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A Large Open Economy Model to Assess Macroeconomic Policy Performance in Japan

Paul D. McNelis

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A Large Open Economy Model to Assess Macroeconomic Policy Performance in Japan

Prepared by Paul D. McNelis [1] [2] [3]

Authorized by Li Lian Ong

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Abstract

This paper develops and estimates a dynamic stochastic general equilibrium (DSGE) model representing several key characteristics of Japan, namely, a large open economy, large fiscal deficits with increasing amounts of debt held by domestic residents, as well as monopolistically-competitive pricing in traded goods. The economy is driven by recurring sets of shocks to productivity, government spending, quality of capital, foreign interest rates, domestic consumption, global demand and export and import markup price setting. The paper compares optimal simple rules for consumption tax rates, a Taylor rule with negative interest rates (and thus not subject to the zero lower bound) and a quantitative easing rule, for reducing government debt held by the banking system. All three rules do very well in mitigating the adverse effects of negative shocks. QE rules have a comparative advantage for reducing debt, while tax-rate rules have the advantage for stabilizing consumption in adverse periods. Both of these policy rules are close to the results prevailing in a Taylor-rule world with no zero bound, with sticky prices.

JEL classification: F34, E44, E52, G28, G32, P52

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[1] Author's e-mail: mcnelis@fordham.edu

[2] Paul D. McNelis is Visiting Professor of Economics at Boston College and Professor Emeritus of Finance and Business Economics at Fordham University. He has been an AMRO Research Fellow under the AMRO Collaboration Research Program funded by the Japan-AMRO Special Trust Fund. The views expressed in the paper are those of the author's and not necessarily the views of Boston College or Fordham University, which should not be held responsible for any consequence arising from the use of the information contained therein.

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Abbreviations

BOJ	Bank of Japan
CES	Constant Elasticity of Substitution
ECB	European Central Bank
GFC	Global Financial Crisis
HSD	Historical Shock Decomposition
MCMC	Monte Carlo Markov Chain

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Economics should be under no illusion that central banking will ever become a science. — Jürg Niehans, *The Theory of Money,* 1978, p. 296.

1 Introduction

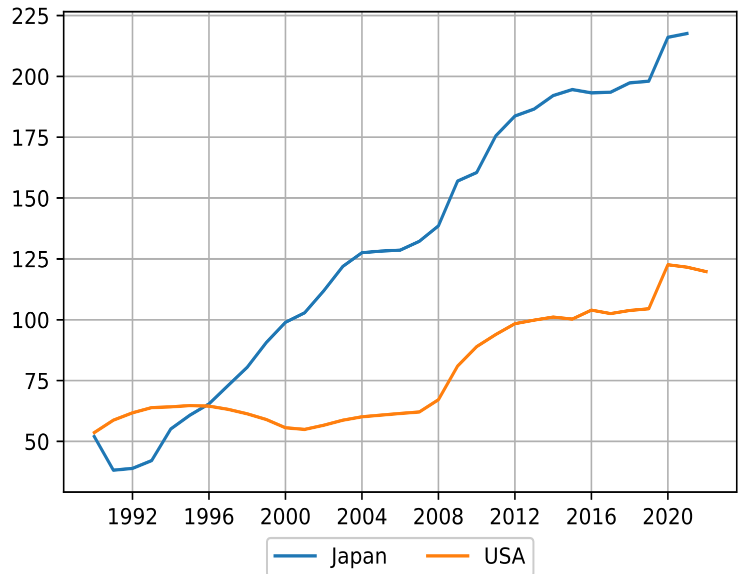
Japan has experienced several adverse and recurring negative shocks over the past two decades. Preceding these periods, the lost decade of the 1990's, with the prolonged recession and ensuing deflation, due to the collapse of the property price bubble, was just the beginning. In the succeeding decades, the effects of the aging population took its toll on the fiscal deficit. The sharp appreciation of the Yen due to uncertainty about the Euro and US Dollar later affected export competitiveness. The loss of life and destruction of capital brought on by the tsunami in March 2011 reduced prospects for a return to higher productivity. The COVID-19 episode came at the end of several difficult decades.

Given the adverse shocks of the past two decades, it is not surprising that the public debt of Japan has outstripped other higher indebted OECD countries.

Figure 1 pictures the gross debt/GDP of Japan. For reference this figure also shows the Debt/GDP ratio of the USA. We see that at the start of the sample in the 1990s, Japan had a lower debt-GDP ratio than that of the USA, but an upward trend from the mid-1990's has brought the current debt ratio to nearly twice as much as that of the USA.

Figure 1. Public Debt Ratios of Japan and USA

(Percent of GDP)

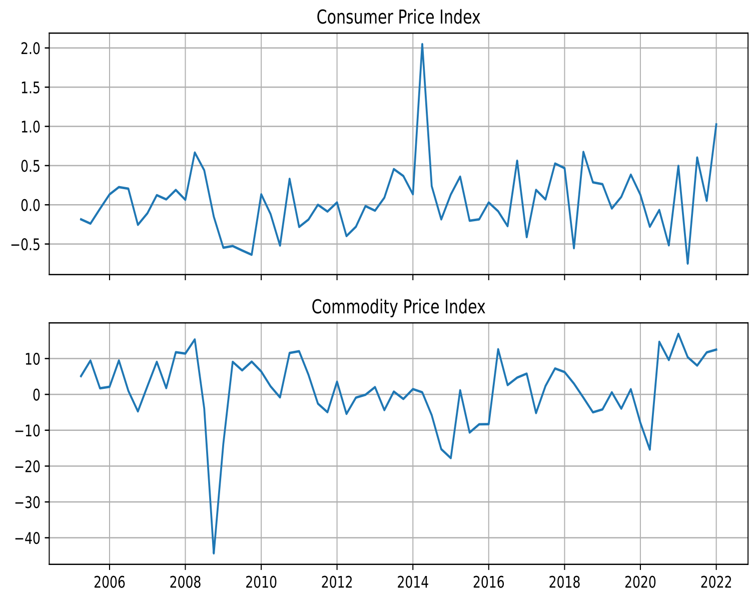


Source: Haver Analytics.

Japan has had episodes of deflation during the past two decades, as shown in Figure 2, after the Global Financial Crisis in 2008, and during the COVID-19 periods. We also see in Figure 2 that the drop in commodity prices after 2008 was correlated with the period of deflation. Similarly the rise in commodity prices after 2020 was correlated with a mild rise in inflation. However, the spike in inflation after 2014 was due to an increase in the sales tax.

Figure 2. Inflation

(Percent year-on-year)



Source: Haver Analytics.

One way to reduce public debt, in the face of declining tax revenue, is to cut government spending. However, the aging population, with the need for increasing government spending on health care and social services, has made this option more difficult, at least as the sole instrument, for fiscal consolidation.

The other option, of course, would be expansionary monetary policy, both to stimulate the economy and reduce the real value of debt through inflation. However, as Figure 3 shows, as economic growth has stagnated, the short-term interest rates have remained at the zero lower bound, For the sake of comparison, Figure 3 also shows the behavior of the US T bill rate over the same period. One of the key policy questions, of course, is how far does the Japanese interest rate deviate from the neutral inflation-target rate or the flexible-price *natural* rate of interest?

The other question, as suggested by Obstfeld (2023): does such a comparison really matter, or put another way, have underlying conditions changed for Japan that would warrant a higher interest rate? The benchmark natural rate of interest is, of course, model dependent. The natural rates of interest may differ between two calibrated DSGE models, one for an open economy subject to external shocks and the other for a closed economy without external shocks.

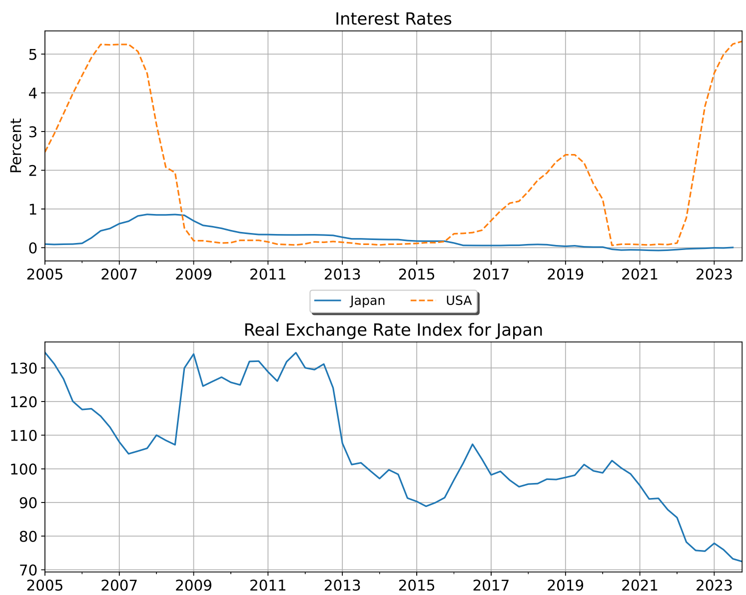
Holston et al. (2017) put forward an alternative approach for measuring the natural rate, based on evaluating trend components of interest rates with

Kalman filtering. These authors found that considerable co-movement of these natural rates across the United States, the Euro Area, Canada and the United Kingdom, suggesting that global rather than domestic factors play key roles for the evolution of the natural rate.

We also see sharp variations in the Real Exchange Rate index for Japan. A fall in this index represents a weakening of the Yen in real terms. It should not be surprising that this is happening at the end of the sample, when the US interest rates are far above the Japan short-term rates. We also see in these figures that as US interest rates came down at the time of the GFC, the Yen appreciated in real terms.

Figure 3. Interest and Exchange Rates

(Percent and Index)



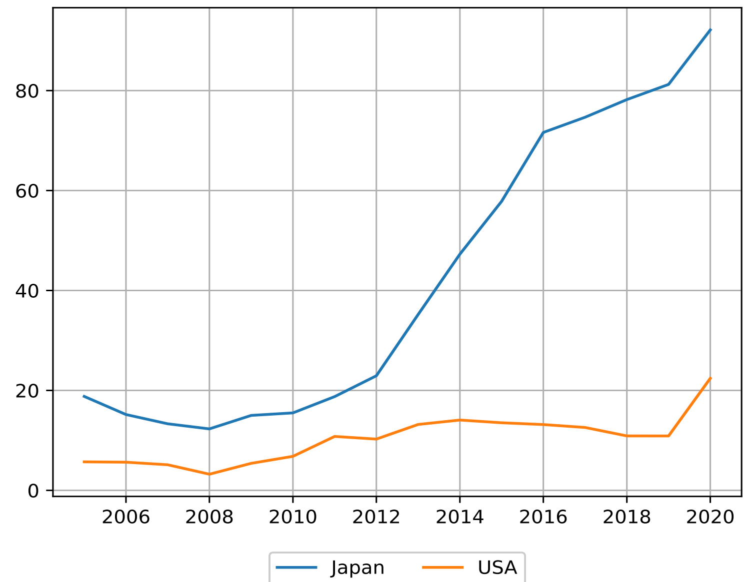
Source: Haver Analytics.

Given that the interest rates in Japan have hovered near the zero lower bound, Quantitative Easing (QE) policies have been in place, in the form of Bank of Japan purchases of public debt and other assets held by financial institutes, as the alternative to Taylor-rule monetary policy responses.

Figure 4 pictures the growth of BOJ asset holdings as a percentage of GDP. For the sake of comparative bench-marking, we also plot the same series for the US Federal Reserve.

Figure 4. Central Bank Asset Purchases: Japan and USA

(Percent of GDP)

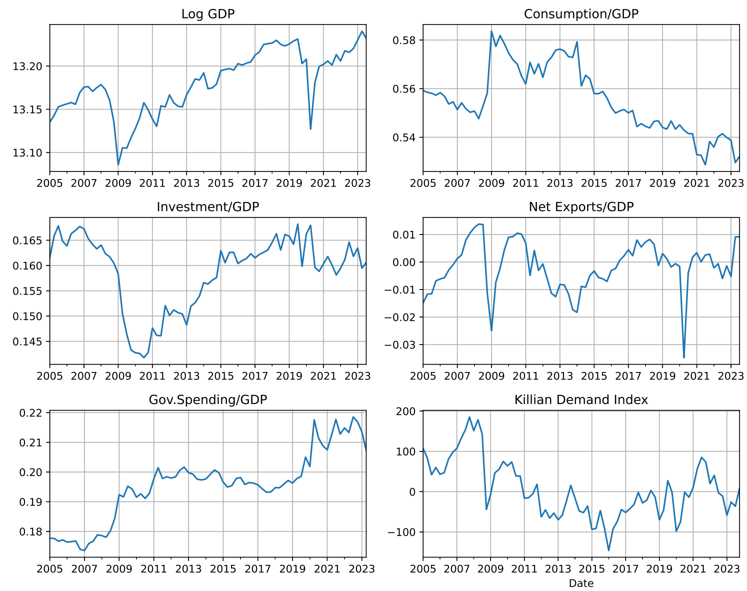


Source: Haver Analytics.

Figure 5 gives a picture of the real macro indicators of Japan, for seasonally adjusted quarterly data, since 2005.

This picture highlights a number of important facts. We plot the logarithm of real GDP, the ratios of private consumption, investment, government spending and net exports relative to GDP and an index of global demand put forward by Kilian (2009). We see that the drop in Net Exports was slightly worse at the time of COVID-19 than at the time of the GFC, but the recovery was faster in the later period.

Figure 5. Macroeconomic Indicators



Source: Haver Analytics.

As noted in a previous paper, McNelis and Yoshino (2016), Yoshino and Taghizadeh-Hesary (2014) have identified two of the three pillars of the Japan policy framework under Prime Minister Abe, as aggressive monetary policy, and fiscal consolidation through increases in the consumption tax rate. As shown above, the Bank of Japan has been aggressive in its asset purchases relative to the Federal Reserve, and as we saw in the inflation chart, in April 2014, the consumption tax rate was increased from 5 percent to 8 percent as part of the fiscal consolidation program. While this rate is small relative to the UK and many Euro Area countries, it is larger than the tax rates in many Asian economies, such as Hong Kong and, until recently, Singapore.

We also pointed out that aggressive monetary expansion began well before the Abe reform package. The first wave of QE in Japan took place in 2001 as effort to stimulate its economy, and was re-implemented in 2010, before the more aggressive policy implemented in 2013, in order to generate an uptick in inflation, as noted by Andolfatto and Li (2014). By contrast, Hayashi and Koeda (2013), making use of a regime-switching structural VAR model for Japan, found that increasing reserves through QE policies increased inflation and output, and terminating QE policies is not by necessity deflationary.

Given that the very high values of the debt/GDP ratio make fiscal expansion difficult, we concentrate in this paper on the Quantitative Easing (QE) and tax-rate rule for consumption as feasible instruments for stabilization in Japan.

As we note, these policies represent alternatives to the use of the short-term interest rate when this rate is at the zero lower bound. To be sure, we are not attempting to assess the actual programs for unconventional monetary policy (in the form of QE) or for the consumption tax-rate changes. These policies were implemented in discrete phases. Our purpose is to assess and compare the effects of optimal consumption tax-rate and optimal QE rules for adjustment, in the wake of prolonged stagnation and increasing government debt.

Of course, unconventional monetary policy, as noted by Sims (2010), involves the central bank engaging in quasi-fiscal monetary policy, in the form of debt management. Normally, the central bank operates at the short-end of maturity structures of debt, while Treasury manages the evolution of longer-term debt. Similarly the use of tax-rate rules as a substitute for the traditional Taylor rule, when the interest rate is at the zero lower bound, is a form of quasi-monetary fiscal policy. The temporary subsidy in the USA for purchases of new cars, for example, dubbed the cash-or-clunkers program, was equivalent to a temporary interest-rate cut. Both would have the effect of moving planned purchases forward in time, which otherwise would have taken place.[1]

In the quantitative easing rule, the central bank increases the reserves of the banking sector by partial buy-backs of the quantity of longer-term government debt held by this sector. In previous work on unconventional monetary policy in the United States, Gertler and Karadi (2011) analyzed optimal QE rules as purchases of private-sector securities by the central bank. However, later QE policies in the United States, known as QE2, involved the purchase of longer-term government bonds from bank and non-bank private financial institutions. Swanson (2011) compared these QE2 policies with Operation Twist in the 1960's, in which the Federal Reserve attempted to lower longer-term bond yields while keeping short-term yields higher (for the sake of the balance of payments account).Swanson (2011) found that the cumulative effects of the six QE2 policy announcements were to lower the long-term yields by 15 basis point, a result consistent with earlier evaluations of Operation Twist by Modigliani and Sutch (1967).

In previous work, Lim and McNelis (2016) compared optimal QE rules (involving purchases of private securities) with optimal tax-rate rules for consumption and labor income, in a closed-economy setting. In their framework, the government budget was balanced, so public debt played no role. What is new in this paper is that we derive optimal rule for QE and tax-rate changes as optimal rules, when there is a growing stock of government debt held by domestic banks, in an open-economy setting.

The use of a consumption tax-rate rule as a substitute for interest-rate changes when the rate is near or at the zero lower bound was noted by Correia et al. (2013). These authors point out that both the interest rate and the consumption tax rate affect the consumption Euler equation and thus directly affect intertemporal decisions on spending. Temporary tax rate changes, like temporary interest-rate changes, affect the decisions to buy now or defer expenditures for future periods in favor of higher saving.

[1]See Mian and Sufi (2012) for further analysis of this program.

We compare the performance of these policy-instrument rules in crisis times, when there are prolonged periods of stagnation and increasing debt. We derive optimal simple rules for QE and consumption tax rates based on welfare optimization and stabilization of public debt. However, as Mendoza (2010) points out, welfare comparisons assessing the effectiveness of such rules, more often than not, show little or no difference, since most of the time, in long simulations, the economy is close to the stochastic mean. We are interested in the performance of these optimal rule during crisis periods, when the economy is far from the steady state, in periods of prolonged stagnation and high debt levels. In a crisis event, to be sure, when it rains it pours, many bad shocks are realized, and key variables fall far away from their stochastic mean values. Of course, in these times, the economy is at the lower tail of the distribution. Optimal policy rules are computed on the basis of minimum variance. It should not be surprising that the performance of optimal rules make a greater difference during crisis events than in normal times under the usual welfare criteria.

This paper thus advocates continued aggressive quantitative easing policies as Japanese debt remains high, as well as the use of consumption tax-rate rules. Hoshi and Ito (2012) have pointed out that Japanese debt, so far, has been defying gravity and stress the dangers of delaying needed reform. Implementing optimal rules for consumption tax rates also remains important for long term efficiency of the economy.

The model of this paper represents a large open economy.This paper is in contrast to most, if not all, of recent DSGE models for Japan. Japan is not a large closed economy so we chose not to use the DSGE framework of Smets and Wouters (2007). But Japan is also not a small open economy, so we do not use the framework of Christiano et al. (2011) for Sweden. In particular we have to incorporate monopolistic competition in export-price setting as well as pricing-to-market in the domestic market for imports.

McNelis and Yoshino (2016) reviewed previous modeling work on Japan with the DSGE approach and we do not repeat it here. The next section summarizes the model. In our Bayesian framework, we do beyond the previous study with a more extensive analysis, in terms of the impulse response analysis, the forecast error variance, the historical shock decomposition, and the use of counterfactual simulations over the past two decades with the smoothed shocks. We also incorporate a larger number of observables and stochastic sources of uncertainty.

These models have concentrated on measurement of the output gap, the importance of stickiness in the labor market, the size of fiscal multipliers, and the relative importance of financial and technology shocks, balance sheet effects, and sunspot equilibria, on investment.

Fueki et al. (2010) used a two-sector framework for measuring potential output and the output gap. They incorporated both growth-rate shocks and investment-specific technology shocks. Potential output was defined in terms of output explained by the pure growth-rate shocks. These authors reported that this approach to potential output and the output gap was more effective for forecasting inflation than more conventional approaches.

More recently, the Bank of Japan has been following a reverse “Operation

Twist", called Yield Curve Control (YCC), in which it is trying to increase the long-term interest rates relative to the short term interest rates, which are now negative. While we do not model a yield curve in this model, we do examine non-traditional instruments, which, of course, affect both the policy rate and long-term returns on capital.

2 The Model

2.1 Households and Budget Constraint

The household consumption at time t, C_t is a Constant Elasticity of Substitution (CES) bundle of both domestic consumption goods, C_t^d and imported consumption goods, C_t^f.

$$C_t = \left[(1-\gamma_1)^{\frac{1}{\theta_1}} \left(C_t^d\right)^{\frac{\theta_1-1}{\theta_1}} + (\gamma_1)^{\frac{1}{\theta_1}} \left(C_t^f\right)^{\frac{\theta_1-1}{\theta_1}}\right]^{\frac{\theta_1}{\theta_1-1}} \tag{1}$$

The parameters γ_1 and $(1-\gamma_1)$ are the relative shares of foreign and domestic goods in the overall consumption index, while θ_1 is the price elasticity of demand for each consumption component.

The demand for each component of consumption is a function of the overall consumption index and the price of the respective component relative to the general price level, P.

$$C_t^d = (1-\gamma_1)\left(\frac{P_t^d}{P_t}\right)^{-\theta_1} C_t \tag{2}$$

$$C_t^f = \gamma_1\left(\frac{P_t^f}{P_t}\right)^{-\theta_1} C_t \tag{3}$$

We assume that government spending G_t is bundled with private consumption to represent effective consumption $\widetilde{C}_t$ through a CES aggregator. We do this to indicate that there is a reason for government spending to take place, that such spending creates externalities for consumption, in the form of services which enhance household marginal utility (such as law enforcement and communication services, as well as health-related services for an aging population). The following equation represents the effective consumption index:

$$\widetilde{C}_t = \left[\phi C_t^{\frac{\kappa-1}{\kappa}} + (1-\phi) G_{t-1}^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{4}$$

This approach was used by Pieschacon (2012) in an analysis of fiscal discipline for resource-exporting economies. The parameter ϕ represents the relative weights of private consumption and government spending for the effective consumption index. The parameter κ is the elasticity of substitution between private consumption and government purchases. As noted by the author, as $\kappa \to 0$, government spending and private consumption become perfect compliments, and as $\kappa \to \infty$,consumption and government spending become perfect substitutes. This formulation of effective consumption in utility, calibrated with a value of $\kappa < 1$, explains the pressures for continued government expenditure even in the fact of declining tax revenue, much as it explains the use of natural resource revenues to finance government spending in OPEC countries.

However, household utility does not simply come from the current effective consumption bundle. Rather, habit persistence applies to this consumption index when it enters the utility function, so that the relevant consumption index is deflated by the habit stock, H_t.

The habit stock is a function of the lagged average consumption bundle, raised to the power ϱ, the habit persistence parameter:

$$H_t = \widetilde{C}_{t-1}^{\varrho} \tag{5}$$

Overall utility is a positive function of the consumption bundle $\widetilde{C}_t$ relative to the habit stock, a negative function of labor L_t, and a positive function of real deposits M_t/P_t:

$$U(\widetilde{C}_t/H_t, , L_t, M_t/P_t) = \frac{\left(\widetilde{C}_t/H_t\right)^{1-\eta}}{1-\eta} - \gamma_t \frac{{L_t}^{1+\varpi}}{1+\varpi} + \mu \frac{\left(\frac{M_t}{P_t}\right)^{1-\vartheta}}{1-\vartheta} \tag{6}$$

The parameter η is the Constant Relative Risk Aversion (CRRA) coefficient, γ is the coefficient of the disutility of labor while the parameter μ is the coefficient of the utility of real deposits The parameters ϖ and ϑ affect the curvature of the utility function with respect to labor and real deposits. As these parameters approach unity, the relationship becomes progressively linear.

Domestically-produced goods are composed of both non-traded services C_t^h and home-produced traded goods C_t^x (some of which are consumed domestically).

The following CES aggregator is used for domestically-produced consumption goods:

$$C_t^d = \left[(1-\gamma_2)^{\frac{1}{\theta_2}} \left(C_t^h\right)^{\frac{\theta_2-1}{\theta_2}} + (\gamma_2)^{\frac{1}{\theta_2}} \left(C_t^x\right)^{\frac{\theta_2-1}{\theta_2}}\right]^{\frac{\theta_2}{\theta_2-1}} \tag{7}$$

The relative demands for the home non-traded goods and the export goods are given by the following equations:

$$C_t^h = (1-\gamma_2)\left(\frac{P_t^h}{P_t^d}\right)^{-\theta_2} C_t^d$$
$$C_t^x = \gamma_2\left(\frac{P_t^x}{P_t^d}\right)^{-\theta_2} C_t^d \tag{8}$$

The parameters γ_2 and $(1-\gamma_2)$ are the shares of the export and non-traded goods in domestic production of consumption goods, and θ_2 is the price elasticity of demand.

The domestically-produced price index is given by the following CES aggregation:

$$P_t^d = \left[(1-\gamma_2)\left(P_t^h\right)^{1-\theta_2} + \gamma_2\left(P_t^x\right)^{1-\theta_2}\right]^{\frac{1}{1-\theta_2}} \tag{9}$$

In the same manner, the overall price index is a CES function of the price of foreign and domestic consumption goods:

$$P_t = \left[(1-\gamma_1)\left(P_t^d\right)^{1-\theta_1} + \gamma_1\left(P_t^f\right)^{1-\theta_1}\right]^{\frac{1}{1-\theta_1}} \tag{10}$$

In addition to buying consumption goods, households put deposits M_t in the bank and receive dividends from all the producing firms as well as from banks. The variable Π_t represents total dividends, from the production of export and home goods, with $\Pi_t = \Pi_t^x + \Pi_t^h$. The representative household pays taxes on labor income $\tau W_t L_t$ and on consumption $\tau_c C_t$.

The household chooses the paths of consumption, labor, deposits, investment and capital, to maximize the present value of its utility function subject to the budget constraint and the law of motion for capital.

Thus, the intertemporal optimization of the household is given by the following expression:

$$\underset{\{C_t, L_t, M_t,\}}{Max} E_t \sum_{\iota=0}^{\infty} \beta^i U(\widetilde{C}_{t+i}/H_{t+i}, L_{t+i}, M_{t+i}/P_{t+1}) \tag{11}$$

The household budget is given by equation (12). To stabilize deposits, there are increasing adjustment costs whenever deposits diverge too far from their steady-state value, $\bar{M}$:

$$\begin{aligned} &W_t L + (1+R_{t-1}^m)M_{t-1} + \Pi_t \\ &= P_t C_t(1+\tau_c) + M_t + .5\varphi_M(M_t - \bar{M})^2 + \tau W_t L_t \end{aligned} \tag{12}$$

The optimization problem is expressed by the intertemporal Lagrangian: $\mathcal{L}_{\mathrm{t}}$:

$$\max_{\{C_t, L_t, M_t\}} \mathcal{L}_{\text{t}\sqcup} = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} U(\widetilde{C}_{t+i}/H_{t+i}, L_{t+i}, M_{t+1}/P_{t+1}) \\ -\Lambda_{t+\iota} \left[\begin{array}{c} P_{t+i}C_{t+i}(1+\tau^c) + M_{t+i} \\ +.5\varphi_M(M_{t+i} - \bar{M})^2 \\ -(1+R^m_{t-1+i})M_{t-1+i} \\ -(\tau^w - 1)W_{t+i}L_{t+i} - \Pi_{t+i} \end{array} \right] \end{array} \right\} \tag{13}$$

Optimizing the Lagrangian equation with respect to the decision variables C_t, L_t, M_t, yields the following set of First-Order Conditions for the representative household:

$$\Lambda_t P_t = \phi \left(\widetilde{C}_t\right)^{1-\kappa-\eta} (H_t)^{\eta-1} (C_t)^{-\kappa-1} \exp(z^c_t) \tag{14}$$

$$\gamma L_t^{\varpi} = \Lambda_t (1 - \tau^w) W_t \tag{15}$$

$$\mu \left(\frac{M_t}{P_t}\right)^{-\vartheta} = \Lambda_t \left[1 + \varphi_M \left(\frac{M_t - \bar{M}}{P_t}\right)\right] + \beta \Lambda_{t+1}(1 + R^m_t) \tag{16}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1}(1 + R^m_t) \tag{17}$$

The variable z^c_t is a shock to the marginal utility of consumption. It is the following stochastic autoregressive process:

$$z^c_t = \rho_c z^c_{t-1} + \epsilon^c_t \tag{18}$$

$$\epsilon^c_t \sim N(0, \sigma^2_c) \tag{19}$$

Equation (14) simply tells us that the marginal utility of wealth is equal to the marginal utility of consumption divided by the price level. The second equation, Equation (15) states that the marginal disutility of labor is equal to the marginal utility of consumption provided by the after-tax wage. The equation for the demand for deposits, 16, equates the marginal utility of money to the opportunity costs of holding money, including transactions costs, plus the expected future utility.

Equation (17) is the Keynes-Ramsey rule for optimal saving: the marginal utility of wealth today should be equal to the discounted marginal utility tomorrow, multiplied by the gross rate of return on saving (in the form of deposits). Under conditions of perfect wage/price flexibility rewriting equation

Equation (20) gives us the stochastic discount rate or natural rate of interest, which serves as a benchmark for assessing interest rates under wage and price markups and frictions, relative to a frictionless economy Under the latter setup, the stochastic discount rate is often used to approximate the natural rate of interest. Obstfeld (2023) and Benigno et al. (2024) present a summary of different approaches for measuring the natural rate and its usefulness as a monetary policy benchmark. Okasaki and Sudo (2018), as we do in this paper, present measures of this rate within a DSGE framework for Japan.

$$\varsigma_t = \frac{\Lambda_t}{\beta \Lambda_{t+1}} \tag{20}$$

2.2 Production and Technology

2.2.1 Nontraded Sector

The non-traded sector (encompassing services and agriculture in the form of rice production and other non-traded goods and services, particularly health care for an aging population) is simply a function of labor L^h and intermediate goods MI:

$$Y_t^h = MI_t^{\alpha_h} (L)^{1-\alpha_h} \tag{21}$$

The coefficient α_h represents the relative factor shares of intermediate goods in the production function of the final goods.

The demand for the home services can be both for domestic consumption, as well for government services:

$$Y_t^h = C_t^h + G_t \tag{22}$$

The variable G_t represents government spending. We assume that such spending takes the form of purchases of non-traded goods. We assume that government spending follows an autoregressive stochastic process:

$$\begin{aligned} G_t &= \overline{G} \cdot \exp(z_t^G) \\ z_t^G &= \rho_G z_{t-1}^G + \epsilon_t^G \\ \epsilon_t^G &\sim N(0, \sigma_G^2) \end{aligned} \tag{23}$$

where $\overline{G}$ is the steady state level of spending, and ρ_g is a smoothing parameter for spending.

We assume that the firm faces a liquidity or working-capital constraint. It must borrow an amount N_t^h from banks each quarter to pay a fraction μ_h of its wage bill, at the borrowing rate R_t^n:

$$N_t^h = \mu_h W_t L \tag{24}$$

The following identity represents the total profits (or dividends) of the home-goods producing firm:

$$\Pi_t^h = P_t^h Y_t^h - (1 + \mu_h R_t^n) W_t L_t^h - P_t^{mi} MI_t \tag{25}$$

where P_t^{mi} is the price of intermediate goods. Maximizing profits with respect to the use of labor and intermediate goods, we have the following first-order conditions for the firm:

$$\begin{aligned} \frac{\partial Y_t^h}{\partial L_t^h} &= (1 + \mu_h R_t^n) \frac{W}{P_t^h} \\ \frac{\partial Y_t^h}{\partial MI_t} &= \frac{P_t^{mi}}{P_t^h} \end{aligned} \tag{26}$$

As with investment goods, we assume intermediate goods MI are both domestically produced and imported from abroad, and that the price P^i is the relevant price for these goods. The investment variable is a CES aggregate of these two investment goods:

$$MI_t = \left[(1 - \gamma_{mi})^{\frac{1}{\theta_{mi}}} \left(MI_t^d \right)^{\frac{\theta_{mi}-1}{\theta_{mi}}} + (\gamma_{mi})^{\frac{1}{\theta_{mi}}} \left(MI_i^f \right)^{\frac{\theta_{mi}-1}{\theta_{mi}}} \right]^{\frac{\theta_{mi}}{\theta_{mi}-1}} \tag{27}$$

The parameters γ_{mi} and $(1 - \gamma_{mi})$ are the relative shares of foreign and domestic goods in the overall investment index, while θ_{mi} is the price elasticity of demand for each investment component.

The demand for each intermediate good component is a function of its relative price:

$$\begin{aligned} MI_t^d &= (1 - \gamma_{mi}) \left(\frac{P_t^x}{P_t^{mi}} \right)^{-\theta_{mi}} MI_t \\ MI_t^f &= \gamma_{mi} \left(\frac{P_t^f}{P_t^{mi}} \right)^{-\theta_{mi}} MI_t \end{aligned} \tag{28}$$

The index P_t^f is the price of imported goods, in domestic currency, while P_t^x is the price of domestic goods-producing forms (which can be exported, or used for domestic consumption and domestic investment). The overall price index for investment goods is given by the following equation:

$$P_t^{mi} = \left[(1 - \gamma_{mi}) \left(P_t^x \right)^{1-\theta_{mi}} + \gamma_{mi} \left(P_t^f \right)^{1-\theta_{mi}} \right]^{\frac{1}{1-\theta_{mi}}} \tag{29}$$

This paper develops and estimates a dynamic stochastic general equilibrium (DSGE) model representing several key characteristics of Japan, namely, a large open economy, with large fiscal deficits and increasing amounts of debt held by domestic residents, as well as monopolistically-competitive pricing in traded goods. The economy is driven through recurring sets of shocks to productivity, commodity prices, labor force participation, domestic consumption, and global demand.

We compare optimal simple rules for consumption tax rates, a Taylor rule with negative interest rates, and a quantitative easing rule, for reducing government debt held by the banking system, as well as optimizing welfare. In times of crisis, we show that the QE policy rule outperforms optimally-derived simple tax-rate rules or Taylor rules with negative interest rates for mitigating the costs of post-crisis adjustment and debt overhang.

Both prices are subject to monopolistically competitive price setting which we discuss below.

2.2.2 Traded Goods

The firms producing goods for export, as well as traded goods for domestic consumption as well as investment and intermediate goods, face a Cobb-Douglas technology, based on labor in this sector, capital and the same labor-augmenting shock which affects production in the non-traded sector, Z_t.

$$Y_t^x = exp(z_t) K_t^{\alpha_x} (L)^{1-\alpha_x} \tag{30}$$

There are shocks to total factor productivity, given by z_t. These shocks following an autoregressive process:

For total factor productivity, we have:

$$z_t = \rho_z z_{t-1} + \epsilon_t^z \tag{31}$$

The shock process is normally distributed:

$$\epsilon_t^z \sim N(0, \sigma_z^2) \tag{32}$$

The foreign demand for exports from the economy is a function of world demand and the relative price of the export good ($P_t^x/S_{t,}$ in foreign prices) relative to the world price P^w. The variable S_t is the exchange rate, and X^* is a proxy for foreign total consumption demand.[2]:

$$X_t = \left(\frac{P_t^x/S_t}{P^w} \right)^{-\theta x} X_t^* \tag{33}$$

[2]The functional form for foreign demand for domestically-produced traded goods follows the same rationale as domestic demand for foreign-produced consumption goods, given by equation 3.

We assume that the world demand follows an autoregressive process:

$$X_t^* = \overline{X}^* exp(z_t^X) \tag{34}$$

$$z_t^X = \rho_X z_{t-1}^X + \epsilon_t^X \tag{35}$$

$$\epsilon_t^X \sim N(0, \sigma_X^2) \tag{36}$$

For simplicity, the world price is normalized at unity. We also set world demand X_t^* at unity. Changes in the demand for exports come from changes in the world export price as well as changes in the exchange rate.

Total demand for the export good is composed of the local demand (for consumption purposes and investment and intermediate goods) as well as the foreign demand:

$$Y_t^x = C_t^x + X_t + I_t^d + MI_t^d \tag{37}$$

These firms face a liquidity constraint for meeting their wage bill:

$$N_t^x = \mu_x W_t L \tag{38}$$

The profits of the export-goods firms are given by the following relation:

$$\Pi_t^x = P_t^x Y_t^x - (1 + \mu_x R_t^n) W_t L_t^x - R_t^k K_t \tag{39}$$

Optimizing profits implies the following first-order condition for cost minimization:

$$\begin{aligned} \frac{\partial Y_t^x}{\partial L_t^x} &= (1 + \mu_x R_t^n) \frac{W}{P_t^x} \\ \frac{\partial Y_t^x}{\partial K_t^x} &= \frac{R_t^k}{P_t^x} \end{aligned} \tag{40}$$

2.2.3 Labor Mobility

We assume that labor can move between the home-goods and export sectors. This implies the following equality for real labor productivity in each sector:

$$\frac{\partial Y_t^x}{\partial L_t^x} \frac{P_t^x}{(1 + \mu_x R_t^n)} = \frac{\partial Y_t^h}{\partial L_t^h} \frac{P_t^h}{(1 + \mu_h R_t^n)} \tag{41}$$

2.2.4 Importing Firms

Imported goods Y^f are used for both consumption C^f and for investment in the goods-producing firms, I^f as well as intermediate goods MI^f for the non-traded sector:

$$Y_t^f = C_t^f + I_t^f + MI_t^f \tag{42}$$

The importing firms do not produce these goods. However, they have to borrow a fraction μ_f of the cost of these imported goods in order to bring them to the home market for domestic consumers and investors:

$$N_t^f = \mu_f (S_t P_t^{f^*} Y_t^f) \tag{43}$$

where $P_t^{f^*}$ is the world price of the import goods. The domestic marginal cost of the imported goods is given by:

$$AF_t = (1 + \mu_f R_t^n)\, S_t P_t^{f^*} \tag{44}$$

2.2.5 Calvo Wage and Price Setting

Wages are modeled as staggered contracts with a fraction $(1 - \xi_w)$ renegotiated each period by households. This fraction j chooses the optimal wage W_t^o by maximizing the expected discounted utility subject to the demand for its labor:

$$L_t^j = \left(\frac{W_t^o}{W_t} \right)^{-\zeta_w} L_t \tag{45}$$

where ζ_w is a parameter governing the degree of substitution.

$$\begin{aligned} W_t^{num} &= (W_t)^{\zeta_w + \zeta_w \varpi} \left(L_t^{1+\varpi} \right) + \xi_w \beta . W_{t+1}^{num} \\ W_t^{den} &= \left[\phi \left(\widetilde{C}_t \right)^{1-\kappa-\eta} (H_t)^{\eta-1} (C_t)^{-\kappa-1} Z_t^c \right] (W_t)^{\zeta} L_t + \xi_w \beta . W_{t+1}^{den} \end{aligned} \tag{46}$$

$$\begin{aligned} (W_t^o)^{1+\zeta_w \varpi} &= \frac{W_t^{num}}{W_t^{den}} \\ W_t &= \left[\xi_w (W_{t-1})^{1-\zeta_w} + (1-\xi_w)(W_t^o)^{1-\zeta_w} \right]^{\frac{1}{1-\zeta_w}} \end{aligned} \tag{47}$$

The symbols W_t^{num} and W_t^{den} represent auxiliary variables in the formulae.

We assume Calvo (1983) pricing for home goods, as well as for imported goods and for exported traded goods, as well as for wage setting. The reason why we make this assumption is to capture Japan as a *large* open economy. In addition to domestic producers exercising monopolistic pricing power in the home-goods sector, foreign exporters exercise monopolistic pricing power in Japan to maintain market share, and Japanese exporters exercise monopolistic pricing power in the foreign markets to maintain market share.

We assume monopolistically competitive firms in the non-traded sector. Let the marginal cost for non-traded home goods at time t be given by the following expression:

$$AH_t = \frac{\left(P^{mi}\right)^{\alpha_h}\left[(1+\mu_1 R_t^n)W_t\right]^{1-\alpha_h}}{Z_t^h} \cdot \frac{1}{\left(\alpha_h\right)^{\alpha_h}\left(1-\alpha_h\right)^{1-\alpha_h}} \tag{48}$$

In the Calvo price setting world, there are forward-looking price setters and backward looking setters. Assuming at time t a probability of persistence of the price at ξ , with demand for the product from firm j given by $Y_t^h\left(P_t^h\right)^{\zeta}$, the expected marginal cost, in recursive formulation, is presented by the expression for A_t^{num}. The expected demand, for the given price, is given by the variable AH_t^{den}.

$$\begin{aligned}
AH_t^{num} &= Y_t^h\left(P_t^h\right)^{\zeta} A_t + \beta\xi A_{t+1}^{num} \\
AH_t^{den} &= Y_t^h\left(P_t^h\right)^{\zeta} + \beta\xi A_{t+1}^{den} \\
P_t^{h,o} &= \frac{A_t^{num}}{A_t^{den}} \\
P_t^h &= \left[\xi_h\left(P_{t-1}^h\right)^{1-\zeta} + (1-\xi_h)\left(P_t^{h,o}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}}
\end{aligned} \tag{49}$$

Calvo pricing for imported goods works in a similar way to Calvo pricing for home goods, $P_{t.}^h$ Given the marginal cost of imported goods, AF_t, the following recursive setup gives us the price setting behavior for imported goods:

$$\begin{aligned}
AF_t^{num} &= Y_t^f\left(P_t^t\right)^{\zeta} AF_t + \beta\xi AF_{t+1}^{num} \\
AF_t^{den} &= Y_t^f\left(P_t^f\right)^{\zeta} + \beta\xi AF_{t+1}^{den} \\
P_t^{f,o} &= \frac{AF_t^{num}}{AF_t^{den}} exp(z_t^f) \\
P_t^{f,b} &= P_{t-1}^f \\
P_t^f &= \left[\xi_i\left(P_t^{f,b}\right)^{1-\zeta} + (1-\xi_i)\left(P_t^{f,o}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}}
\end{aligned} \tag{50}$$

We also assume monopolistic pricing for domestically-produced traded goods. The marginal cost of these goods is given by the following expression:

$$AX_t = \frac{\left(R_t^k\right)^{\alpha_x}\left[(1+\mu_x R_t^n)W_t\right]^{1-\alpha_x}}{Z_t^x} \cdot \frac{1}{(\alpha_x)^{\alpha_x}(1-\alpha_x)^{1-\alpha_x}} \tag{51}$$

In a symmetric manner with the pricing of home goods, we use the following recursive setup:

$$\begin{aligned}
AX_t^{num} &= Y_t^x \left(P_t^x\right)^{\zeta} AX_t + \beta\xi AX_{t+1}^{num} \\
AX_t^{den} &= Y_t^x \left(P_t^x\right)^{\zeta} + \beta\xi AX_{t+1}^{den} \\
P_t^{x,o} &= \frac{A_t^{num}}{A_t^{den}} \exp(z_t^*) \\
P_t^{x,b} &= P_{t-1}^x \\
P_t^x &= \left[\xi_x \left(P_t^{x,b}\right)^{1-\zeta} + (1-\xi_x)\left(P_t^{x,o}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}}
\end{aligned} \tag{52}$$

The terms z_t^f, z_t^* represent markup shocks to the pricing of imported and export goods sold in the home market. The shocks follow the same stochastic autoregressive processes:

$$z_t^f = \rho_f z_{t-1}^f + \epsilon_t^f \tag{53}$$

$$z_t^* = \rho_* z_{t-1}^* + \epsilon_t^* \tag{54}$$

$$\epsilon_t^f \sim N(0, \sigma_f^2) \tag{55}$$

$$\epsilon_t^* \sim N(0, \sigma_*^2) \tag{56}$$

2.3 The Financial Sector

The financial sector consists of banks who accept deposits and lend to firms, as well as to the government and borrow or lend internationally.

2.3.1 Banks

Following Gertler and Karadi (2011) and Dedola et al. (2013), we assume a subset of householders are bankers.

Banks own and rent capital K for rental to firms which produce traded-goods. Capital for rental to the firms depreciates at the rate δ. When bankers accumulate or decumulate capital beyond the steady state level, they pay adjustment costs. The following law of motion is specified for capital, with adjustment costs given by AC_t, and ϕ is the adjustment cost parameter. The variable $\overline{K}$ is the steady state level of the capital stock for domestic goods producing firms and I_t is investment.

Capital accumulation has the following law of motion and adjustment costs:

$$\begin{aligned} K_t &= (1-\delta)exp(z_t^i)K_{t-1} + I_t \\ AC_t &= \left(\frac{\phi\left(I_t - \delta\overline{K}\right)^2}{2K_t}\right) \end{aligned} \tag{57}$$

The stochastic shock to the quality of capital, given by the term z_t^i, has the following specification:

$$\begin{aligned} z_t^i &= \rho_i z_{t-1}^i + \epsilon_t^i \\ \epsilon_t^i &\sim N(0, \sigma_i^2) \end{aligned} \tag{58}$$

Shocks to the quality of capital, given by the term Z^i have been incorporated into several models of Japan. This formulation follows Gertler and Karadi (2011). As these authors note, such a shocks affect obsolescence or depreciation of capital. We thus have two types of shocks to factors of production, one to the productivity of labor, and one to the quality of capital. Given the changing demographics of Japan and the natural disasters involving the physical destruction of capital, both of these shocks are relevant to our model for Japan.

Since Japan is a large economy, we assume that investment goods are both domestically produced and imported from abroad, and that the price P^i is the relative price for these goods. The investment variable is a CES aggregate of these two investment goods:

$$I_t = \left[(1-\gamma_i)^{\frac{1}{\theta_i}}\left(I_t^d\right)^{\frac{\theta_i-1}{\theta_i}} + (\gamma_i)^{\frac{1}{\theta_i}}\left(I_i^f\right)^{\frac{\theta_i-1}{\theta_i}}\right]^{\frac{\theta_i}{\theta_i-1}} \tag{59}$$

The parameters γ_i and $(1-\gamma_i)$ are the relative shares of foreign and domestic goods in the overall investment index, while θ_i is the price elasticity of demand for each investment component.

The demand for each investment component is a function of its relative price:

$$I_t^d = (1-\gamma_i)\left(\frac{P_t^x}{P_t^i}\right)^{-\theta_i} I_t$$
$$I_t^f = \gamma_i\left(\frac{P_t^f}{P_t^i}\right)^{-\theta_i} I_t \tag{60}$$

The index P_t^f, equal to $S_t P_t^{f^*}$ is the price of imported goods, in domestic currency, while P_t^x is the price of domestic traded goods-producing firms (which can be exported, or used for domestic consumption and domestic investment). The overall price index for investment goods is given by the following equation:

$$P_t^i = \left[(1-\gamma_i)\left(P_t^x\right)^{1-\theta_i} + \gamma_i\left(P_t^f\right)^{1-\theta_i}\right]^{\frac{1}{1-\theta_i}} \tag{61}$$

Banks also lend to all three types of firms for working capital, to pay for labor costs:

$$N_t = N_t^x + N_t^h + N_t^f \tag{62}$$

In addition to these firms, the banks lend to the government B_t^g and receive a interest rate $R_{t.}^g$. They can also borrow and lend in a Fed-funds market, B_t^{ff} at a risk free Fed-Funds rate, R_t. This assumption, in which we differentiate the risk-free rate R_t from the government bond yield R_t^g, is especially relevant to the case of Japan. While the risk-free Fed funds rate has been at the zero lower bound, the government bond yields have not always have hit the lower bound.

We assume that accumulation of government bonds and foreign debt involve adjustment costs, when their levels falling above or below their steady-state levels. As Schmitt-Grohe and Uribe (2003) noted, assuming adjustment costs on foreign debt accumulation is one way to close open-economy models. We assume that for government debt, these costs take the form of a term premium for longer-term government debt.

In addition to paying deposits the interest rate R_t^m we assume that banks are also required to set aside a required ratio of reserves on outstanding deposits, $\phi_M M_t$. The relevant opportunity cost of holding these reserves is of course the amount the banks can earn by holding risk-free government bonds, $\phi_M R_t M_{t.}$ In addition, banks are required to set aside a fraction of capital against their outstanding loans, $\phi_{N,t} N_{t.}$. As in the case of the required reserves against deposits, the opportunity cost is given by $\phi_N R_t N_{t.}$

The banks also receive federal funds (or reserves) from the central bank, but of course have to pay interest rate at the risk free rate R_t. The following equation gives the gross profit of the banking sector.

$$\begin{aligned}
\Pi_t^B = {} & (1+R_t^k)K_t - P_{t+i}^i I_{t+i} - P_{t+i}^i \frac{\phi\left(I_{t+i} - \delta\overline{K}\right)^2}{2K_{t+i}^x} \\
& + (1+R_t^g)B_t^g - B_{t+1}^g + (1+R_t^n)N_t - N_{t+1} + S_{t+1}B_{t+1}^f - (1+R_t^*)B_t^f S_t \\
& - (1+R_t^m)M_t + M_{t+1} - \phi_M R_t M_t - \phi_N R_t N_t \\
& - .5\varphi_g(B_t^g - \bar{B}^g)^2 - .5\varphi_f S_{t-1}(B_t^f - \bar{B}^f)^2 \\
& - (1+R_t)FF_t + FF_{t+1}
\end{aligned} \tag{63}$$

The banking sector receives cash inflow from interest payments capital leased to firms, and incurs costs from purchasing new investment goods as well as adjustment costs on new investment. It also receives returns from outstanding loans to the government $(1+R_t^g)B_t^g$ and firms $(1+R_t^n)N_t$, as well as new cash deposits and borrowing from abroad, given by M_{t+1} and $S_{t+1}B_{t+1}^t$, where S_t is the exchange rate. It also receives injections of federal funds from the central bank, FF_{t+1}. The cash outflows of the private banking sector take the form of purchases of new government bond issues (B_{t+1}^g) , new loans to the private sector (N_{t+1}), gross interest payments on foreign debt $(1+R_t^*)B_t^f S_t$, as we as on deposits, $(1+R_t^m)M_t$, as well as interest rates on outstanding federal funds, $(1+R_t)FF_t$. The balance sheet also takes into account the costs of holding reserves on deposits as well as the costs of capital/asset ratios, respectively, given by $\phi_M R_t M_t$and $\phi_N R_t N_t$, as well as the adjustment costs for public and foreign debt diverging from steady-state levels, represented by $.5\varphi_g(B_t^g - \bar{B}^g)^2$ and $.5\varphi_f(B_t^f - \bar{B}^f)^2$.

The bank maximizes the present discounted value of its profits, given by V_t^B, with respect to its portfolio of assets (purchasing and accumulating capital, extending loans to the government and firms, B_{t+1}^g and N_{t+1}) and liabilities (deposits from households, borrowing from foreign financial centers, and from the central bank, given by M_{t+1} ,B_{t+1}^f, FF_{t+1} :

$$\max_{\{K_{t+1}, I_t, B_{,t+1}^g, N_{t+1}, M_{t+1}, B_{t+1}^f, FF_{t+1}\}} V_t^B = \Pi_t^B + \beta V_{t+1}^B \tag{64}$$

This intertemporal optimization is subject to the law of motion for capital goods, given by the system of equation system (57):

The first-order conditions lead to the familiar expressions for Tobin's Q and optimal investment, the spreads for interest rates, and the interest-parity equation:

$$
\begin{aligned}
Q_t &= \beta E_t\left(\Lambda_{t+1}\left(R^k_{t+1} + \beta P^i_{t+1}\frac{\left(\phi\left[I_{t+1} - \delta\overline{K}\right]\right)^2}{2\left(K_t\right)^2}\right) + Q_{t+1}(1-\delta)\right)\\
I_t &= \delta\overline{K} + \frac{K_t}{\phi}\left(\frac{Q_t}{\Lambda_t} - P^i_t\right)\\
1+R_t &= 1+R^n_t - \phi_N\\
1+R_t &= 1+R^m_t + \phi_M\\
1+R_t &= 1+R^g_t - \varphi_g(B^g_{t+1} - \bar{B}^g)\\
(1+R_t)S_t &= [1+R^f_t + \varphi_g(B^g_{t+1} - \bar{B}^g)]S_{t+1}\\
\frac{S_{t+1}}{S_t} &= \frac{1+R_t + \varphi_g(B^g_{t+1} - \bar{B}^g)}{1+R^r_t + \varphi_f(B^f_{t+1} - \bar{B}^f)}
\end{aligned}
\tag{65}
$$

We assume that the foreign interest rate evolves according to the following stochastic process, where $\bar{R}^f$ represents the steady-state foreign rate of interest:

$$R^f_t = \bar{R}^f exp(z^{R^r}_t) \tag{66}$$

$$z^{R^f}_t = \rho_{R^f} z^{R^f}_{t-1} + \epsilon^{R^f}_t \tag{67}$$

$$\epsilon^{R^f}_t \sim N(0, \sigma^2_{R^f}) \tag{68}$$

2.3.2 Monetary Policy

We assume that the monetary policy, in normal times, follows an inflation-targeting regime based on the familiar Taylor rule, or is constant, slightly above the zero lower bound, in crisis times:

$$
\begin{aligned}
R_t &= \rho_r R_{t-1} + (1-\rho_\pi)\rho_\pi\widehat{\pi}_t + (1-\rho_\pi)\rho_y\widehat{y}_t + (1-\rho_r)\overline{R}\\
&= \bar{R}
\end{aligned}
\tag{69}
$$

The coefficients ρ_r and ρ_π are, respectively, the smoothing parameter and the inflation and output-gap coefficients, with $0 < \rho_r < 1$ and $\rho_\pi > 1, \rho_y > 0$. The variable $\overline{R}$ is the steady state interest rate, equal to the steady state foreign interest rate $\overline{R}^f$, while $\widehat{\pi}_t$ is the deviation of actual inflation from the target rate of inflation and $\widehat{y}_t$ is the deviation of output growth from the target growth rate.

Given that the central bank sets the interest rate, it provides Federal Funds to, or takes reserves from, the banking sector, to ensure banking-sector solvency.

$$\Delta FF_t = P^i_{t+i} I_{t+i} + P^i_{t+i} \frac{\phi\left(I_{t+i} - \delta\overline{K}\right)^2}{2K^x_{t+i}} - (1 + R^k_t) K_t$$
$$+ N_t + B_t + (1 + R^*_t + \Phi_{t-1}) B^f_{t-1} S_{t-1} \tag{70}$$
$$+ (1 + R_t - \phi_M - \phi_M R_t) M_{t-1} - B^f_t S_t$$
$$- (1 + R_t + \phi_N - \phi_N R_t) N_{t-1} - M_t - (1 + R_t) B_{t-1}$$

In a Quantitative Easing QE_t policy, the monetary authority increases the reserves of the banking sector by purchasing bonds, thus replacing a fraction of the banking-sector portfolio holding of government interest-bearing assets with reserves.

$$QE_t = \psi_t(B_{t-1}) \tag{71}$$

The QE parameter, ψ_t,is time-varying: The QE parameter, ψ_t, is time-varying:

$$\begin{aligned} \psi_t &= \rho_\psi \psi_{t-1} + (1 - \rho_\psi)\overline{\psi} + (1 - \rho_\psi)\rho_{\psi B}\phi^B_{t-1} + (1 - \rho_\psi)\rho_{\psi D}\phi^D_{t-1} \\ 0 &< \rho_\psi < 1 \\ \rho_{\psi B} &> 0 \\ \rho_{\psi D} &> 0 \end{aligned} \tag{72}$$

The variables ϕ^B_t and ϕ^D_trepresent the government debt and primary government deficit/GDP ratios relative to target ratios:

$$\phi^B_t = \frac{B_t}{P_t Y_t} - \overline{\phi}^B \tag{73}$$

$$\phi^D_t = \frac{P^h_t G_t - TAX_t}{P_t Y_t} - \overline{\phi}^D \tag{74}$$

To be sure, as Gertler and Karadi (2011) and Dedola et al. (2013) explored, QE policies can also involve the purchase of private-sector capital owned by the banking system. In this case the returns on such capital, including capital gains, would accrue to the central bank. These authors assume that such returns are used by the consolidated government sector to finance additional spending. In our QE framework, the government debt is purchased by the central bank in return for additional holdings of federal funds.

2.3.3 Domestic and Foreign Debt

The government takes in taxes from the households and engages in spending on non-traded services. The government budget constraint takes the following form, in nominal terms:

$$TAX_t = \tau^w W_t L_t + \tau_t^c P_t C_t$$
$$B_t^g = (1 + R_{t-1}^g) B_{t-1}^G + P_t^h G_t - TAX_t \tag{75}$$

However the government debt held by the banking system is given by the following expression, which of course is different from total debt when quantitative easing is taking place:

$$B_t^g = (1 + R_{t-1} - \psi) B_{t-1}^g + P_t^h G_t - TAX_t \tag{76}$$

For the optimal simple rule for the tax rate on consumption, we specify the following adjustment, similar to the adjustment specified in equation 72. Both rules have a smoothing coefficient as well as a response to both the debt and deficit to GDP ratios:

$$\tau_t^c = \rho_{\tau^c} \tau_{t-1}^c + (1 - \rho_{\tau^c}) \overline{\tau^c} + (1 - \rho_{\tau^c}) \rho_{\tau^c B} \phi_{t-1}^B + (1 - \rho_{\tau^c}) \rho_{\tau^c D} \phi_{t-1}^D \tag{77}$$

$$0 < \rho_{\tau^c} < 1$$
$$\rho_{\tau^c B} > 0$$
$$\rho_{\tau^c D} > 0$$

We find the optimal values for the coefficients on the basis of a linear quadratic control problem, in which we minimize the discounted quadratic terms for utility, the debt/gdp and deficit/gdp ratios, with weights of .1 on utility and .4 on the debt and deficit ratios. We also add a penalty term for changes in the the QE parameter ψ_t and the consumption tax rate τ_c. The solution for the optimal rules were fairly robust with respect to the choice of weights for the objectives.

The aggregate foreign borrowing or asset accumulation evolves through the following identity:

$$S_t B_t^f = [1 + R_{t-1}^*] S_t B_{t-1}^f + P_t^f (C_t^f + I_t^f) - P_t^x X_t \tag{78}$$

3 Calibration and Bayesian Estimation

We calibrate one subset of parameters, namely, those which affect the steady-state and long-run properties of the model. We use Bayesian estimation for the parameters which affect the stochastic processes and the dynamics of the model, such as the Calvo parameters, as well as the standard deviations for the shocks to labor productivity, marginal cost pricing, and government spending.

3.1 Parameters

We calibrate the parameters in accordance with the steady state by using the Japanese data from the beginning of 2005 through the end of 2022. The parameters which affect the dynamics of the system for the shocks as well as for the price-setting behavior, as well as the standard deviations of the stochastic shocks in the model, are obtained from Bayesian estimation for the same sample period. Our sample period is thus framed by two crises, the Global Financial Crisis of 2008 and the COVID-19 Pandemic beginning in 2020.

The calibrated parameter values appear in Table 1. The discount parameter β follows the value used by most conventional models, in order to produce a steady-state quarterly interest rate near the lower bound (in this case, 50 basis points). The habit persistence parameter ϱ is consistent with most of the empirical estimations.[3] The depreciation rate, δ, is set at .022 for a quarterly rate. This implies a slightly higher rate of depreciation calibrated by Fujiwara et al. (2005), but his model was estimated well before the Tsunami in 2011. Similarly the coefficients for capital and for intermediate goods, in the production of traded and non-traded goods, α_x and α_h, are set at .3 to assure a constant labor share across all sectors of the economy.

For capital goods, we specify a higher value for the depreciation than the conventional value, and a relatively lower value for the adjustment cost parameter ϕ.[4] The parameter θ_1, set at a higher value than θ_2, indicates a higher intratemporal elasticity between consumption of home and foreign goods in the total consumption index than the elasticity of intratemporal substitution between consumption of domestically-produced traded and home goods in the domestic consumption index.

For investment, we assume an equal share of domestic and imported goods, with $\gamma_i = .5$. The elasticity parameter θ_i is set at 2.5, equal to the elasticity parameter for home and foreign goods in the consumption aggregators.

The ratios of consumption of foreign goods in the aggregate consumption, γ_1 and the share of export-goods consumption in the total domestic consumption basket, γ_2, are assumed to be 0.2 and 0.1 respectively, for an approximate characterization of the Japanese consumption pattern. In this model, the steady-state values are quite sensitive to the tax rates. The income and consumption tax rates, τ, τ_C, are set close to applicable average tax rates in Japan, at 0.2 on average for the income tax and 0.05 for consumption tax respectively. The parameters generate the steady-state consumption share in GDP to be 0.54, close to what we observe in our sample The ratio of government spending-to-GDP ratio of .16, is below the observed average ratio of the sample, as shown in Figure 5.[5] The domestic investment/gdp ratio implied by our parameters is

[3]According to Teo (2006), the estimated habit persistence parameter of the Taiwan Province of China is approximately 0.8. Smets and Wouters (2003a) and Smets and Wouters (2007) report estimates for habit persistence close to this range for the Euro Area and the USA.

[4]We choose these values to capture the structural changes taking place in Japan due to the aging population, such as replacement of housing structures which are no longer needed.

[5]In our sample, the government spending index is for government consumption.

.18, also close but above the observed ratio of .15.

Since the financial system is well established in Japan, we specify relatively low financial friction parameters. The parameters μ_i, $i = 1, ..., 3$, representing the borrowing needs of the export, home-goods and importing firms, were all set equal at a value of .5. Finally the banking reserve and lending cost parameters ϕ_M, ϕ_N, are set to replicate observed low spreads in the financial sector.

The financial sector parameters, in tandem with the household budget constraint, imply a deposit/gdp ratio of 7.65. These deposits are the only financial assets held by the households. The steady-state ratio is larger than the actual household real financial asset/gdp ratio of 3.08.

The adjustment cost parameters for investment and for the financial sector with respect to government bonds and foreign assets/debt, are set at .005, for investment and .05 for the financial assets. The latter are set at values to keep the model stationary. We set the share of imported investment goods to total investment goods, γ_i at .5, in order to reflect the growing importance of imported components from China, for Japanese-owned exporting firms based in Japan.[6]

[6]See the web site, http://web-japan.org/factsheet/en/pdf/e05_trade.pdf

Table 1. Calibration of Structural Parameters

Parameter	Symbol	Value	Source/Target
	Utility parameters and household budget constraint		
β	Discount	0.995	Low interest-rate economy
ρ	Habit persistence	0.8	Smets and Wouters (2003b)
γ_1	Foreign con/total con ratio	0.2	Standard
γ_2	Traded con/domestic C ratio	0.1	Match data
θ_1	Intratermporal sub.-total consumption goods	2.5	Standard value
θ_2	Intratermporal sub.-consumption goods	2.5	Standard value
η	CRRA Parameter	3.0	Generate precautionary saving
ϕ_C	Consumption coefficient in CES aggregator	0.95	Match correlation of G,C
κ	CES consumption parameter	0.1	Match correlation of G,C
γ_L	Disutility of Labor Coefficient	1	Standard value
ϖ	Frisch elasticity	1	Standard value
γ_M	Utility of Money Coefficient	1	Standard value
ϑ	Elasticity of deposits in utility	3	Link money utility with consumption
τ^w, τ^c	Labor and income tax rates	.2, .05	Match data
φ_M	Adjustment costs for deposits	.01	Ensure stability
	Banking sector parameters		
ϕ	Capital adj. cost	0.005	Standard value
δ	Depreciation rate (quarterly)	0.022	Hansen and Imrohoroglu (2013)
φ_g, φ_f	Adj. Cost: domestic and foreign bonds	[0.05, 0.05]	Ensure stability
μ_h, μ_x, μ_f	Lending/working capital parameters	[1,1,1]	Generate bank lending channel
ϕ_M, ϕ_N	Deposit and lending parameters	[0.1, 0.15]	Generate spreads with risk free rate
γ_i	Imported invest/total invest.	0.5	Increased trade in intermediate goods
θ_i	Intratemporal sub.-investment goods	2.5	Standard value
	Production Function Coefficients		
α_x	Capital coefficient in traded goods	0.30	Match labor income share
α_h	Intermediate goods coefficient in non-traded sector	0.30	Match labor income share
	Calvo Pricing		
ς, ς_w	Intratemporal substitution for pricing and wages	[6,6]	Standard value

Source: Author calculations.

3.2 Bayesian Estimates

For estimating the model, we used quarterly data. We use logarithmic first differences of data for GDP, consumption, investment, government spending, the real exchange rate, world demand, foreign commodity prices and foreign interest rates. There are either stochastic shocks: for total factor productivity, labor productivity, the quality of capital, the marginal utility of consumption, real government spending, as well as world demand, commodity prices and the foreign interest rate.

Admittedly our DSGE estimation is smaller in scope than many models, such as those of Smets and Wouters (2007). While the use of more observables gives greater detail and realism, the inclusion of more observables requires the incorporation of additional stochastic shocks. Chari et al. (2009) note that many of the stochastic shocks incorporated in DSGE models are not properly structural shocks, and thus may be misleading when such models are used for policy evaluation. For this reason, we prefer to have less rather than more observables and stochastic shocks.

Table 2 pictures the Bayesian estimates for the dynamic parameters as well as the standard deviations of the stochastic shocks. There is also a high degree of persistence in government spending, with a counter-cyclical component, rather than a pro-cyclical component, relating spending to lagged GDP. There are also high persistence effects for labor productivity and the quality of capital. There is higher inertia in the Calvo parameters of import-pricing and wages than for home-goods and for export-good pricing. The volatility estimates for productivity and quality of capital are higher than the estimated volatility for government spending.

Table 2. Bayesian Estimates

Parameter	Name	Dist.	Prior	Prior	Posterior	Inf	Sup
Coefficients	Name		Mean	Std Dev	Mean	.025	.975
ρ_g	Govt Spending	Beta	.5	.2	0.963	0.963	0.963
ρ_i	Cap Quality	Beta	.5	.2	0.978	0.978	0.979
ρ_z	TFP	Beta	.5	.2	0.927	0.927	0.927
ρ_{Rf}	Foreign Interest	Beta	.5	.2	0.248	0.248	0.248
ρ_X	World Demand	Beta	.5	.2	0.510	0.510	0.510
ρ_{Pf}	Import Markup	Beta	.5	.2	0.813	0.813	0.813
ρ_{PX}	Export Markup	Beta	.5	.2	0.503	0.503	0.813
ρ_c	Marginal Utility	Beta	.5	.2	0.918	0.918	0.918
ρ_R	Taylor Rule Lag	Beta	.5	.2	0.922	0.922	0.922
ρ_π	Taylor Inflation	Normal	1.5	.2	1.596	1.595	1.596
ρ_y	Taylor Output Gap	Beta	.5	.2	0.444	0.444	0.444
ξ_w	Calvo labor	Beta	.5	.2	0.522	0.522	0.522
ξ_h	Calvo home good	Beta	.5	.2	0.999	0.999	0.999
ξ_i	Calvo import	Beta	.5	.2	0.148	0.148	0.148
ξ_x	Calvo export	Beta	.5	.2	0.974	0.974	0.974
Volatility	Name	Dist	Mean	St. Dev.	Mean	Lower	Upper
σ_G	Govt Spending				0.0006	0.0006	0.0006
σ_i	Cap Quality	Inv Gamma	.05	4	3.4703	3.4652	3.4753
σ_{Px}	Export Markup	Inv Gamma	.05	4	0.3580	0.3547	0.3606
σ_z	TFP	Inv Gamma	.05	4	0.1658	0.1639	0.1679
σ_X	World Demand	Inv Gamma	.05	4	0.1379	0.1378	0.1379
σ_{Pf}	Import Markup	Inv Gamma	.05	4	1.4540	1.4522	1.4561
σ_c	Marginal Utility	Inv Gamma	.05	4	2.1784	2.1759	2.1810
σ_{Rf}	Foreign Interest	Inv Gamma	.05	4	6.0756	6.0657	6.0840

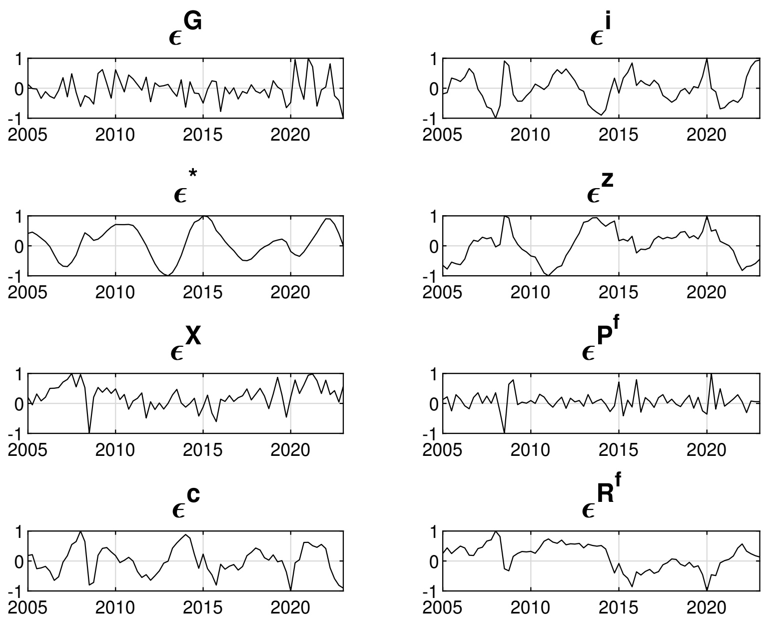
Source: Author calculations.

3.3 Smoothed Shocks

The normalized smoothed shocks generated by the Bayesian estimates appear in Figure 6. We picture the shocks for government spending, ϵ^G, quality of capital, ϵ^i, export-price markups, ϵ^*, total factor productivity, ϵ^z, world demand, ϵ^X, import-price markups, ϵ^{P^f}, marginal utility of consumption, ϵ^c, and foreign interest rates, ϵ^{R^f}. Not surprisingly, we see a large negative shock to productivity and the quality of capital at the time of the 2011 earthquake. We also see a positive shock to marginal utility, at the time of COVID-19.

Figure 6. Smoothed Shocks, 2005-2022

(Percent)



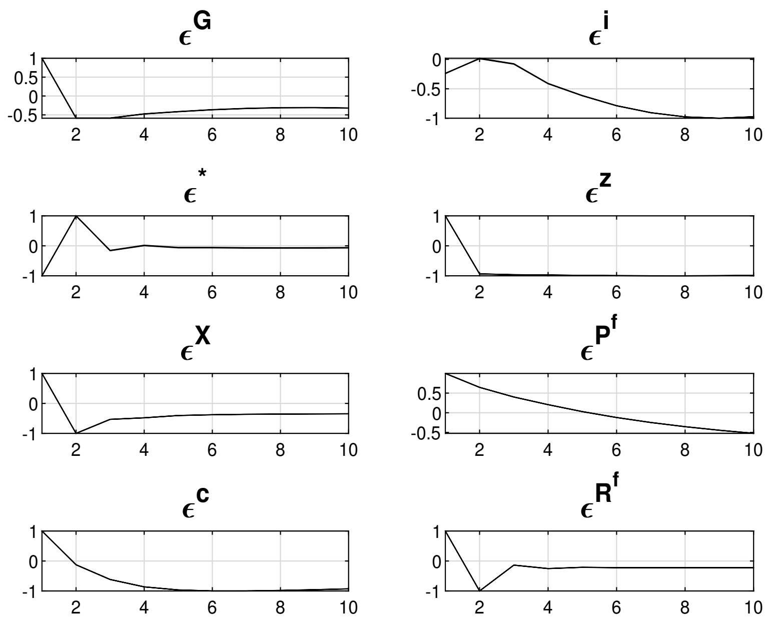
Source: Author calculations.

3.4 Impulse Response and Variance Decomposition Analysis

Figure 7 pictures the impulse response paths of output growth for the same shocks. This figure shows that shocks to government spending, productivity, world demand, and marginal utility effects have fast and positive effects on GDP. While a shock to the marginal utility of consumption reduces consumption in our CRRA framework, it stimulates investment through increased saving.

Figure 7. GDP Growth: Bayesian Impulse Response Paths

(Percentage deviation from steady state)

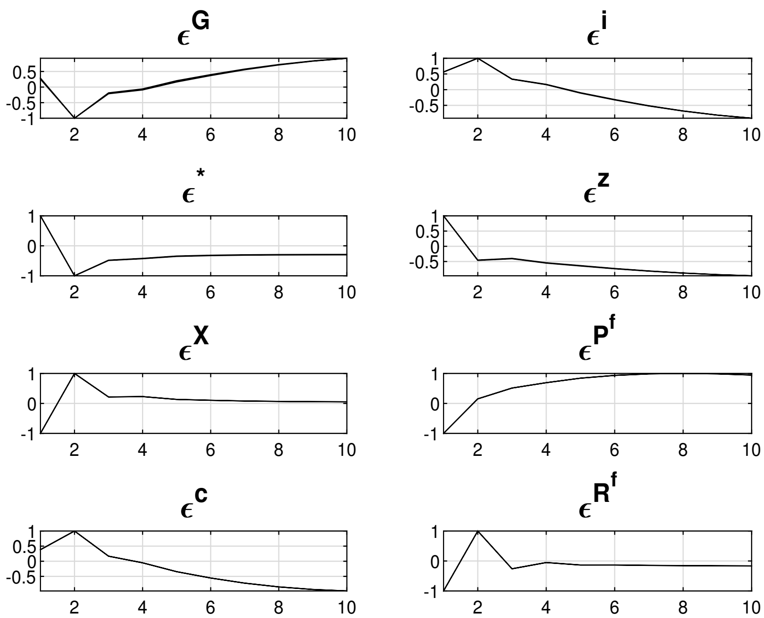


Source: Author calculations.

Figure 8 pictures the adjustment of the real exchange rate for each of the shocks. We see that productivity and capital quality, given by ϵ^z, ϵ^i, lead to an improvement improvement of the real exchange rate, while, as expected, foreign interest rates, given by ϵ^{Rf} and import prices, ϵ^{P^f}, lead to a fall in this rate.

Figure 8. Real Exchange Rate: Bayesian Impulse Response Paths

(Percentage deviation from steady state)

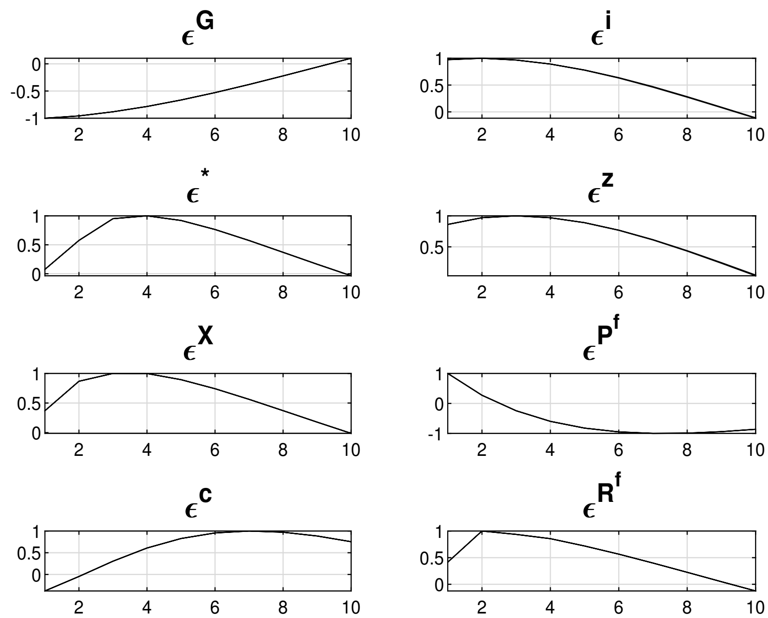


Source: Author calculations.

Figure 9 gives the impulse response of the fiscal balance, including debt service, in response to the same shocks. We see that the quality of capital, productivity and world demand improve the fiscal balance, as does a shock to the marginal utility of consumption, while an increase in government spending leads to a fall in the fiscal balance, as expected.

Figure 9. Fiscal Balance: Bayesian Impulse Response Paths

(Percentage deviation from steady state)



Source: Author calculations.

Table 3 gives the conditional variance decomposition for GDP growth at one, four, eight, twelve and sixteen quarters.

This table shows that government spending has little effect on output growth. Productivity explains more than 55 percent of the variance of output growth, followed by quality of capital and export prices, at the 16th quarter.

Table 3. Conditional Variance Decomposition of GDP Growth

(Unit)

	Quarter:				
Source:	1	4	8	12	16
σ_G	0.000	0.000	0.000	0.000	0.000
σ_i	0.018	0.055	0.189	0.293	0.324
σ_{P^x}	0.059	0.092	0.077	0.065	0.062
σ_z	0.852	0.763	0.659	0.578	0.554
σ_X	0.027	0.033	0.027	0.023	0.021
σ_{Pf}	0.000	0.000	0.000	0.000	0.000
σ_λ	0.010	0.009	0.009	0.008	0.007
σ_{Rf}	0.034	0.048	0.039	0.033	0.031

Source: Author calculations.

Table 4 gives the conditional variance decomposition of the real exchange rate. We see that world demand and the quality of capital are the key drivers of this variable.

Table 4. Conditional Variance Decomposition of the Real Exchange Rate

(Unit)

					Quarter:
Source:	1	4	8	12	16
σ_G	0.000	0.000	0.000	0.000	0.000
σ_i	0.007	0.020	0.022	0.034	0.046
σ_{P^x}	0.963	0.945	0.943	0.929	0.915
σ_z	0.017	0.013	0.013	0.015	0.017
σ_X	0.006	0.007	0.007	0.007	0.007
σ_{P^f}	0.000	0.000	0.000	0.000	0.000
σ_λ	0.000	0.000	0.000	0.000	0.000
σ_{R^f}	0.008	0.015	0.015	0.015	0.015

Source: Author calculations.

Table 5 gives information about the conditional variance of the fiscal balance. We see that capital quality shocks play a greater role at longer horizons, while in the shorter term, productivity and export-price shocks are the most important drivers.

Table 5. Conditional Variance Decomposition of the Fiscal Balance

(Unit)

					Quarter:
Source:	1	4	8	12	16
σ_G	0.000	0.000	0.000	0.000	0.000
σ_i	0.103	0.065	0.159	0.610	0.831
σ_{P^x}	0.163	0.304	0.302	0.144	0.061
σ_z	0.658	0.538	0.451	0.201	0.087
σ_X	0.023	0.031	0.028	0.013	0.006
σ_{P^f}	0.000	0.000	0.000	0.000	0.000
σ_λ	0.024	0.022	0.026	0.017	0.009
σ_{R^f}	0.030	0.040	0.032	0.015	0.006

Source: Author calculations.

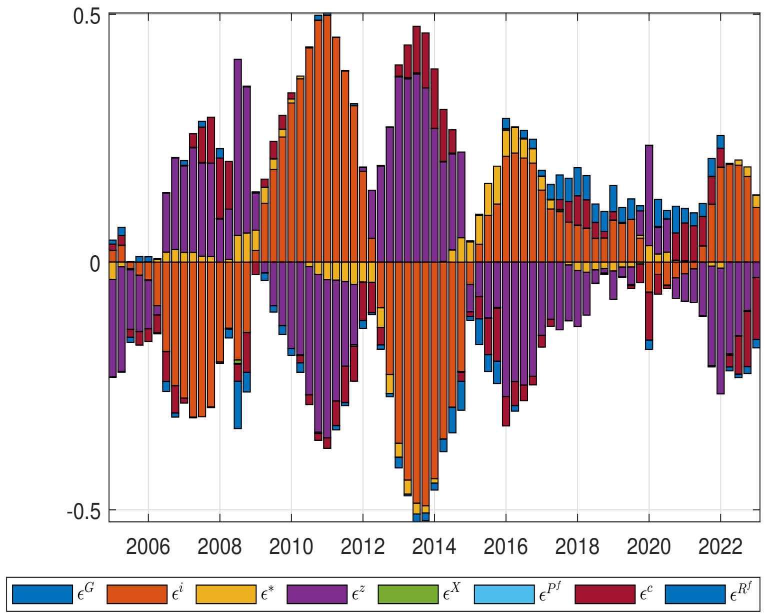
3.5 Historical Shock Decomposition

The next three figures picture the historical shock decomposition for GDP growth, the real exchange rate, and the fiscal balance. during the sample period. This gives us a picture of which types of shocks are more important at specific times in the sample.

Figure 10 gives us information on the contributions of the shocks to GDP growth over time.

Figure 10. Historical Shock Decomposition: Output Growth

(Unit contributions to quarterly change)

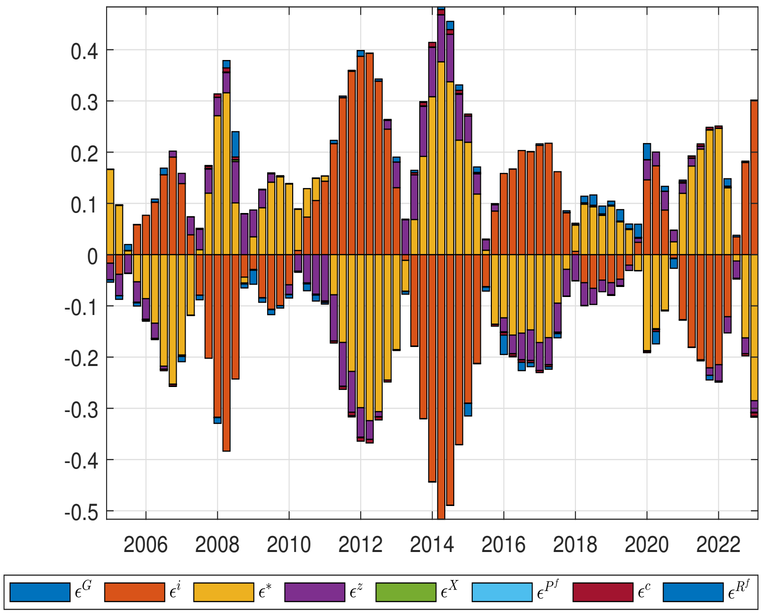


Source: Author calculations.

Figure 11 shows the historical shock decomposition for the real exchange rate.

Figure 11. Historical Shock Decomposition: Real Exchange Rate

(Unit contribution to quarterly change)

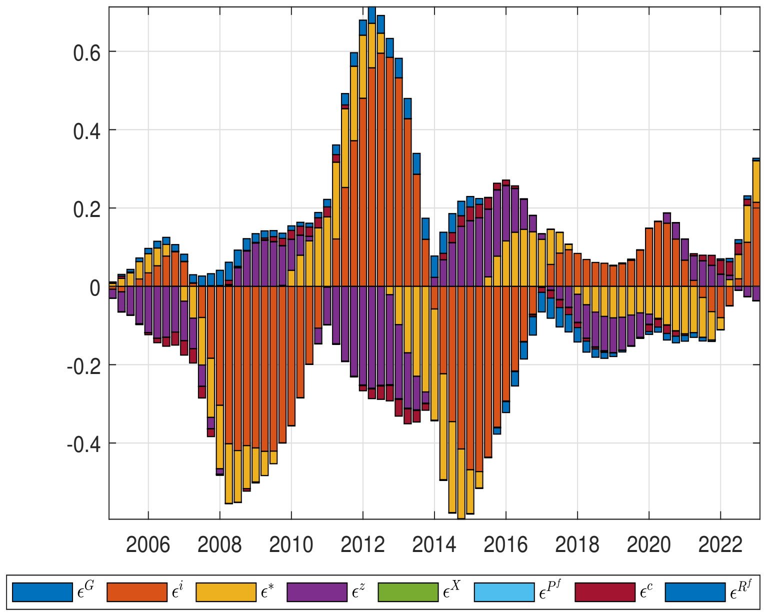


Source: Author calculations.

Figure 12 gives the corresponding information about the fiscal balances.

Figure 12. Historical Shock Decomposition: Fiscal Balance

(Unit contribution to quarterly change)



Source: Author calculations.

4 Simulation Results

4.1 The natural rate of interest and cost of distortions

To understand the usefulness of the estimated model, we first make use of the simulated model for understanding the costs of nominal price by comparing the stochastic discount rates under flexible wages and prices and the stochastic discount rate with price and wage rigidities. The simulation was for T=10,000. As noted above, the stochastic discount rate under a flexible wage and price system is one way to approximate the natural rate of interest. The stochastic discount rate is given in equation (20). While the mean and median values are equal for both, since the model is centered around the steady state, we see that there is higher volatility and skewness under Calvo pricing but higher kurtosis in a flexible price world.[7]

[7]See Obstfeld (2023) for a further discussion of the differences between the natural and neutral interest rates

Table 6. Distributional Parameters of the Stochastic Discount Rate

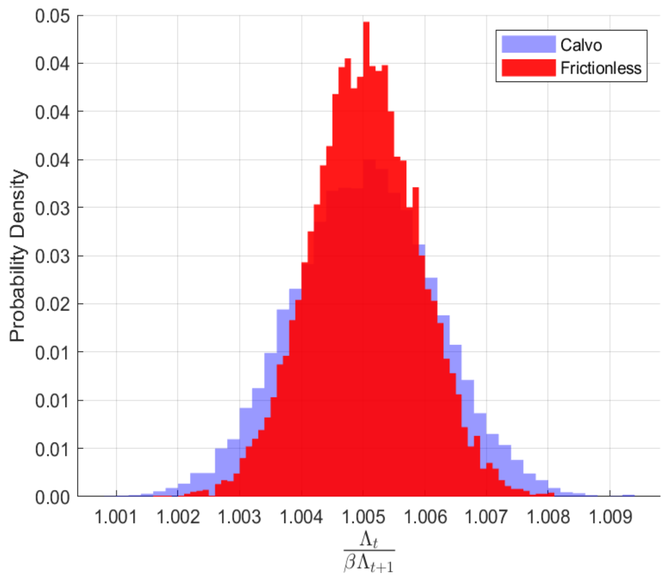
(Index)

	Mean	Median	Std Dev.	Skewness	Kurtosis
Frictionless	1.0050	1.0050	0.0009	0.0037	3.0403
Calvo	1.0050	1.0050	0.0012	0.0041	2.9293

Source: Author calculations.

Figure 13 pictures the distribution of the Stochastic Discount Rates under the two regimes.

Figure 13. Distribution of the Stochastic Discount Rate

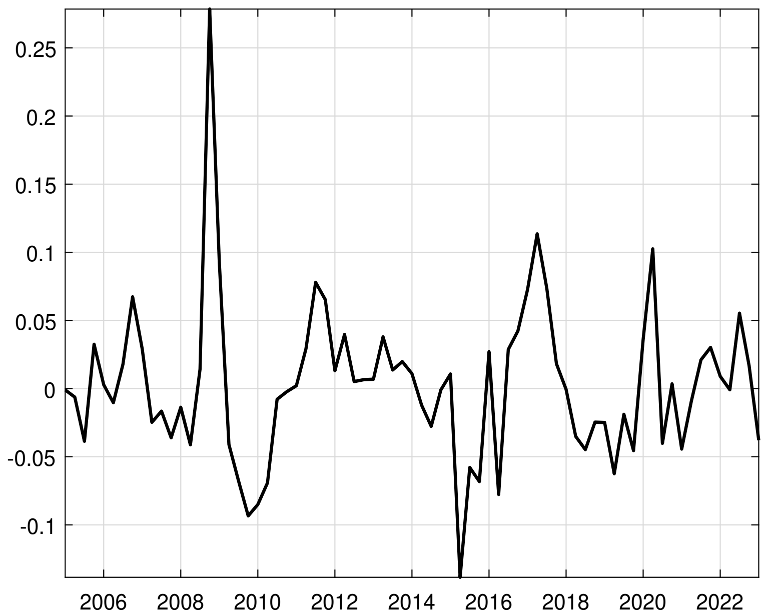


Source: Author calculations.

Figure 14 pictures the differences between the two discount rates with the realized or smoothed shocks for the sample period. We see that for the realized shocks, the natural rate was much higher than the rate under the actual Calvo regime.

We see that at the time of the GFC in 2008, the Fukushima earthquake in 2011, and COVID-19 in 2020, that the natural rate was 5 to 25 percent higher than the rates implied by the estimated model. This means, of course, the the interest rates were artificially low at the time of these crises.

Figure 14. Difference Between Stochastic Discount Rates under Frictionless and Calvo Regimes



Source: Author calculations.

The question of course, is if the policy instruments can move the outcomes of the economy closer to that of a frictionless flexible price world. Erceg et al. (2000) noted that optimal monetary price cannot guide the economy back to a Pareto optimal world (with full price/wage flexibility) with only one instrument. At best, optimal monetary policy, in the form of a Taylor rule, can approximate the outcome of a second-best outcome under a Ramsey rule. Of course a Ramsey rule is complex, requiring full knowledge of all of the state variables and shocks. These authors show that a Taylor rule based on lagged wage inflation, rather than on lagged price inflation, closely matches a Ramsey outcome.

These authors worked in a closed-economy framework. We are working in an open large-economy framework, with four sources of price frictions: for home goods, imported good, export goods and wages. We examine two instruments: a tax-transfer rule for expenditures and a quantitative easing policy, in which the central bank purchases government debt from the banking sector. We call the former a quasi-monetary fiscal policy and the latter a quasi-fiscal monetary policy. The setup for the optimal rules for these policy are given in equations (72) and (77). We estimate the optimal parameters only in the Calvo world of price and wage stickiness.

4.2 Optimal Taylor Rule and QE/Tax-Transfer Parameters

We calculate optimal simple rules for the above model for the tax rate, the QE policy and the Taylor rule. The parameters are based on Linear Quadratic optimization of welfare, given the estimated model structure. Schmitt-Grohe and Uribe (2007) advocate the use of such rules over more complex Ramsey (1927) rules, since the variables in these rules are readily observable. The parameter values are constrained to be positive and for the Taylor rule, to ensure determinacy, the inflation coefficient, ρ_π is constrained to be greater than one.

Table 7 gives the parameter values for the optimal simple rules. This table shows a number of interesting results. First we see that the optimal rules for both the consumption tax rate and quantitative easing call for a mild smoothing coefficient, with lower values than that of smoothing coefficient for the Taylor rule. Secondly, we see that the tax-rate and the QE rules put slightly more weight on the debt/GDP ratio than the deficit/GDP ratio.

Table 7. Taylor Rule Parameters and Optimal Rules for Taxes and Quantitative Easing

Parameter	Taylor Rule	Tax-Rate Rule	QE Rule	Tax-Rate/QE Rule
ρ_r	.864	–	–	–
ρ_π	1.0102	–	–	–
ρ_y	.441	–	–	–
ρ_{τ^c}	–	0.495	–	0.495
$\rho_{\tau^c B}$	–	2.955	–	2.955
$\rho_{\tau^c D}$	–	2.044	–	2.044
ρ_ψ	–	–	.476	.476
$\rho_{\psi B}$	–	–	3.020	3.020
$\rho_{\psi D}$	–	–	2.044	2.044

Source: Author calculations.

4.2.1 Dark Corner Dynamics: Calvo Pricing and the Zero Lower Bound

To evaluate thee effectiveness of the optimal Taylor rule without a zero lower bound, as well as the optimal tax/transfer and QE rules, we first examine the behavior of GDP, Consumption, the Real Wage, Tobin's Q, the Real Exchange Rate, the Stochastic Discount Rate, and Domestic Debt and Foreign Asset to GDP ratios under "dark corner" dynamics.

Following the methodology of Mendoza (2010) we use a crisis-event analysis, since we are interested in the dynamic behavior of key variables, pre-, during and post- crisis events, where the crisis events have been generated by a sequence of adverse shocks in the home country. Following Kaminsky et al. (2005), we are interested in the adjustment process not just when it rains but when it pours.

The key advantage of this approach, as noted by Mendoza (2010), is that welfare comparisons of alternative policy regimes are based on the stochastic mean value of welfare components, which often show little differences. What is more important is to compare how alternative regimes work during crisis events.

Following this approach, we first examine the adjustment for five quarter before and five quarters after the worst crisis events in the long simulation, when GDP is at its absolute minimum value. We examine the median values of key variables for all of the instances when GDP is two standard deviations below its stochastic mean.

We take 100,000 quarterly observations generated by our stochastic simulations and, emulating the empirical literature on crisis events or sudden stops, identify particular sudden stop episodes. We then go backward and forward by five quarters and obtain the median values of key variables leading up to and following the crisis event. To understand the relative change in each variable, we normalize the value of each variable leading up to the crisis event or sudden stop at unity.

As noted by Mendoza (2010), looking at welfare measures over the full period of simulation, based on averages, will not help us see how these rules perform when things get bad, as they do, for all economies, some of the time.

The time scale is in reference to the "crisis event" or GDP bottoming out at time t=0. Figure 15 shows that the median drop in GDP at the crisis event t=0 is almost 40%. Due to both forward-looking expectations and habit persistence, consumption starts to fall before the crisis event, and continues to drop. Due to Calvo wage/price frictions, the drop in the real wage is slow, and actually starts to rise after the crisis. Tobin's Q starts to fall in anticipation of the crisis while the real exchange rate starts to depreciate slightly before the crisis. As expected, the stochastic discount rate rises due to the high risk in the economy.

Not surprisingly, the domestic debt/GDP ratio rises while the foreign asset/GDP ratio falls.

The issue for us is how would a system of no zero lower bound on interest, coupled with an optimal Taylor, or a tax/transfer rule for consumption, or a QE rule, perform when we hit dark corners with Calvo sticky prices?

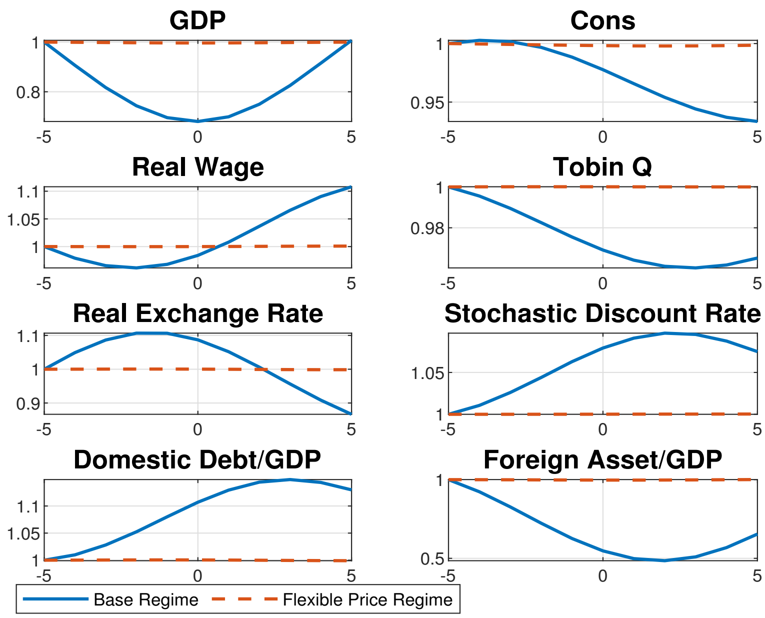
In Figure 15 we picture the "Dark Corner" dynamics of GDP, Consumption, the real wage, Tobin's Q, the real exchange rate, the stochastic discount rate, and the domestic debt and foreign asset to GDP ratios.

The smooth curve represents dynamics under Calvo pricing and a zero lower bound, while the red broken line shows that adjustment under a system of perfect price flexibility and a Taylor rule with no zero lower bound

We see that in a perfectly flexible world, both the interest rates and prices adjust to the negative shocks, so that there is little movement in the real variables. For the same set of shocks, the sticky price and ZLB world show that the adverse shocks generate a large fall in GDP, a more protracted but smaller fall in consumption (due to habit persistence), a fall and then a rise in the real wage, coupled with a fall in Tobin's Q. The real exchange rate depreciates at the time of the crisis event while the discount rate increases as the economy becomes a more risk environment. Finally we see that the domestic debt rises and foreign assets fall, relative to GDP.

Figure 15. Dark Corner Dynamics with Calvo Pricing and the Zero Lower Bound

(T-5 quarters = 1)



Source: Author calculations.

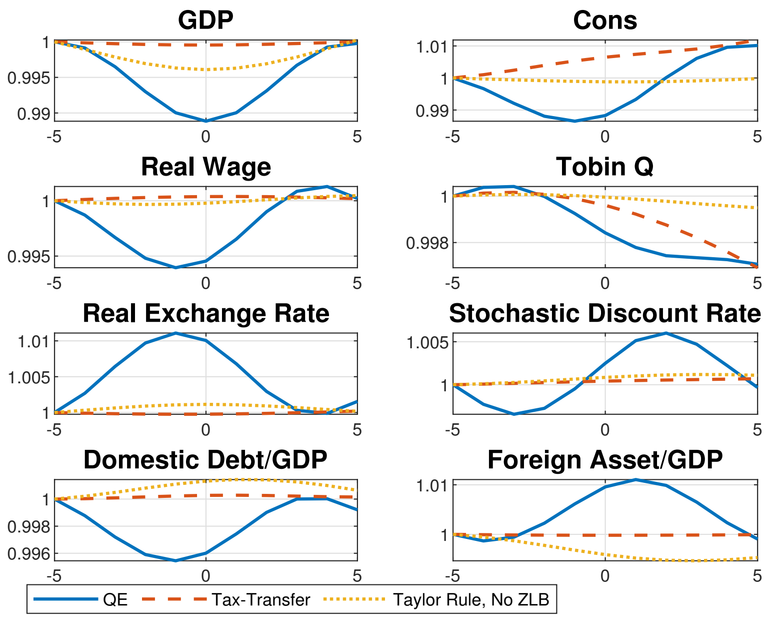
4.2.2 Optimal Policy Rules

Figure 16 pictures the normalized pre- and post-crisis dynamics under the optimal policy regimes. The solid curve shows adjustment under the QE policy rule, the broken curve the paths generated by the Tax-Transfer rule and the dotted curve the dynamics associated by an optimal Taylor rule with no zero lower bound.

We see that the three optimal rules are quite effective for stabilizing the key macroeconomic variables during dark-corner periods. We should remember that the optimal rules represent policy instruments are examples of what these rules can do when there are no limits on the tentative values of interest rates and when there are no limits on the extent of tax/transfer payments or monetary expansion coming from central bank asset purchases. In practice, the implementation of such rules is much more limited in scope. But Figure 16 tells us that the use of such rules, even in a more limited way, has desirable effects on the adjustment of key macroeconomic variables in "dark corner" periods.

Figure 16. Optimal Policy Rules in Dark Corners

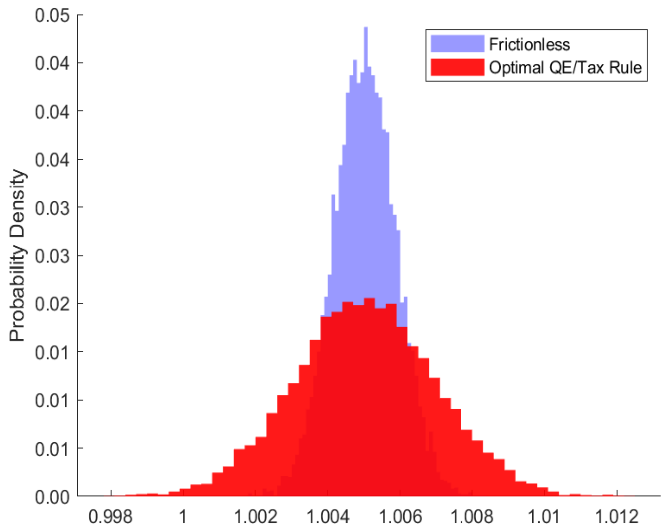
(T-5 quarters = 1)



Source: Author calculations.

To assess the role of the natural rate as an indicator of the performance of the optimal rules, we compare the histogram of the Stochastic Discount Rate in a frictionless world with that of the same rate in a world governed by the optimal mix of QE and Tax rules, set by the parameters in the last column of Table 7. The histograms appear in Figure 17. We see that even under the optimal rules for both tax rates and quantitative easing, the relevant discount rate has a different distribution than that of the natural rate or stochastic discount rate in a frictionless world with no zero bound. In particular, there is greater variance and lower kurtosis of the discount rate under the optimal rules than in a frictionless world. Again this should not be surprising. The world is always more volatile and risky under sticky prices and wages, and even optimal policies do not completely eliminate this risk, relative to a frictionless world.

Figure 17. The Natural Rate as Benchmark



$\frac{\Lambda_t}{\beta\Lambda_{t+1}}$

Source: Author calculations.

5 Conclusion

This paper developed and estimated a model of a large open economy model, intended to capture key characteristics of Japan. The model simulations for crisis events was able to replicate lost-decade phenomena, when the zero lower bound is a binding constraint on monetary policy.

We then compared the adjustment of key variables in a Taylor rule setting with no zero lower bound (with negative interest rates), as well as optimal rules for the consumption tax rate and for quantitative easing, in the form of purchase of government bonds by the central bank from the private banking system.

The QE policy has the advantage for reducing debt while the Tax rule has faster effects for consumption stabilization. While these rules compare favorably with an optimal Taylor rule with no zero lower bound, we also note that even the optimal rules do not bring the relevant discount rate to the natural rate of interest prevailing in a frictionless world.

While our results show that QE is effective in times of crisis, like the fiscal instruments for tax rates or negative interest rates, we do not explore the dangers to QE policies in normal times (such as inflation and loss of credibility). We caution that the QE policy is an emergency policy, to be used in times of prolonged crisis.

A useful extension would be an examination of a process of "tapering" from a QE rule, over a given horizon. Should the tapering process be gradual or abrupt? This is an open question for which models of this type can be put to work.

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Address: 10 Shenton Way, #15-08
MAS Building, Singapore 079117
Website: www.amro-asia.org
Tel: +65 6323 9844
Email: enquiry@amro-asia.org
LinkedIn | Twitter | Facebook | YouTube