the public consumption shock $\nu^{iG}_{i}(t)$ (which directly influences the public consumption) follows stationary first order autoregressive process, with normally distributed innovations $\epsilon_{\nu^{iG}_{i}}(t) \sim \text{iid } \mathcal{N}(0, \sigma^{\nu^{iG}_{i}}_{i})$:

$$\nu^{iG}_{i}(t) = \rho \nu^{iG}_{i}(t-1) + \epsilon_{\nu^{iG}_{i}}(t)$$

(Appendix.L.23)

Equation (Appendix.L.25) shows the linear deviation of the corporate income tax rate shock $\nu^{iT}_{i}(t)$ (which directly influences the corporate income tax rate) follows stationary first order autoregressive process, with normally distributed innovations $\epsilon_{\nu^{iT}_{i}}(t) \sim \text{iid } \mathcal{N}(0, \sigma^{\nu^{iT}_{i}}_{i})$:

$$\nu^{iT}_{i}(t) = \rho \nu^{iT}_{i}(t-1) + \epsilon_{\nu^{iT}_{i}}(t)$$

(Appendix.L.25)

Equation (Appendix.L.26) shows the linear deviation of the labor income tax rate shock $\nu^{iL}_{i}(t)$ (which directly influences the labor income tax rate) follows stationary first order autoregressive process, with normally distributed innovations $\epsilon_{\nu^{iL}_{i}}(t) \sim \text{iid } \mathcal{N}(0, \sigma^{\nu^{iL}_{i}}_{i})$:
\[ \nu_i^{\tau L}(t) = \rho_{\nu \tau} \nu_i^{\tau L}(t-1) + \varepsilon_i^{\nu \tau L}(t) \]  
(Appendix.L.26)

Since we want to further incorporate the structure of import tariff into this model, we also add the import tariff rate shock to this model. The following innovatively added equation (Appendix.L.27) shows the linear deviation of the import tariff rate shock \( \nu_i^{\tau M}(t) \) (which directly influences the import tariff rate) follows stationary first order autoregressive process, with normally distributed innovations \( \varepsilon_i^{\nu \tau M}(t) \sim \text{iid } \mathcal{N}(0, \sigma_i^{\nu \tau M}) \):

\[ \nu_i^{\tau M}(t) = \rho_i^{\nu \tau} \nu_i^{\tau M}(t-1) + \varepsilon_i^{\nu \tau M}(t) \]  
(Appendix.L.27)

Here \( \rho_i^{\nu \tau} \) represents the country differentiated coefficient for the intertemporal change of country differentiated import tariff rate shock, \( \sigma_i^{\nu \tau M} \) represents the standard deviation of the normally distributed innovations \( \varepsilon_i^{\nu \tau M}(t) \) to reflect the country differentiated tax reduction intensity and volatility in the tariff terms of for example RCEP, which are all estimated based on the specific tariff reduction commitments of each country.

Equation (Appendix.L.28) shows the linear deviation of the transfer payment shock \( \nu_i^{T}(t) \) (which directly influences the ratio of discretionary lump sum transfer payments by the fiscal authority to nominal output) follows stationary first order autoregressive process, with normally distributed innovations \( \varepsilon_i^{\nu T}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\nu T}) \):

\[ \nu_i^{T}(t) = \rho_i^{\nu T} \nu_i^{T}(t-1) + \varepsilon_i^{\nu T}(t) \]  
(Appendix.L.28)

Equation (Appendix.L.29) shows the linear deviation of the bank capital requirement shock \( \nu_i^{\kappa}(t) \) (which directly influences the regulatory bank capital ratio requirement) follows stationary first order autoregressive process, with normally distributed innovations \( \varepsilon_i^{\nu \kappa}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\nu \kappa}) \):

\[ \nu_i^{\kappa}(t) = \rho_i^{\nu \kappa} \nu_i^{\kappa}(t-1) + \varepsilon_i^{\nu \kappa}(t) \]  
(Appendix.L.29)

Equation (Appendix.L.30) shows the linear deviation of the mortgage loan to value limit shock \( \nu_i^{\phi D}(t) \) (which directly influences the regulatory mortgage loan to value ratio limit) follows normally distributed white noise process, and \( \varepsilon_i^{\nu \phi D}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\nu \phi D}) \):

\[ \nu_i^{\phi D}(t) = \varepsilon_i^{\nu \phi D}(t) \]  
(Appendix.L.30)

Equation (Appendix.L.31) shows the linear deviation of the corporate loan to value limit shock \( \nu_i^{\phi F}(t) \) (which directly influences the regulatory corporate loan to value ratio limit) follows normally distributed white noise process, and \( \varepsilon_i^{\nu \phi F}(t) \sim \text{iid } \mathcal{N}(0, \sigma_{\nu \phi F}) \):

\[ \nu_i^{\phi F}(t) = \varepsilon_i^{\nu \phi F}(t) \]  
(Appendix.L.31)
M. International Commodity Markets

Equation (Appendix.M.1) shows the logarithmic deviation of the price of internationally homogeneous energy commodities denominated in US dollars in the international commodity market $\ln P^e_{Y}(t)$ not only depends on a weighted average of its past and expected future values, driven by the world output weighted average of the logarithmic deviation of the contemporaneous real labor cost per unit of output of any specific economy $\sum_{j=1}^{N} \omega_{i}^Y \left( \ln W_{i}^j(t) + \ln L_{i}^j(t) - \ln P_{i}^j(t) - \ln Y_{i}^j(t) \right)$, but also depends on the world output weighted average of the difference between the logarithmic deviation of the contemporaneous US dollar denominated core price level of any specific economy and the logarithmic deviation of the contemporaneous price of internationally homogeneous energy commodities denominated in US dollars in the international commodity market $\sum_{j=1}^{N} \omega_{i}^Y \left( \ln R_{i}^j(t) - \ln R_{eY}^j(t) \right)$, and the world output weighted average of the contemporaneous, past and expected future values of the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by any specific economy to US dollar $\sum_{j=1}^{N} \omega_{i}^Y \left( \frac{1}{1+\beta} \ln E_{i,1}^j(t) - \frac{\beta}{1+\beta} E_t \ln E_{i,1}^j(t+1) \right)$, and is also influenced by the logarithmic deviation of the contemporaneous energy commodity price markup $\ln \hat{Y}_{e}^Y(t)$, according to internationally homogeneous energy commodity price Phillips curve:

$$
\ln P^e_{Y}(t) = \frac{1}{1+\beta} \ln P^e_{Y}(t-1) + \frac{\beta}{1+\beta} E_t \ln P^e_{Y}(t+1) + \frac{(1-\omega_{i}^Y)(1-\omega_{i}^Y)}{\omega_{i}^Y(1+\beta)} \sum_{j=1}^{N} \omega_{i}^Y \left( \ln W_{i}^j(t) + \ln L_{i}^j(t) - \ln P_{i}^j(t) - \ln Y_{i}^j(t) \right) + \ln \hat{Y}_{e}^Y(t) + \lambda^Y \left( \ln P_{i}^j(t) - \ln E_{i,1}^j(t) - \ln P^e_{Y}(t) \right) - \sum_{j=1}^{N} \omega_{i}^Y \left( \ln E_{i,1}^j(t) - \ln \hat{E}_{i,1}^j(t-1) - \frac{\beta}{1+\beta} E_t \ln E_{i,1}^j(t+1) \right)
$$

(Appendix.M.1)

Equation (Appendix.M.2) shows the logarithmic deviation of the price of internationally homogeneous nonenergy commodities denominated in US dollars in the international commodity market $\ln P^Y_{ne}(t)$ not only depends on a weighted average of its past and expected future values, driven by the world output weighted average of the logarithmic deviation of the contemporaneous real labor cost per unit of output of any specific economy $\sum_{j=1}^{N} \omega_{i}^Y \left( \ln W_{i}^j(t) + \ln L_{i}^j(t) - \ln P_{i}^j(t) - \ln Y_{i}^j(t) \right)$, but also depends on the world output weighted average of the difference between the logarithmic deviation of the contemporaneous US dollar denominated core price level of any specific economy and the logarithmic deviation of the contemporaneous price of internationally homogeneous nonenergy commodities denominated in US dollars in the international commodity market $\sum_{j=1}^{N} \omega_{i}^Y \left( \ln R_{i}^j(t) - \ln R^Y_{ne}(t) \right)$, and the world output weighted average of the contemporaneous, past and expected future values of the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by any specific economy to US dollar $\sum_{j=1}^{N} \omega_{i}^Y \left( \frac{1}{1+\beta} \ln E_{i,1}^j(t) - \frac{\beta}{1+\beta} E_t \ln E_{i,1}^j(t+1) \right)$, and is also influenced by the logarithmic deviation of
the contemporaneous nonenergy commodity price markup $\ln \delta_{\text{ne}}^Y(t)$, according to internationally homogeneous nonenergy commodity price Phillips curve:

$$
\ln P_{\text{ne}}^Y(t) = \frac{1}{1+\beta} \ln P_{\text{ne}}^Y(t-1) + \frac{\beta}{1+\beta} E_t \ln P_{\text{ne}}^Y(t+1) + \left(1 - \omega_{\text{ne}}^Y \right) \left(1 - \omega_{\text{ne}}^Y \beta \right) \sum_{j=1}^{N} \omega_j^Y \left( \ln \delta_{j,1}^Y(t) - \ln R_{j}(t) - \ln Y_{j}(t) \right) + \lambda^Y \left( \ln \delta_{j,1}^Y(t) - \ln R_{j}(t) - \ln Y_{j}(t) \right) - \sum_{j=1}^{N} \omega_j^Y \left( \ln \delta_{j,1}^Y(t) - \frac{1}{1+\beta} \ln R_{j}(t) - \beta E_t \ln R_{j}(t+1) \right)
$$

(Appendix.M.2)

Equation (Appendix.M.3) requires that the world output weighted average of the linear deviation of the ratio of the contemporaneous trade balance of any specific economy (denominated in US dollars) to its contemporaneous nominal output denominated in US dollars $\sum_{j=1}^{N} \omega_j^Y \frac{TB_{j}(t)}{E_{j}(t)P_j^Y(t)Y_j(t)}$ equals 0, in order to guarantee the multilateral consistency in nominal trade flows in the international commodity markets:

$$
\sum_{j=1}^{N} \omega_j^Y \frac{TB_{j}(t)}{E_{j}(t)P_j^Y(t)Y_j(t)} = 0
$$

(Appendix.M.3)

**N. All Exogenous Shocks for International Commodity Markets**

Equation (Appendix.N.1) shows the logarithmic deviation of the energy commodity price markup $\ln \delta_{e}^Y(t)$ (which directly influences the price of internationally homogeneous energy commodities denominated in US dollars in the international commodity market) follows normally distributed white noise process, and $\varepsilon_{e}^\delta(t) \sim \text{iid } N(0, \sigma_{e}^{2})$:

$$
\ln \delta_{e}^Y(t) = \varepsilon_{e}^\delta(t)
$$

(Appendix.N.1)

Equation (Appendix.N.2) shows the logarithmic deviation of the nonenergy commodity price markup $\ln \delta_{\text{ne}}^Y(t)$ (which directly influences the price of internationally homogeneous nonenergy commodities denominated in US dollars in the international commodity market) follows normally distributed white noise process, and $\varepsilon_{\text{ne}}^\delta(t) \sim \text{iid } N(0, \sigma_{\text{ne}}^{2})$:

$$
\ln \delta_{\text{ne}}^Y(t) = \varepsilon_{\text{ne}}^\delta(t)
$$

(Appendix.N.2)
Appendix III: Calibration or Estimation of Parameters

In this part of the appendix, we provide users with the details around the calibration or estimation of parameters of the AGMFM. Specifically, which parameters need to be calibrated or estimated and what data sources are used, are described in the rest parts of this appendix. Given the significant differences in data sources, the following first part describes the calibration or estimation of whole-economy-level parameters, and users are also suggested to refer to Vitek (2015, 2018) for relevant calibration or estimation details about those whole-economy-level parameters, while the second part describes the calibration or estimation of industry-level parameters. In fact, there is still a third category of non-economy-specific and non-industry-specific parameters (e.g., discount factor, consumption habitat persistence, elasticities, etc.), however, since their calibration or estimation and their specific values are discussed in detail by Vitek (2015, 2018), we will not repeat the same introduction here.

A. Whole-Economy-Level Parameters

Parameters that correspond to the whole economy, rather than the 45 industries under the whole economy, are called whole-economy-level parameters. The Haver Analytics or the CEIC database could be selected to perform the calibration or estimation of most whole-economy-level parameters of the AGMFM.

The values of all the following five types of parameters need to be calibrated or estimated in advance, so as to make the model ready to be directly used for simulation.

First, if n is the number of selected economies in the AGMFM, the data file should contain all the following n*1 vectors of parameters:

1. AGPYRAT, whose i-th element is assigned the ratio of the accumulated public financial wealth possessed by the i-th economy's fiscal authority (also called the i-th economy's net government asset) to its nominal GDP, and AGPYRAT=BLGPYRAT+BSGPYRAT.
2. BLGPYRAT, whose i-th element is assigned the ratio of the accumulated public financial wealth that is allocated to domestic long term bonds by the i-th economy's fiscal authority to its nominal GDP.
3. BSGPYRAT, whose i-th element is assigned the ratio of the accumulated public financial wealth that is allocated to domestic short term bonds by the i-th economy's fiscal authority to its nominal GDP.
4. CYRAT, whose i-th element is assigned the ratio of the i-th economy's real consumption to its real GDP (expenditure-based GDP accounting).
5. GCYRAT, whose i-th element is assigned the ratio of the i-th economy's real public consumption to its real GDP (expenditure-based GDP accounting).
6. GIYRAT, whose i-th element is assigned the ratio of the i-th economy's real public investment to its real GDP (expenditure-based GDP accounting).
7. GYRAT, whose i-th element is assigned the ratio of the i-th economy's real domestic public demand to its real GDP (expenditure-based GDP accounting), and GYRAT=GCYRAT+GIYRAT.
IHYRAT, whose i-th element is assigned the ratio of the i-th economy’s real residential investment to its real GDP (expenditure-based GDP accounting).

IKYRAT, whose i-th element is assigned the ratio of the i-th economy’s real business investment to its real GDP (expenditure-based GDP accounting).

IYRAT, whose i-th element is assigned the ratio of the i-th economy’s real investment to its real GDP (expenditure-based GDP accounting), and IYRAT=IHYRAT+IKYRAT.

MCMRAT, whose i-th element is assigned the ratio of the i-th economy’s both energy commodity type and nonenergy commodity type real imports (also called internationally homogeneous energy or nonenergy commodities in the international commodity markets) to its total real imports, and MCMRAT=MEMRAT+MNMRAT.

MEMRAT, whose i-th element is assigned the ratio of the i-th economy’s energy commodity type real imports (also called internationally homogeneous energy commodities in the international commodity market) to its total real imports.

MNMRAT, whose i-th element is assigned the ratio of the i-th economy’s nonenergy commodity type real imports (also called internationally homogeneous nonenergy commodities in the international commodity market) to its total real imports.

MYRAT, whose i-th element is assigned the ratio of the i-th economy’s real imports to its real GDP (expenditure-based GDP accounting).

TPYRAT, whose i-th element is assigned the i-th economy’s broad level of taxation (as a proportion of the i-th economy's nominal GDP).

WA, whose i-th element is assigned the ratio of the i-th economy’s capital market capitalization to the total capital market capitalization of all n economies.

WC, whose i-th element is assigned the ratio of the domestic mortgage loan issued by banks of the i-th economy to both domestic mortgage loan and domestic and foreign nonfinancial corporate loan issued by banks of the i-th economy.

WLPYRAT, whose i-th element is assigned the ratio of the i-th economy’s total labor income to its nominal GDP (income-based GDP accounting).

WY, whose i-th element is assigned the ratio of the i-th economy’s nominal GDP denominated in US dollars to total nominal GDP denominated in US dollars of all n economies.

XCXRAT, whose i-th element is assigned the ratio of the i-th economy’s both energy commodity type and nonenergy commodity type real exports (also called internationally homogeneous energy or nonenergy commodities in the international commodity markets) to its total real exports, and XCXRAT=XEXRAT+XNXRAT.

XEXRAT, whose i-th element is assigned the ratio of the i-th economy’s energy commodity type real exports (also called internationally homogeneous energy commodities in the international commodity market) to its total real exports.

XNXRAT, whose i-th element is assigned the ratio of the i-th economy’s nonenergy commodity type real exports (also called internationally homogeneous nonenergy commodities in the international commodity market) to its total real exports.
(23) XYRAT, whose i-th element is assigned the ratio of the i-th economy's real exports to its real GDP (expenditure-based GDP accounting), and CYRAT+IYRAT+GYRAT+XYRAT-MYRAT=1.

(24) GDP, whose i-th element is assigned the i-th economy's nominal GDP denominated in US dollars.

**Second**, the data file should contain all the following n*n matrices of parameters:

1. WB, whose i-th row and j-th column element is assigned the ratio of the i-th economy's investment in the j-th economy's vintage diversified long term bonds to the i-th economy's investment in all the n economies' vintage diversified long term bonds.

2. WCbi, whose i-th row and j-th column element is assigned the ratio of the i-th economy's nonfinancial corporate lending to the j-th economy to the i-th economy's nonfinancial corporate lending to all the n economies.

3. WF, whose i-th row and j-th column element is assigned the ratio of the i-th economy's nonfinancial corporate borrowing from the j-th economy to the i-th economy's nonfinancial corporate borrowing from all the n economies.

4. WM, whose i-th row and j-th column element is assigned the ratio of the i-th economy's imports from the j-th economy to the i-th economy's imports from all the n economies.

5. WS, whose i-th row and j-th column element is assigned the ratio of the i-th economy's investment in the j-th economy's industry diversified and firm diversified stocks to the i-th economy's investment in all the n economies' industry diversified and firm diversified stocks.

6. WT, whose i-th row and j-th column element is assigned the ratio of the i-th economy's imports and exports with the j-th economy to the i-th economy's imports and exports with all the n economies.

7. WX, whose i-th row and j-th column element is assigned the ratio of the i-th economy's exports to the j-th economy to the i-th economy's exports to all the n economies.

**Third**, the data file should contain the following n*2 matrix of parameters:

1. LAMBDA, its i-th row and 1-st column element is assigned the i-th economy's interbank market contagion level, the value of this element is set in the interval [1,3], with 1 for the lowest level of interbank market contagion (for example China) and 3 for the highest level of interbank market contagion (for example the United Kingdom and the United States), while its i-th row and 2-nd column element is assigned the i-th economy's capital market contagion level, the value of this element is also set in the interval [1,3], with 1 for the lowest level of capital market contagion (for example Brazil and India).

For the assignments of LAMBDA, users are strongly advised to refer to the latest IMF Annual Report on Exchange Arrangements and Exchange Restrictions (for example, IMF 2021 currently) and most of the search results from the IMF's website with "International Financial Contagion" as the search term.
Fourth, the data file should also contain the following \( n\times T \) matrices of parameters, and \( T \) is the number of quarters in which the user pays attention to the change of tariff rates:

1. **TARIFFRAT**, whose \( i \)-th row and \( j \)-th column element is assigned the tariff rate constant \( \tau_i^M \) for the \( i \)-th economy’s imports in the \( j \)-th period.

2. **TARIFFCOEF1**, whose \( i \)-th row and \( j \)-th column element is assigned the value of the coefficient \( \rho_i \) in the following equation, which is specifically estimated for the change of the tariff rate variable \( \tau_i^M(j) \) for the \( i \)-th economy’s imports from the \((j-1)\)-th period to the \( j \)-th period:

\[
(\tau_i^M(j) - \tau_i^M) = \rho_i (\tau_i^M(j-1) - \tau_i^M) + \nu_{i}^M(j)
\]

3. **TARIFFCOEF2**, whose \( i \)-th row and \( j \)-th column element is assigned the value of the coefficient \( \rho_i \) in the following equation, which is specifically estimated for the change of the tariff rate shock \( \nu_{i}^M(j) \) for the \( i \)-th economy from the \((j-1)\)-th period to the \( j \)-th period:

\[
\nu_{i}^M(j) = \rho_i \nu_{i}^M(j-1) + \epsilon_{i}^M(j)
\]

4. **SIGMA**, whose \( i \)-th row and \( j \)-th column element is assigned the value of the standard deviation of the normally distributed innovations \( \epsilon_{i}^M(j) \) in the above equation, which is also specifically estimated for the change of the tariff rate shock \( \nu_{i}^M(j) \) for the \( i \)-th economy from the \((j-1)\)-th period to the \( j \)-th period.

The model builder downloads and further calculates the TARIFFRAT matrices of parameters for the AGMFM, based on the tariff rate data in the annexes of the RCEP Agreement (China FTA Network 2020). While all the above TARIFFCOEF1, TARIFFCOEF2 and SIGMA matrices of parameters are estimated based on the period-by-period change of the above TARIFFRAT matrices of parameters.

Fifth, since the AGMFM further incorporates the structure of international direct investment, including the structure of FDI and ODI, the data file should additionally contain the following \( n \times n \) or \( n \times 3 \) matrix of parameters:

1. **totalFDIRatio**, whose \( j \)-th row and \( i \)-th column element is assigned the ratio of the US dollar denominated foreign or domestic direct investment from the \( j \)-th economy to the \( i \)-th economy to the US dollar denominated foreign or domestic direct investment from all economies to the \( i \)-th economy.

2. **BOP**, whose \( i \)-th row and 1-st column element is assigned the ratio of the \( i \)-th economy’s current account balance denominated in US dollars to the \( i \)-th economy’s nominal GDP denominated in US dollars; whose \( i \)-th row and 2-nd column element is assigned the ratio of the \( i \)-th economy’s direct investment balance denominated in US dollars to the \( i \)-th economy’s nominal GDP denominated in US dollars; whose \( i \)-th row and 3-rd column element is assigned the ratio of the rest of capital and financial account balance of the \( i \)-th economy denominated in US dollars to the \( i \)-th economy’s nominal GDP denominated in US dollars.

In addition, if the users find that the settings in the AGMFM related to the following four vectors of parameters (EconomyName, ExchangeRateArrangements, ExchangeRateAnchor, and SingleCurrencyUnion) have changed in reality, they could modify the relevant settings in the model by themselves.
(1) EconomyName is a text type vector, and its i-th element should be the name of the i-th economy selected. Here it is worth noting that the 1st economy of the AGMFM is mandated to be set to the United States, only because the quotation currency for transactions in the foreign exchange market is issued by the United States. If the names of some economies have changed in reality, users can modify their name text in the model file directly.

(2) ExchangeRateArrangements is a vector with n elements, and its i-th element is the assignment of values to the specific kind of exchange rate arrangements of the i-th economy selected. To be more specific, the model file has assigned a value of 1 to a free floating exchange rate and flexible inflation targeting arrangement, a value of 2 to a managed exchange rate arrangement, and a value of 3 to a fixed exchange rate arrangement (including the situation where the economy belongs to a currency union but is not the primary economy of this union).

(3) ExchangeRateAnchor is also a vector with n elements, and only when the i-th economy has a fixed exchange rate arrangement, and chooses the currency issued by another economy (for this reason that economy issuing anchor currency must also be included in this model, for example we assume it is listed as the j-th economy) as the exchange rate anchor of its currency (including the situation where the economy belongs to a currency union but is not the primary economy of this union, then it is considered as if this economy chooses the currency issued by the primary economy of this union as the exchange rate anchor of its currency), users should assign j to the i-th element of ExchangeRateAnchor. In addition, the assignments for other economies with either free floating exchange rate and flexible inflation targeting arrangements or managed exchange rate arrangements can be arbitrary values, because the assignments of these economies in the vector ExchangeRateAnchor are not used by the model file.

(4) SingleCurrencyUnion is also a vector with n elements, and only when the i-th economy belongs to a currency union that is composed of several economies and uses the same currency issued by the primary economy j in this union, users should assign j to the i-th element of SingleCurrencyUnion, the assignments for other economies can be arbitrary values since they are not used by the model file.

If the exchange rate arrangement, inflation targeting arrangement, exchange rate anchor, or currency union arrangement of some economies have changed in reality, users should adjust the equation (Appendix.F.1a), (Appendix.F.1b) or (Appendix.F.1c), and (Appendix.J.1a), (Appendix.J.1b) or (Appendix.J.1c) of these economies in the AGMFM. Please refer to the Subsection F and Subsection J in Appendix II for more details around these equations.

For the updates or changes of exchange rate arrangement, inflation targeting arrangement, exchange rate anchor, or currency union arrangement of all economies covered by the AGMFM in reality, users are strongly advised to refer to the latest Annual Report on Exchange Arrangements and Exchange Restrictions published by the IMF (for example, IMF 2021 currently).

B. Industry-Level Parameters

Parameters that correspond to the 45 industries under the whole economy are called industry-level parameters. The model builder currently selected the latest version ICIQ tables prepared by OECD, to perform the calibration or estimation of all industry-level parameters of the AGMFM (OECD 2021).
The values of the following two types of industry-level parameters need to be calibrated or estimated in advance, so as to make the AGMFM ready to be directly used for simulation.

First, all the following industry-level vectors or matrices should be prepared in advance, for directly applying the complete AGMFM to various research topics:

(1) **inputratio1**, whose \((45^*(i-1)+k)\)-th element is assigned the ratio of industrial real output minus all intermediate inputs of the k-th industry of the i-th economy to the real output of the k-th industry of the i-th economy, in other words, it is the ratio of the GVA part of the k-th industry of the i-th economy, produced by capital and labor as input factors, to the real output of the k-th industry of the i-th economy.

(2) **inputratio2**, whose \((45^*(j-1)+m)\)-th row and \((45^*(i-1)+k)\)-th column element is assigned the ratio of the intermediate goods or services from the m-th industry of the j-th economy that are adopted as intermediate inputs of the k-th industry of the i-th economy to the real output of the k-th industry of the i-th economy.

(3) **outputratio1**, whose \((45^*(i-1)+k)\)-th row and \((3^*(j-1)+1)\)-th column element is assigned the ratio of the final goods or services from the k-th industry of the i-th economy for private consumption of the j-th economy to the real output of the k-th industry of the i-th economy; whose \((45^*(i-1)+k)\)-th row and \((3^*(j-1)+2)\)-th column element is assigned the final goods or services from the k-th industry of the i-th economy for private investment of the j-th economy to the real output of the k-th industry of the i-th economy; whose \((45^*(i-1)+k)\)-th row and \((3^*j)\)-th column element is assigned the ratio of the final goods or services from the k-th industry of the i-th economy for public purchase of the j-th economy to the real output of the k-th industry of the i-th economy.

(4) **outputratio2**, whose \((45^*(i-1)+k)\)-th row and \((45^*(j-1)+m)\)-th column element is assigned the ratio of the intermediate goods or services from the k-th industry of the i-th economy that are adopted as intermediate inputs of the m-th industry of the j-th economy to the real output of the k-th industry of the i-th economy.

(5) **totaloutputratio**, whose k-th row and i-th column element is assigned the proportion of the real output of the k-th industry of the i-th economy to the real output of all industries of the i-th economy.

(6) **exportratio1**, whose \((45^*(i-1)+k)\)-th row and \((3^*(j-1)+1)\)-th column element is assigned the ratio of the exported final goods or services from the k-th industry of the i-th economy for private consumption of the j-th economy to the total exports of the k-th industry of the i-th economy; whose \((45^*(i-1)+k)\)-th row and \((3^*(j-1)+2)\)-th column element is assigned the exported final goods or services from the k-th industry of the i-th economy for private investment of the j-th economy to the total exports of the k-th industry of the i-th economy; whose \((45^*(i-1)+k)\)-th row and \((3^*j)\)-th column element is assigned the ratio of the exported final goods or services from the k-th industry of the i-th economy for public purchase of the j-th economy to the total exports of the k-th industry of the i-th economy.

(7) **exportratio2**, whose \((45^*(i-1)+k)\)-th row and \((45^*(j-1)+m)\)-th column element is assigned the ratio of the exported intermediate goods or services from the k-th industry of the i-th economy that are adopted as intermediate inputs of the m-th industry of the j-th economy to the total exports of the k-th industry of the i-th economy.
A totalexportratio, whose k-th row and i-th column element is assigned the proportion of the total exports of the k-th industry of the i-th economy to the total exports of all industries of the i-th economy.

An importratio, whose (4*n*(k-1)+j)-th row and i-th column element is assigned the ratio of the imported intermediate goods or services from the k-th industry of the j-th economy that are adopted as intermediate inputs for all industries of the i-th economy to the total imports of the i-th economy from foreign k-th industry; whose (4*n*(k-1)+n+3*(j-1)+1)-th row and i-th column element is assigned the ratio of the imported final goods or services from the k-th industry of the j-th economy for private consumption of the i-th economy to the total imports of the i-th economy from foreign k-th industry; whose (4*n*(k-1)+n+3*(j-1)+2)-th row and i-th column element is assigned the ratio of the imported final goods or services from the k-th industry of the j-th economy for private investment of the i-th economy to the total imports of the i-th economy from foreign k-th industry; whose (4*n*(k-1)+n+3*j)-th row and i-th column element is assigned the ratio of the imported final goods or services from the k-th industry of the j-th economy for public purchase of the i-th economy to the total imports of the i-th economy from foreign k-th industry.

A totalimportratio, whose k-th row and i-th column element is assigned the proportion of the total imports of the i-th economy from foreign k-th industry to the total imports of the i-th economy from all foreign industries.

A consumptionratio, whose (n*(k-1)+j)-th and i-th column element is assigned the ratio of final goods or services from the k-th industry of the j-th economy for the i-th economy's private consumption, private investment and public purchase to final goods or services from the k-th industry of all economies including the i-th economy for the i-th economy's private consumption, private investment and public purchase.

A totalconsumptionratio, whose k-th row and i-th column element is assigned the i-th economy's private consumption, private investment and public purchase of final goods or services from the k-th industry of all economies including the i-th economy for the i-th economy's private consumption, private investment and public purchase of all industries of all economies including the i-th economy.

The model builder calculates all the above industry-level vectors or matrices of parameters for the AGMFM, based on each column or each row of the latest version OECD ICIO tables. For detailed structure of the OECD ICIO tables, please refer to the OECD webpage for more details (OECD 2021).

Second, the following K sets of n*T matrices of parameters should also be prepared in advance for model application, here K is the number of industries that the user pays attention to the change of their industry-level tariff rates, and T is the number of quarters in which the user pays attention to the change of these industry-level tariff rates:

(1) A TARIFFRATind, whose i-th row and j-th column element of the k-th set of matrix is assigned the tariff rate constant $\tau_{ik}^M$ for the total imports of the i-th economy from the k-th industry of all other economies except the i-th economy in the j-th period.

(2) A TARIFFCOEF1ind, whose i-th row and j-th column element of the k-th set of matrix is assigned the value of the coefficient $p_{ik}^M$ in the following equation, which is specifically estimated for the change of the industry-level tariff rate variable $\tau_{ik}^M(j)$ for the total imports of the i-th economy from the k-th industry of all other economies except the i-th economy, from the (j-1)-th period to the j-th period:
\((\tau_{i,j}^M(j) - \tau_{i,j}^M) = \rho_{i,k}^M (\tau_{i,j}^M(j - 1) - \tau_{i,j}^M) + \nu_{i,k}^M(j)\)

(3) TARIFFCOEF2ind, whose i-th row and j-th column element of the k-th set of matrix is assigned the value of the coefficient \(\rho_{i,k}^M\) in the following equation, which is specifically estimated for the change of the industry-level tariff rate shock \(\nu_{i,k}^M(j)\) for the total imports of the i-th economy from the k-th industry of all other economies except the i-th economy, from the (j-1)-th period to the j-th period:

\(\nu_{i,k}^M(j) = \rho_{i,k}^M \nu_{i,k}^M(j - 1) + \varepsilon_{i,k}^M(j)\)

(4) SIGMAind, whose i-th row and j-th column element of the k-th set of matrix is assigned the value of the standard deviation of the normally distributed innovations \(\varepsilon_{i,k}^M(j)\) in the above equation, which is also specifically estimated for the change of the industry-level tariff rate shock \(\nu_{i,k}^M(j)\) for the total imports of the i-th economy from the k-th industry of all other economies except the i-th economy, from the (j-1)-th period to the j-th period.

The model builder downloads and further calculates the TARIFFRATind matrices of parameters for the AGMFAM, based on the industry-level tariff rate data in the annexes of the RCEP Agreement (China FTA Network 2020). The above TARIFFCOEF1ind, TARIFFCOEF2ind and SIGMAind matrices of parameters are all estimated based on the period-by-period change of the above TARIFFRATind matrices of parameters.
Appendix IV: List of All Variables in the AGMFM

This part of the appendix lists all those economy-specific variables, economy-specific and industry-specific variables, and international market variables, as the actual variables to be solved in the final computable large-scale AGMFM.

In the following list, after each colon is both the specific variable contained in those equations that constitute the final computable large-scale AGMFM in Section IV and Appendix II, and the brief description of this specific variable. Before each colon is the specific code symbol corresponding to this variable in the model program, which is convenient as an easy reference for users when they need to add various model codes to do various types of DSGE analyses, in order to meet the needs of specific research topics.

For the meaning of the linear deviation or the logarithmic deviation of variables, please refer to the specific explanation at the beginning of Section IV.

A. Household Sector

(1) \pi_{C_i} : the linear deviation of the consumption price inflation \bar{\pi}_C(t)

(2) \lnP_{C_i} : the logarithmic deviation of the domestic currency denominated consumption price level \ln\bar{P}_C(t)

(3) \lnC_{i} : the logarithmic deviation of the total consumption by all three kinds of households (bank intermediated households, capital market intermediated households, and credit constrained households) \ln\bar{C}(t)

(4) \lnC_{C_i} : the logarithmic deviation of the consumption by credit constrained households \ln\bar{C}_C(t)

(5) \bar{\pi}_{A_B_H_i} : the linear deviation of the nominal property return (property balances of bank intermediated households are distributed across the values of bank deposits and domestic real estate portfolio) \bar{\pi}_{A_B_H}(t)

(6) \bar{\tau}_{A_A_H_i} : the linear deviation of the nominal portfolio return (portfolio balances of capital market intermediated households are allocated across the values of internationally diversified short term bonds, internationally diversified and vintage diversified long term bonds, and internationally diversified and industry diversified and firm diversified stocks) \bar{\tau}_{A_A_H}(t)

B. Labor Supply Sector

(1) \bar{\pi}_{W_i} : the linear deviation of the nominal wage inflation \bar{\pi}_W(t)

(2) \lnW_i : the logarithmic deviation of the domestic currency denominated nominal wage \ln\bar{W}_i(t)

(3) \lnN_i : the logarithmic deviation of the total labor force \ln\bar{N}_i(t)

(4) \bar{u}_L_i : the linear deviation of the unemployment rate \bar{u}_L(t)
C. Construction Sector

(1) \( I_{H_i} \): the logarithmic deviation of the residential investment \( \ln I_{H_i}(t) \)

(2) \( Q_{H_i} \): the logarithmic deviation of the domestic currency denominated shadow price of housing \( \ln Q_{H_i}(t) \)

(3) \( \iota_{H_i} \): the logarithmic deviation of the domestic currency denominated rental price of housing \( \ln \iota_{H_i}(t) \)

(4) \( H_i \): the logarithmic deviation of the housing stock \( \ln H_i(t) \)

(5) \( V_{H_i} \): the logarithmic deviation of the domestic currency denominated price of housing \( \ln V_{H_i}(t) \)

(6) \( PI_{H_i} \): the logarithmic deviation of the real estate developer profits \( \ln PI_{H_i}(t) \)

D. Production Sector

(1) \( pi_{Y_i} \): the linear deviation of the output price inflation \( \pi_{Y_i}(t) \)

(2) \( P_{Y_i} \): the logarithmic deviation of the domestic currency denominated output price level \( \ln P_{Y_i}(t) \)

(3) \( L_i \): the logarithmic deviation of the employed labor force \( \ln L_i(t) \)

(4) \( I_{K_i} \): the logarithmic deviation of the business investment from the \( i \)-th economy to the production sector of all economies including the \( i \)-th economy itself \( \ln I_{K_i}(t) \)

(5) \( Q_{K_i} \): the logarithmic deviation of the domestic currency denominated shadow price of private physical capital \( \ln Q_{K_i}(t) \)

(6) \( u_{K_i} \): the logarithmic deviation of the capital utilization rate \( \ln u_{K_i}(t) \)

(7) \( K_i \): the logarithmic deviation of the private physical capital stock \( \ln K_i(t) \)

(8) \( V_{S_i} \): the logarithmic deviation of the domestic currency denominated price of corporate equity \( \ln V_{S_i}(t) \)

(9) \( PI_{S_i} \): the logarithmic deviation of the corporate profits \( \ln PI_{S_i}(t) \)

(10) \( Y_{\text{caret}}_i \): the logarithmic deviation of the output gap \( \ln Y_{\text{caret}}_i(t) \)

(11) \( Y_{\text{tilde}}_i \): the logarithmic deviation of the potential output (the inferred total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \ln Y_{\text{tilde}}_i(t) \)

(12) \( \text{ALPHA}_i \): the logarithmic deviation of the labor productivity \( \ln \text{ALPHA}_i(t) \)
(13) \( \text{ALPHA}_{\text{tilde}}_i \) : the logarithmic deviation of the trend labor productivity (which exhibits partial adjustment dynamics of labor productivity) \( \ln \tilde{\Lambda}_i(t) \)

(14) \( \text{DELTA}_{\text{ALPHA}_{\text{tilde}}_i} \) : the intertemporal change in the logarithmic deviation of the trend labor productivity \( \Delta \ln \tilde{\Lambda}_i(t) \)

E. Banking Sector

(1) \( M_S_i \) : the logarithmic deviation of the bank money stock (also called the aggregate bank funding) \( \ln M^S_i(t) \)

(2) \( B_C_B_i \) : the logarithmic deviation of the aggregate bank assets (also called the bank credit stock) \( \ln B^{CB}_i(t) \)

(3) \( B_C_D_i \) : the logarithmic deviation of the domestic currency denominated total final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate developers \( \ln B^{CD}_i(t) \)

(4) \( B_C_F_i \) : the logarithmic deviation of the domestic currency denominated total final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( \ln B^{CF}_i(t) \)

(5) \( i_M_i \) : the linear deviation of the domestic nominal mortgage loan rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate developers \( i^M_i(t) \)

(6) \( i_C_E_i \) : the linear deviation of the weighted average of domestic and foreign nominal corporate loan rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy \( i^{CE}_i(t) \)

(7) \( i_C_i \) : the linear deviation of the domestic nominal corporate loan rate for the economy specific local currency denominated final corporate loans provided by domestic and foreign global final banks \( i^C_i(t) \)

(8) \( I_B_i \) : the logarithmic deviation of the bank retained earnings \( \ln I^B_i(t) \)

(9) \( Q_B_i \) : the logarithmic deviation of the domestic currency denominated shadow price of bank capital \( \ln \tilde{Q}^B_i(t) \)

(10) \( K_B_i \) : the logarithmic deviation of the aggregate bank capital \( \ln K^B_i(t) \)

(11) \( \text{delta}_B_i \) : the linear deviation of the bank capital destruction rate \( \delta^B_i(t) \)

(12) \( \text{delta}_M_i \) : the linear deviation of the domestic mortgage loan default rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate developers \( \delta^M_i(t) \)
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(13) \(\delta_{C,E_i}^{\text{CE}}(t)\): the linear deviation of the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks

(14) \(\delta_{C,i}^{\text{C}}(t)\): the linear deviation of the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy

**F. Foreign Exchange Sector**

(1) \(\varepsilon_{i,1}^{\text{E}}(t)\): the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to US dollar (which is the quotation currency for transactions in the foreign exchange market issued by the 1-st economy ("United States")).

(2) \(\varepsilon_{i,j}(t)\) (here \(j \neq 1\)): the logarithmic deviation of the nominal bilateral exchange rate of the currency issued by the i-th economy to the currency issued by the j-th economy.

(3) \(\varepsilon_{i}^{\text{E}}(t)\): the logarithmic deviation of the nominal effective exchange rate of the currency issued by the i-th economy (which is the trade weighted average price of foreign currency in terms of the i-th economy's currency).

(4) \(\omega_{i,1}^{\text{E}}(t)\): the logarithmic deviation of the real bilateral exchange rate of the currency issued by the i-th economy relative to US dollar (which is the relative price of the 1-st economy's output in terms of the i-th economy's output).

(5) \(\omega_{i}^{\text{E}}(t)\): the logarithmic deviation of the real effective exchange rate of the currency issued by the i-th economy (which is the trade weighted average price of foreign output in terms of the i-th economy's output).

**G. Export and Import Sectors**

(1) \(\ln X_i(t)\): the logarithmic deviation of the total exports.

(2) \(\ln M_i(t)\): the logarithmic deviation of the total imports.

(3) \(\tau_i(t)\): the logarithmic deviation of the terms of trade (which measures the i-th economy's relative price of exports to imports, and also takes into account the logarithmic deviation of the global terms of trade shifter to ensure multilateral consistency in nominal trade flows in the international commodity markets).

(4) \(\tau_{X_i}^{\text{X}}(t)\): the logarithmic deviation of the internal terms of trade (which measures the price of exports relative to the core price level).

(5) \(\tau_{M_i}^{\text{M}}(t)\): the logarithmic deviation of the external terms of trade (which measures the post-tariff price of imports relative to the core price level).

(6) \(\pi_{X_i}^{\text{X}}(t)\): the linear deviation of the export price inflation.
(7) \( P_{X_i} \) : the logarithmic deviation of the domestic currency denominated price of exports \( \ln P_X^i(t) \)

(8) \( \pi_{M_i} \) : the linear deviation of the post-tariff import price inflation \( \hat{\pi}_M^i(t) \)

(9) \( P_{M_i} \) : the logarithmic deviation of the domestic currency denominated post-tariff price of imports \( \ln P_M^i(t) \)

(10) \( \pi_{Mt_i} \) : the linear deviation of the pre-tariff import price inflation \( \hat{\pi}_M^i, T(t) \)

(11) \( P_{Mt_i} \) : the logarithmic deviation of the domestic currency denominated pre-tariff price of imports \( \ln P_M^{i, T}(t) \)

H. Balance of Payments Sector

(1) \( CA_i \) : the linear deviation of the ratio of the current account balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{CA_i(t)}{\varepsilon_1, i(t) \cdot P_Y^i(t) \cdot Y_i(t)} \)

(2) \( A_i \) : the linear deviation of the ratio of the contemporaneous net foreign asset position (which equals the sum of the financial wealth of the household sector, the construction sector, the production sector, the banking sector, the export sector, the import sector, and the government sector of the i-th economy) to past nominal output of the i-th economy \( \frac{\hat{A}_i(t)}{P_Y^i(t-1) \cdot Y_i(t-1)} \)

(3) \( TB_i \) : the linear deviation of the ratio of the trade balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{TB_i(t)}{\varepsilon_1, i(t) \cdot P_Y^i(t) \cdot Y_i(t)} \)

(4) \( CFA_D_i \) : the linear deviation of the ratio of the direct investment balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{CFA_D^i(t)}{\varepsilon_1, i(t) \cdot P_Y^i(t) \cdot Y_i(t)} \)

(5) \( I_{Ki_in_i} \) : the logarithmic deviation of the domestic and foreign business investment for the production sector of the i-th economy \( \ln I_{Ki,in}^i(t) \)

(6) \( CFA_{Rest_i} \) : the linear deviation of the ratio of the rest of capital and financial account balance (denominated in US dollars) to nominal output of the i-th economy denominated in US dollars \( \frac{CFA_{Rest_i}(t)}{\varepsilon_1, i(t) \cdot P_Y^i(t) \cdot Y_i(t)} \)

I. Virtual “Absorption Sector”

(1) \( \pi_i \) : the linear deviation of the core inflation \( \hat{\pi}_i(t) \)

(2) \( P_i \) : the logarithmic deviation of the domestic currency denominated core price level \( \ln \hat{P}_i(t) \)

(3) \( Y_i \) : the logarithmic deviation of the total output \( \ln Y_i(t) \)
(4) \( D_i \) : the logarithmic deviation of the total domestic demand \( \ln D_i(t) \)

(5) \( I_i \) : the logarithmic deviation of the total investment (including residential investment for the construction sector and business investment for the production sector) \( \ln I_i(t) \)

J. Government Sector

(1) \( i_P_i \) : the linear deviation of the nominal policy interest rate \( \hat{i}_i^P(t) \)

(2) \( \kappa_i \) : the linear deviation of the bank capital ratio \( \hat{\kappa}_i(t) \)

(3) \( \kappa_{Ri} \) : the linear deviation of the regulatory bank capital ratio requirement \( \hat{\kappa}_{Ri}(t) \)

(4) \( \phi_{Di} \) : the linear deviation of the regulatory mortgage loan to value ratio limit \( \hat{\phi}_{Di}(t) \)

(5) \( \phi_{Fi} \) : the linear deviation of the regulatory corporate loan to value ratio limit \( \hat{\phi}_{Fi}(t) \)

(6) \( G_i \) : the logarithmic deviation of the domestic public demand (which is the sum of public consumption and public investment) \( \ln G_i(t) \)

(7) \( G_{Ci} \) : the logarithmic deviation of the public consumption \( \ln G_{Ci}(t) \)

(8) \( G_{Ii} \) : the logarithmic deviation of the public investment \( \ln G_{Ii}(t) \)

(9) \( K_{Gi} \) : the logarithmic deviation of the public physical capital stock \( \ln K_{Gi}(t) \)

(10) \( \tau_{Ki} \) : the linear deviation of the corporate income tax rate \( \hat{\tau}_i^K(t) \)

(11) \( \tau_{Li} \) : the linear deviation of the labor income tax rate \( \hat{\tau}_i^L(t) \)

(12) \( (\tau_{Mi} - \tau_{M_i}^*) \) : the linear deviation of the import tariff rate \( \hat{\tau}_i^M(t) \), here \( \tau_{M_i}^* \) is the tariff rate constant for the \( i \)-th economy’s imports in period \( t \)

(13) \( T_{Ci} \) : the linear deviation of the ratio of both nondiscretionary and discretionary lump sum transfer payments by the fiscal authority to nominal output \( \frac{T_{Ci}(t)}{P_i(t)Y_i(t)} \)

(14) \( T_{CN_i} \) : the linear deviation of the ratio of nondiscretionary lump sum transfer payments by the fiscal authority to nominal output (a budget neutral nondiscretionary lump sum transfer program that redistributes national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households) \( \frac{T_{CN_i}(t)}{P_i(t)Y_i(t)} \)

(15) \( T_{CD_i} \) : the linear deviation of the ratio of discretionary lump sum transfer payments by the fiscal authority to nominal output (a discretionary lump sum transfer program that only provides income support to credit constrained households) \( \frac{T_{CD_i}(t)}{P_i(t)Y_i(t)} \)

(16) \( FB_i \) : the linear deviation of the ratio of the fiscal balance of the fiscal authority to nominal output \( \frac{FB_i(t)}{P_i(t)Y_i(t)} \)
(17) \( PB_i \) : the linear deviation of the ratio of the primary fiscal balance of the fiscal authority to nominal output \( \frac{PB_i(t)}{P_i^Y(t)} \)

(18) \( T_i \) : the logarithmic deviation of the ratio of the tax revenue of the fiscal authority to nominal output \( \ln \frac{T_i(t)}{P_i^Y(t)} \)

(19) \( A_{G_i} \) : the linear deviation of the ratio of the contemporaneous net government asset of the fiscal authority (also called accumulated public financial wealth) to past nominal output \( \frac{A_{G_i}(t)}{P_i^Y(t-1)} \)

K. Combination of Domestic Interbank, Money, Bond and Equity Markets and International Financial Markets

(1) \( i_B_i \) : the linear deviation of the nominal interbank loans rate \( i^B_i(t) \)

(2) \( \upsilon_{i_B_i} \) : the logarithmic deviation of the weighted average of domestic and foreign liquidity risk premium for the i-th economy (the above nominal interbank loans rate is adjusted by this liquidity risk premium) \( \ln \upsilon_{i_B_i}(t) \)

(3) \( i_S_i \) : the linear deviation of the nominal short term bond yield \( i^S_i(t) \)

(4) \( r_S_i \) : the linear deviation of the real short term bond yield \( r^S_i(t) \)

(5) \( \upsilon_{i_S_i} \) : the logarithmic deviation of the weighted average of domestic and foreign credit risk premium for the i-th economy (the above nominal short term bond yield is adjusted by this credit risk premium) \( \ln \upsilon_{i_S_i}(t) \)

(6) \( i_L_i \) : the linear deviation of the nominal long term bond yield \( i^L_i(t) \)

(7) \( r_L_i \) : the linear deviation of the real long term bond yield \( r^L_i(t) \)

(8) \( \upsilon_B_i \) : the logarithmic deviation of the weighted average of domestic and foreign duration risk premium for the i-th economy (the above nominal long term bond yield is adjusted by this duration risk premium) \( \ln \upsilon_B_i(t) \)

(9) \( \mu_B_i \) : the logarithmic deviation of the weighted average of domestic and foreign term premium for the i-th economy \( \ln \mu_B_i(t) \)

(10) \( \upsilon_S_i \) : the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the i-th economy (the domestic currency denominated price of corporate equity is adjusted by this equity risk premium) \( \ln \upsilon_S_i(t) \)

(11) \( i_L_E_i \) : the linear deviation of the nominal effective long term market interest rate (the ratio of the fiscal balance of the fiscal authority to nominal output is adjusted by this domestic nominal effective long term market interest rate) \( i^{LE}_i(t) \)
L. All Whole-Economy-Level Exogenous Variables

(1) \( \nu_{NU_i} \) : the logarithmic deviation of the labor supply shock \( \ln \nu_{NU_i}(t) \) (which directly influences the unemployment rate)

(2) \( \nu_{C_i} \) : the logarithmic deviation of the consumption demand shock \( \ln \nu_{C_i}(t) \) (which directly influences the total consumption by all three kinds of households)

(3) \( \nu_{I_H_i} \) : the logarithmic deviation of the residential investment demand shock \( \ln \nu_{I_H_i}(t) \) (which directly influences the residential investment)

(4) \( \nu_{I_K_i} \) : the logarithmic deviation of the business investment demand shock \( \ln \nu_{I_K_i}(t) \) (which directly influences the business investment)

(5) \( \theta_{Y_i} \) : the logarithmic deviation of the output price markup shock \( \ln \theta_{Y_i}(t) \) (which directly influences the domestic currency denominated core price level)

(6) \( \theta_{L_i} \) : the logarithmic deviation of the wage markup shock \( \ln \theta_{L_i}(t) \) (which directly influences the nominal wage)

(7) \( \nu_{i_B_i} \) : the logarithmic deviation of the liquidity risk premium shock \( \ln \nu_{i_B_i}(t) \) (which directly influences the nominal interbank loans rate)

(8) \( \nu_{H_i} \) : the logarithmic deviation of the housing risk premium shock \( \ln \nu_{H_i}(t) \) (which directly influences both the nominal property return and the domestic currency denominated price of housing)

(9) \( \nu_{i_S_i} \) : the logarithmic deviation of the credit risk premium shock \( \ln \nu_{i_S_i}(t) \) (which directly influences the nominal short term bond yield)

(10) \( \nu_{B_i} \) : the logarithmic deviation of the duration risk premium shock \( \ln \nu_{B_i}(t) \) (which directly influences the nominal long term bond yield)

(11) \( \nu_{S_i} \) : the logarithmic deviation of the equity risk premium shock \( \ln \nu_{S_i}(t) \) (which directly influences the domestic currency denominated price of corporate equity)

(12) \( \nu_{EPSILON_i} \) : the logarithmic deviation of the currency risk premium shock \( \ln \nu_{EPSILON_i}(t) \) (which directly influences the nominal bilateral exchange rate, the nominal effective exchange rate, the real bilateral exchange rate, and the real effective exchange rate)

(13) \( \theta_{C_D_i} \) : the logarithmic deviation of the mortgage loan rate markup shock \( \ln \theta_{C_D_i}(t) \) (which directly influences the domestic nominal mortgage loan rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate developers)
(14) \( \theta_{C_F_i} \): the logarithmic deviation of the corporate loan rate markup shock \( \ln \theta_{C_F_i}(t) \) (which directly influences both the domestic nominal corporate loan rate for economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks, and the weighted average of domestic and foreign nominal corporate loan rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy)

(15) \( \nu_{\Delta M_i} \): the linear deviation of the mortgage loan default shock \( \nu_{\Delta M_i}(t) \) (which directly influences the domestic mortgage loan default rate for the domestic currency denominated final mortgage loans issued by domestic final banks of the i-th economy to domestic real estate developers)

(16) \( \nu_{\Delta C_i} \): the linear deviation of the corporate loan default shock \( \nu_{\Delta C_i}(t) \) (which directly influences both the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the i-th economy to firms of the i-th economy, and the weighted average of domestic and foreign corporate loan default rate for the economy specific local currency denominated final corporate loans provided by domestic final banks of the i-th economy to both domestic and foreign global final banks)

(17) \( \nu_{iP_i} \): the linear deviation of the monetary policy shock \( \nu_{iP_i}(t) \) (which directly influences the nominal policy interest rate)

(18) \( \nu_{GC_i} \): the linear deviation of the public consumption shock \( \nu_{GC_i}(t) \) (which directly influences the public consumption)

(19) \( \nu_{\tau K_i} \): the linear deviation of the corporate income tax rate shock \( \nu_{\tau K_i}(t) \) (which directly influences the corporate income tax rate)

(20) \( \nu_{\tau L_i} \): the linear deviation of the labor income tax rate shock \( \nu_{\tau L_i}(t) \) (which directly influences the labor income tax rate)

(21) \( \nu_{\tau M_i} \): the linear deviation of the tariff rate shock \( \nu_{\tau M_i}(t) \) (which directly influences the import tariff rate for the total imports of the i-th economy from the k-th industry of all other economies)

(22) \( \nu_T_i \): the linear deviation of the transfer payment shock \( \nu_T_i(t) \) (which directly influences the ratio of discretionary lump sum transfer payments by the fiscal authority to nominal output)

(23) \( \nu_{\kappa_i} \): the linear deviation of the bank capital requirement shock \( \nu_{\kappa_i}(t) \) (which directly influences the regulatory bank capital ratio requirement)

(24) \( \nu_{\phi_D_i} \): the linear deviation of the mortgage loan to value limit shock \( \nu_{\phi_D_i}(t) \) (which directly influences the regulatory mortgage loan to value ratio limit)
(25) \( \nu_{\phi F i} \) : the linear deviation of the corporate loan to value limit shock \( \psi_{F i}^k(t) \) (which directly influences the regulatory corporate loan to value ratio limit) Since the following six whole-economy-level exogenous shocks existing in the GFM introduced in Vitek (2015, 2018) could be completely replaced by the weighted sum of the corresponding industry-level exogenous shocks listed in the previous Section IV, they no longer appear in the AGMFM. However, if all industry-level equations in the AGMFM are removed to return to a simplified whole-economy-level DSGE model without the intra-industry and inter-industry structure under the whole economy, the following six whole-economy-level exogenous shocks should be re-added to the model, to maintain the integrity of that simplified AGMFM.

(26) \( \nu_{\alpha i} \) : the logarithmic deviation of the labor productivity shock \( \ln \nu_{\alpha i}^A(t) \) (which directly influences the labor productivity and the trend labor productivity)

(27) \( \nu_{X i} \) : the logarithmic deviation of the export demand shock \( \ln \nu_{X i}^X(t) \) (which directly influences the total exports)

(28) \( \nu_{M i} \) : the logarithmic deviation of the import demand shock \( \ln \nu_{M i}^M(t) \) (which directly influences the total imports)

(29) \( \theta_{X i} \) : the logarithmic deviation of the export price markup shock \( \ln \theta_{X i}^X(t) \) (which directly influences the domestic currency denominated price of exports)

(30) \( \theta_{M i} \) : the logarithmic deviation of the import price markup shock \( \ln \theta_{M i}^M(t) \) (which directly influences the domestic currency denominated price of imports)

(31) \( \nu_{G I i} \) : the linear deviation of the public investment shock \( \psi_{G I i}^I(t) \) (which directly influences the public investment)

M. All Industry-Level Endogenous Variables

(1) \( Y_{k i} \) : the logarithmic deviation of the real output of the k-th industry of the i-th economy \( \ln Y_{1,k}(t) \)

(2) \( \tilde{Y}_{k i} \) : the logarithmic deviation of the potential output of the k-th industry of the i-th economy (the inferred industry-level total output given full utilization of private physical capital, effective labor and potential-output-level intermediate inputs from all domestic and foreign industries) \( \ln \tilde{Y}_{1,k}(t) \)

(3) \( X_{k i} \) : the logarithmic deviation of the total exports of the k-th industry of the i-th economy \( \ln X_{1,k}(t) \)

(4) \( M_{k i} \) : the logarithmic deviation of the total imports of the i-th economy from the k-th industry of all other economies \( \ln M_{1,k}(t) \)

(5) \( P_{Y k i} \) : the logarithmic deviation of the domestic currency denominated output price level for the output of the k-th industry of the i-th economy \( \ln \bar{P}_{1,k}(t) \)
(6) $P_{C_k_i}$: the logarithmic deviation of the domestic currency denominated consumption price level in the i-th economy for the final goods or services from the domestic and foreign k-th industry $\ln P_{C_k_i}^C(t)$

(7) $pi_{X_k_i}$: the linear deviation of the export price inflation for the exported output of the k-th industry of the i-th economy $\pi_{X_k_i}^X(t)$

(8) $P_{X_k_i}$: the logarithmic deviation of the price of exports for the exported output of the k-th industry of the i-th economy $\ln P_{X_k_i}^X(t)$

(9) $pi_{M_k_i}$: the linear deviation of the post-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies $\pi_{M_k_i}^M(t)$

(10) $P_{M_k_i}$: the logarithmic deviation of the post-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies $\ln P_{M_k_i}^M(t)$

(11) $pi_{Mt_k_i}$: the linear deviation of the pre-tariff import price inflation for the total imports of the i-th economy from the k-th industry of all other economies $\pi_{Mt_k_i}^{M,T}(t)$

(12) $P_{Mt_k_i}$: the logarithmic deviation of the pre-tariff price of imports for the total imports of the i-th economy from the k-th industry of all other economies $\ln P_{Mt_k_i}^{M,T}(t)$

(13) $(\tau_{M_k_i} - \tau_{tariff\_constant_k_i})$: the linear deviation of the import tariff rate for the total imports of the i-th economy from the k-th industry of all other economies $(\tau_{M_k_i}^M(t) - \tau_{Mt_k_i}^{M,T}(t))$, here $\tau_{Mt_k_i}^{M,T}$ is the tariff rate constant for the total imports of the i-th economy from the k-th industry of all other economies except the i-th economy in period t

(14) $ALPHA_{k_i}$: the logarithmic deviation of the labor productivity for the k-th industry of the i-th economy $\ln \Lambda_{k_i}^l(t)$

(15) $G_{I_k_i}$: the logarithmic deviation of the public investment for the k-th industry of the i-th economy $\ln G_{I_k_i}^I(t)$

(16) $K_G_{k_i}$: the logarithmic deviation of the public physical capital stock for the k-th industry of the i-th economy $\ln K_G_{k_i}^G(t)$

(17) $PI_S_{k_i}$: the logarithmic deviation of the corporate profits of the k-th industry of the i-th economy $\ln PI_S_{k_i}^S(t)$

(18) $V_S_{k_i}$: the logarithmic deviation of the domestic currency denominated price of corporate equity of the k-th industry of the i-th economy $\ln V_S_{k_i}^S(t)$

(19) $upsilon_S_{k_i}$: the logarithmic deviation of the weighted average of domestic and foreign equity risk premium for the k-th industry of the i-th economy (the domestic currency denominated price of corporate equity of the k-th industry of the i-th economy is adjusted by this industry-level equity risk premium) $\ln upsilon_S_{k_i}^S(t)$
(20) \( \text{delta}_C_{k_i} \) : the linear deviation of the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the \( i \)-th economy to firms of the \( k \)-th industry of the \( i \)-th economy \( \delta_{C_{k_i}}(t) \)

N. All Industry-Level Exogenous Variables

(1) \( \text{nu}_Y_{\text{input}_m\rightarrow j\rightarrow k_i} \) : the logarithmic deviation of the intermediate input shock for the intermediate goods or services from the \( m \)-th industry of the \( j \)-th economy to the \( k \)-th industry of the \( i \)-th economy \( \ln \nu_{Y_{\text{input}_m\rightarrow j\rightarrow k_i}}(t) \) (which directly influences the quantity of intermediate goods or services from the \( m \)-th industry of the \( j \)-th economy to the \( k \)-th industry of the \( i \)-th economy)

(2) \( \text{nu}_Y_{\text{output}_m\rightarrow j\rightarrow k_i} \) : the logarithmic deviation of the output demand shock for the intermediate or final goods or services from the \( k \)-th industry of the \( i \)-th economy that would serve as intermediate inputs for domestic and foreign production sector or final output goods or services for domestic and foreign absorption sector \( \ln \nu_{Y_{\text{output}_m\rightarrow j\rightarrow k_i\rightarrow all\_economies}}(t) \) (which directly influences the real output of the \( k \)-th industry of the \( i \)-th economy)

(3) \( \text{nu}_Y_{\text{output}_m\rightarrow j\rightarrow k_i\rightarrow j_m} \) : the logarithmic deviation of the output demand shock for the intermediate goods or services from the \( k \)-th industry of the \( i \)-th economy demanded by the \( m \)-th industry of the \( j \)-th economy \( \ln \nu_{Y_{\text{output}_m\rightarrow j\rightarrow k_i\rightarrow j_m}}(t) \) (which directly influences the intermediate goods or services output from the \( k \)-th industry of the \( i \)-th economy to the \( m \)-th industry of the \( j \)-th economy)

(4) \( \text{nu}_Y_{\text{output}_m\rightarrow j\rightarrow k_i\rightarrow i_k} \) : the logarithmic deviation of the output demand shock for the intermediate goods or services from the \( k \)-th industry of the \( j \)-th economy demanded by all industries of the \( i \)-th economy \( \ln \nu_{Y_{\text{output}_m\rightarrow j\rightarrow k_i\rightarrow i_k}}(t) \) (which directly influences the intermediate goods or services output from the \( k \)-th industry of the \( j \)-th economy to all industries of the \( i \)-th economy)

(5) \( \text{nu}_C_{j_k} \) : the logarithmic deviation of private consumption shock for final goods or services used for private consumption in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy \( \ln \nu_{C_{j_k}}(t) \) (which directly influences the quantity of final goods or services used for private consumption in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy)

(6) \( \text{nu}_I_{j_k} \) : the logarithmic deviation of private investment shock for final goods or services used for private investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy \( \ln \nu_{I_{j_k}}(t) \) (which directly influences the quantity of final goods or services used for private investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy)

(7) \( \text{nu}_G_{j_k} \) : the logarithmic deviation of public purchase shock for final goods or services used for public consumption or public investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy \( \ln \nu_{G_{j_k}}(t) \) (which directly influences the quantity of final goods or services used for public consumption or public investment in the \( j \)-th economy from the \( k \)-th industry of the \( i \)-th economy)
\(\theta_{Y_ki}\) : the logarithmic deviation of the output price markup shock for the output of the \(k\)-th industry of the \(i\)-th economy \(\ln \hat{\theta}_{Y_ki}(t)\) (which directly influences the domestic currency denominated output price of the \(k\)-th industry of the \(i\)-th economy)

\(\theta_{X_ki}\) : the logarithmic deviation of the export price markup shock for the exports of the \(k\)-th industry of the \(i\)-th economy \(\ln \hat{\theta}_{X_ki}(t)\) (which directly influences the domestic currency denominated export price for exports from the \(k\)-th industry of the \(i\)-th economy)

\(\theta_{M_ki}\) : the logarithmic deviation of the import price markup shock for the imports of the \(i\)-th economy from foreign \(k\)-th industry \(\ln \hat{\theta}_{M_ki}(t)\) (which directly influences the domestic currency denominated import price for imports of the \(i\)-th economy from the \(k\)-th industry of all other economies)

\(\nu_{\tau_Mk_i}\) : the linear deviation of the import tariff rate shock for the total imports of the \(i\)-th economy from the \(k\)-th industry of all other economies \(\nu_{\tau_Mk_i}(t)\) (which directly influences the import tariff rate for the total imports of the \(i\)-th economy from the \(k\)-th industry of all other economies)

\(\nu_{\text{ALPHA}_k_i}\) : the logarithmic deviation of the labor productivity shock for the production of the \(k\)-th industry of the \(i\)-th economy \(\ln \hat{\nu}_{\text{ALPHA}_k_i}(t)\) (which directly influences the labor productivity for the \(k\)-th industry of the \(i\)-th economy)

\(\nu_{G_I_k_i}\) : the linear deviation of the public investment shock for the \(k\)-th industry of the \(i\)-th economy \(\nu_{G_I_k_i}(t)\) (which directly influences the public investment for the \(k\)-th industry of the \(i\)-th economy)

\(\nu_{S_k_i}\) : the logarithmic deviation of the equity risk premium shock for the \(k\)-th industry of the \(i\)-th economy \(\ln \hat{\nu}_{S_k_i}(t)\) (which directly influences the domestic currency denominated equity price of listed companies in the \(k\)-th industry of the \(i\)-th economy)

\(\nu_{\text{delta}_C_k_i}\) : the linear deviation of the corporate loan default shock for the domestic currency denominated final corporate loans issued by global final banks of the \(i\)-th economy to firms of the \(k\)-th industry of the \(i\)-th economy \(\nu_{\text{delta}_C_k_i}(t)\) (which directly influences the domestic corporate loan default rate for the domestic currency denominated final corporate loans issued by global final banks of the \(i\)-th economy to firms of the \(k\)-th industry of the \(i\)-th economy)

### O. All Endogenous Variables for International Commodity Markets

\(P_{Ye}\) : the logarithmic deviation of the price of internationally homogeneous energy commodities denominated in US dollars in the international commodity market \(\ln \hat{P}_{Ye}(t)\)

\(P_{Yne}\) : the logarithmic deviation of the price of internationally homogeneous nonenergy commodities denominated in US dollars in the international commodity market \(\ln \hat{P}_{Yne}(t)\)
(3) \( \upsilon_{\text{TAU}} \): the logarithmic deviation of the global terms of trade shifter (which is adopted for international trade adjustment to ensure multilateral consistency in nominal trade flows in the international commodity markets) \( \ln \upsilon_{\text{TAU}}(t) \)

**P. All Exogenous Variables for International Commodity Markets**

(1) \( \theta_{Y_e} \): the logarithmic deviation of the energy commodity price markup \( \ln \theta_{Y_e}(t) \) (which directly influences the price of internationally homogeneous energy commodities denominated in US dollars in the international commodity market)

(2) \( \theta_{Y_{ne}} \): the logarithmic deviation of the nonenergy commodity price markup \( \ln \theta_{Y_{ne}}(t) \) (which directly influences the price of internationally homogeneous nonenergy commodities denominated in US dollars in the international commodity market)
References


